Solution of Ill-Posed Inverse Problem of Distributed Generation Using Complex-Valued Network Inversion

Takehiko Ogawa, Kyosuke Nakamura, and Hajime Kanada*

***Abstract*— Network inversion has been studied as a neural network based solution of inverse problems. Complex-valued network inversion has been proposed as the extension of this inversion to the complex domain. Further, regularization is considered for solving ill-posed inverse problems. On the other hand, the estimation of the parameters of a distributed generation from observed data is a complex-valued inverse problem with ill-posedness. In this paper, we propose the application of a complex-valued network inversion with regularization to the inverse estimation of a distributed generation.**

***Index Terms*— Complex-valued neural networks, ill-posed inverse problems, distributed generation.**

I. Introduction

The inverse problem of estimating causes from observed results has been studied in various engineering fields [1]. Network inversion has been studied as a neural network based solution of inverse problems [2]. While the original network inversion method has been applied to usual real-valued neural networks, the complex-valued network inversion method has been proposed to solve inverse problems on a complex-valued neural network [3]. Inverse problems are generally ill-posed, which implies that the existence, uniqueness, and stability of their solution are not guaranteed. Regularization imposes specific conditions on an ill-posed inverse problem to convert it into a well-posed problem [4]. In the case of complex-valued network inversion, regularization has been examined on simple inverse problems [5].

Distributed generation is an important technique that involves natural power sources or fuel cells. In distributed generation, it is important to estimate the parameters of several power supplies to control them. The problem of estimating the parameters from a large amount of observed data is an ill-posed inverse problem featuring complex numbers [6]. In this paper, we propose the application of complex-valued network inversion with regularization to the ill-posed inverse estimation of the distributed generation. Specifically, we consider the problem of estimating the voltage of the power supply from the observed output voltage and current of a circuit with two power supplies. The problem of ill-posedness concerning the uniqueness of the solution appears in the inverse estimation of the distributed generation. To show the effect of the proposed method, we carry out a simulation using the complex-valued network inversion with regularization.

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II. Inverse Problems and Network Inversion

The inverse problem refers to the problem of estimating the cause from the observed phenomenon. The cause is estimated from the fixed model and given result in the inverse problem. The solution of inverse problems is important in various engineering fields [1].

Network inversion is a method for solving inverse problems using multilayer neural networks. In this method, we estimate the corresponding input from a given output using a trained network. The network is typically trained using the error back-propagation method. In the trained network, we provide the observed output with fixed trained weights. The input can then be updated according to the calculated input correction signal. Essentially, the input is estimated from the output using an iterative update of the input based on the output error, as shown in Fig. 1. By doing so, the inverse problem of estimating input from the given output is solved using the multilayer neural network.

To solve the inverse problem using network inversion, the network is used in two phases: forward training and inverse estimating. In the training phase, the weights w are updated using

$$w(n+1) = w(n) - \varepsilon_t \frac{\partial E}{\partial w} \tag{1}$$

where x, y, E, and ε_t are the training input, training output, output error, and training gain, respectively. It is assumed that the output error is caused by the inaccurate adjustments of the weights in the training phase. This is the procedure employed in the usual error back-propagation method. In the inverse estimation phase, the input x is updated using

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$$x(n+1) = x(n) - \varepsilon_e \frac{\partial E}{\partial x} \tag{2}$$

where x, y, E, and ε_e are the random input, provided output, output error, and input update gain, respectively. It is assumed that the output error is caused by the maladjustments of the input during inverse estimation. By repeating this updating procedure, the input is estimated from the provided output [2].

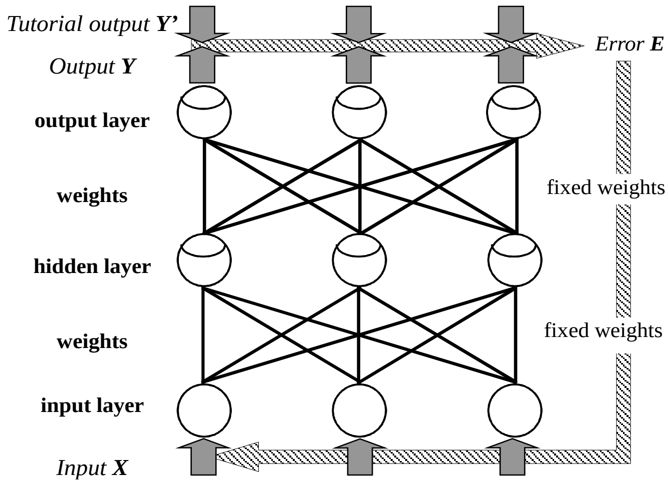

Fig. 1. Iterative update of input using network inversion.

A. Complex-Valued Network Inversion

The original network inversion procedure is a solution for inverse problems using a typical real-valued multilayer neural network. Recently, an extension of the multilayer neural network to the complex domain has been studied [7]. In addition, the complex-valued network inversion has been proposed to solve the general inverse problem involving complex values.

Complex-valued network inversion involves the use of a complex-valued multilayer neural network. It is an extension of the usual network inversion to the complex domain. The complex input is estimated from the provided complex output using a trained network.

In the training phase, the complex weight $w=w_R+iw_I$ is updated using

$$\begin{aligned} w_R(n+1) &= w_R(n) - \varepsilon_t \left(\frac{\partial E_R}{\partial w_R} + \frac{\partial E_I}{\partial w_R} \right) \\ w_I(n+1) &= w_I(n) - \varepsilon_t \left(\frac{\partial E_I}{\partial w_I} - \frac{\partial E_R}{\partial w_I} \right) \end{aligned} \tag{3}$$

where $x = x_R + ix_I$, $y = y_R + iy_I$, $E = E_R + iE_I$, and ε_t are the complex training input, training output, output error, and training gain, respectively. By repeating this updating procedure, a forward relation is obtained. This is the usual complex error back-propagation method. In the inverse estimation phase, the complex input $x = x_R + ix_I$ is updated using

$$\begin{aligned} x_R(n+1) &= x_R(n) - \varepsilon_e \left(\frac{\partial E_R}{\partial x_R} + \frac{\partial E_I}{\partial x_R} \right) \\ x_I(n+1) &= x_I(n) - \varepsilon_e \left(\frac{\partial E_I}{\partial x_I} - \frac{\partial E_R}{\partial x_I} \right) \end{aligned} \tag{4}$$

where $x = x_R + ix_I$, $y = y_R + iy_I$, $E = E_R + iE_I$, and ε_e are the complex random input, provided output, output error, and input update gain, respectively. By repeating this procedure, the complex input approaches the corresponding value of the provided output. When the error becomes sufficiently small, the input correction is completed and the obtained complex input becomes a solution. As a result, the complex input can be inversely estimated from the complex output using the trained complex weights [3, 5].

B. Ill-Posedness and Regularization

In the inverse problem, the existence, uniqueness, and stability of solution are not guaranteed. The problem that does not satisfy these three conditions is referred to as being ill-posed. The ill-posedness of a problem is an important issue in complex-valued network inversion.

In this paper, we consider the regularization for complex-valued network inversion. The method is based on Tikhonov regularization [4]. We use a regularization functional that is minimized in accordance with the output error in the inverse estimation phase in order to impose the constraint condition. For real-valued network inversion, we define the energy function E with the regularization functional $K(x)$ as

$$E = \frac{1}{2} \sum_r (y'_r - y_r)^2 + \lambda \sum_k K(x_k) \tag{5}$$

where $y'_r = y'_{rR} + iy'_{rI}$, $y_r = y_{rR} + iy_{rI}$, and $x_k = x_{kR} + ix_{kI}$ are the r-th tutorial output, r-th network output, and k-th input, respectively. The first and second terms represent the output error and regularization functional, respectively. The parameter λ is the regularization coefficient.

Next, we extend the regularization to complex-valued network inversion as

$$\begin{aligned} E_R &= \frac{1}{2} \sum_r (y'_{rR} - y_{rR})^2 + \lambda \sum_k K(x_{kR}) \\ E_I &= \frac{1}{2} \sum_r (y'_{rI} - y_{rI})^2 + \lambda \sum_k K(x_{kI}) \end{aligned} \tag{6}$$

where $y'_r = y'_{rR} + iy'_{rI}$, $y_r = y_{rR} + iy_{rI}$, and $x_k = x_{kR} + ix_{kI}$ are the r-th complex provided output, r-th complex network output, and k-th complex input, respectively [5].

Dynamic regularization is also applicable to the complex-valued network inversion. We consider the decay of the regularization coefficient $\lambda(t)$ from $\lambda(0)$ to zero with the epoch number t.

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III. Inverse Estimation of Distributed Generation

One example of distributed generation is the small-scale power supply allocated near a consumer. The distributed generation system can be expected to decrease the environmental load and power cost and to increase the stability of the power supplies. However, it is difficult to maintain the quality of the electric power and to detect the incidents in case of numerous power supplies.

In this study, we consider the inverse problem of estimating the voltage of the power supplies from the observed voltage and current data. We compose an inverse estimation problem using a simple AC electric circuit model that consists of several power supplies. In this circuit, we assume that the impedance element is driven by many AC power supplies. Concretely, we use a model that includes two power supplies with complex impedance and output complex impedance, as shown in Fig. 2. The circuit parameters are as follows. The values of each impedance are Z1 = Z2 = Z3 = 1 + i [Ω]. The parameters in the power supplies VG1 and VG2 are varied to obtain the training data. In addition, we prepare the test data by changing the output voltage and current. The amplitudes of VG1 and VG2 are varied from 30 to 180 [V] in steps of 30 [V]. Their phase is also varied from -150° to 180° in 30° steps.

The problem of ill-posedness, which concerns the uniqueness of the solution, arises in the inverse estimation of the power supplies VG1 and VG2. This problem is ill-posed because we cannot distinguish between VG1 and VG2 and hence treat them as the same performances. In this study, we examine the ill-posed inverse estimation of the parameters of the power supply using complex-valued network inversion under conditioning using regularization. We use the following regularization term

$$K(x_{kR}) = \frac{1}{2}\sum_k \left(x'_{kR} - x_{kR}\right)^2$$
$$K(x_{kI}) = \frac{1}{2}\sum_k \left(x'_{kI} - x_{kI}\right)^2 \qquad (7)$$

where k is the number of neurons that we impose on a specific condition . Here, we provide the correct value of VG1 or VG2 as $x_k' = x_{kR}' + ix_{kI}'$. This implies that the restraint condition that VG1 or VG2 is correct is considered to be the regularization. The input of the complex-valued network inversion with regularization is also updated based on equation (4). The input has to be updated in the real and imaginary parts of the complex-valued network inversion. The ill-posedness in the complex domain can be reduced by iterative correction of the input.

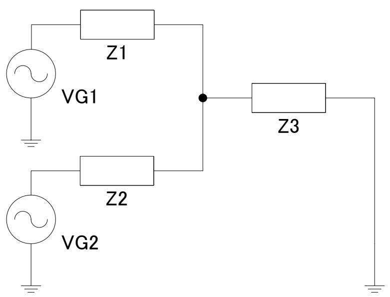

Fig. 2. Circuit model of distributed generation for simulation.

IV. Simulation

We carry out the inverse estimation of the power supply parameters of the distributed generation circuit shown in Fig. 4. We use a complex-valued neural network with two input neurons and two output neurons, which correspond to the complex voltages of the two sources and the measured complex current and voltage, respectively. These values are normalized by each maximum and minimum value and used as an input and output value for the network. The network architecture and network parameters are shown in Fig. 3 and Table 1, respectively.

To confirm the regularization, we perform the following simulation. First, we examine the inverse estimation without the regularization. Next, we perform regularization for VG1 and VG2. We carry out each simulation five times and show the plots of the five estimated results.

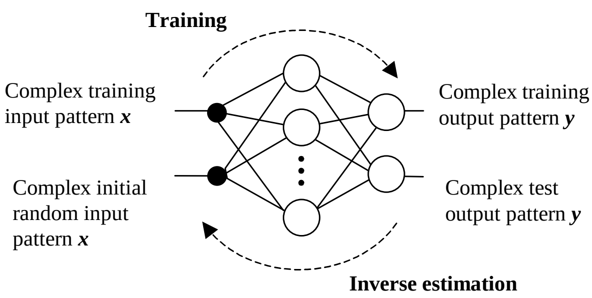

Fig. 3. The network architecture used in simulation

Table 1 Network Parameters

Number of input neurons	2
Number of hidden neurons	10
Number of output neurons	2
Training rate ε_t	0.0001
Input correcting rate ε_e	0.0001
Max. number of training epochs	10000
Max. number of estimating epochs	10000
Training error to be attained	0.0001
Estimation error to be attained	0.0001

A. Results

The inversely estimated inputs without regularization are shown in Fig. 4. Fig. 4(a) and 4(b) shows the estimated results of VG1 and VG2, respectively. We found that both the estimated inputs are not the correct voltages of VG1 and VG2. This is because the solution cannot be computed in the correct manner because of the ill-posedness of the problem with respect to the uniqueness of the solution.

The estimated inputs with regularization are shown in Figs. 5 and 6. Fig. 5(a) and 5(b) shows the estimated results of VG1 and VG2, respectively, when the limitation is imposed to VG2 by the regularization. Similarly, Fig. 6(a) and 6(b) shows the estimated results of VG2 and VG2, respectively, when the limitation is imposed on VG1. The results showed that the input correction is sufficiently suitable and the input is

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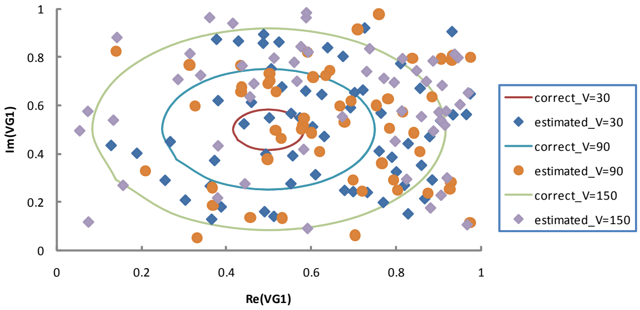
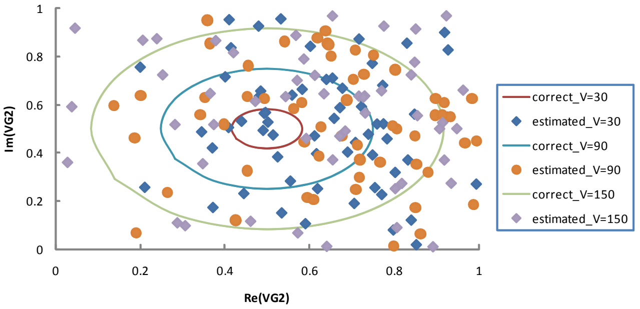
correctly estimated when the limitation is imposed on one of the inputs. Therefore, the problem of ill-posedness is not the initial-value problem, and we confirmed the effectiveness of the regularization method.

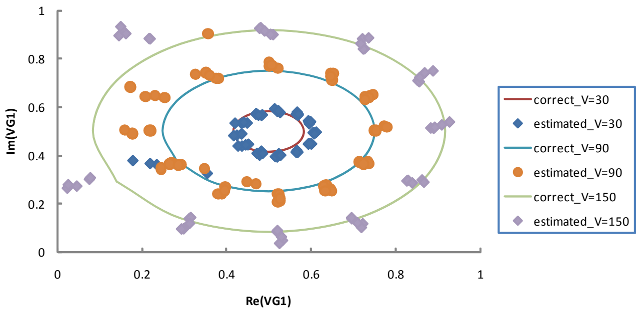


(a)

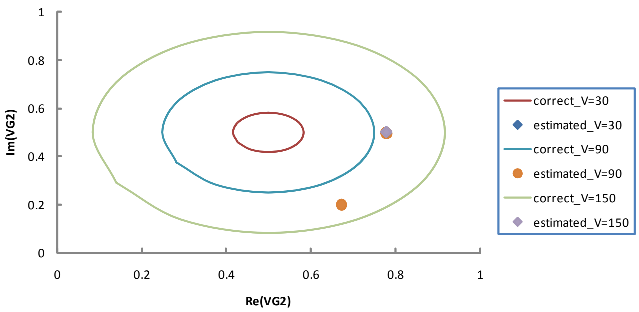


(b)

Fig. 4 Estimated results of the power supplies (a) VG1 and (b) VG2 without regularization.

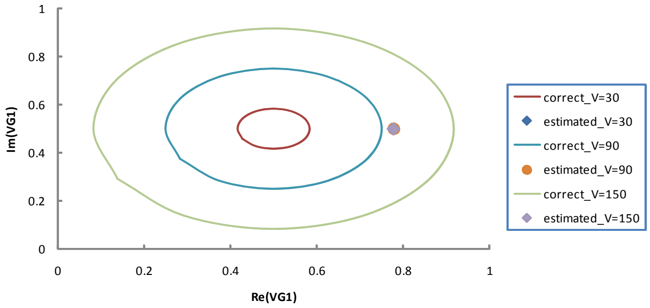


(a)

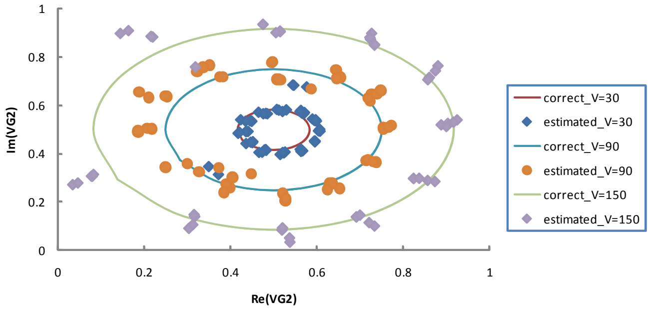


(b)

Fig. 5 Estimated results of the power supplies (a) VG1 and (b) VG2 with regularization for VG2.



(a)



(b)

Fig. 6 Estimated results of the power supplies (a) VG1 and (b) VG2 with regularization for VG1.

V. Conclusion

In this study, we proposed the application of a complex-valued network inversion to the ill-posed inverse estimation of the distributed generation. We carried out the simulation of the inverse estimation of the voltage source using a simple distributed generation model. To investigate the effect of ill-posedness, we examined the complex-valued network inversion with regularization. Consequently, we confirmed that the complex-valued network inversion method with regularization was effective in solving the ill-posed inverse estimation problem of distributed generation. In future studies, we will attempt to increase the number of voltage sources and improve their estimation accuracy.

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