

Multiple Particle Swarm Optimizers with Inertia Weight with Diverive Curiosity

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Abstract — In this paper we propose a newly multiple particle swarm optimizers with inertia weight with diverive curiosity (MPSOIW $_{\alpha}$ /DC) for improving the search performance and intelligent processing of a plain MPSOIW. It has the following outstanding features: (1) Decentralization in multi-swarm exploration with hybrid search (MPSOIW $_{\alpha}$), (2) Concentration in evaluation and behavior control with diverive curiosity (DC), and (3) Their effective combination. For inspecting the effectiveness of the proposal, computer experiments on a suite of 5-dimensional benchmark problems are carried out. We examine its intrinsic characteristics, and compare the search ability with other methods. The obtained results indicate that the search performance of the MPSOIW $_{\alpha}$ /DC is superior to the PSOIW/DC, EPSOIW, PSOIW, OPSO, and RGA/E for the given problems.

Keywords: cooperative particle swarm optimization, curiosity, hybrid search, swarm intelligence

1 Introduction

In recent years, a lot of studies and investigations on cooperative PSO in relation to symbiosis, group behavior, and sensational synergy are in the researchers' spotlight. Various kinds of methods such as hybrid PSO, multi-layer PSO, multiple PSO with decision-making strategy etc. were published [5, 11, 16] for attaining high-performance. In contrast to those methods operating a singular particle swarm, many attempts, plans, and strategies can be perfected, which mainly focus on the information propagation and intelligent processing within the whole multi-swarm.

Needless to say, the approach of group searching, parallel and intelligent processing has become one of extremely important ways to treat with different optimization problems. For improving the search performance of a plain multiple particle swarm optimizers with inertia weight (MPSOIW), in this paper we propose a new method of cooperative PSO – multiple particle swarm optimizers with inertia weight with diverive curiosity (MPSOIW $_{\alpha}$ /DC).

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In comparison with the plain MPSOIW, the proposed method has the following outstanding characters: (1) Decentralization in multi-swarm exploration with hybrid search (MPSOIW $_{\alpha}$), (2) Concentration in evaluation and behavior control with diverive curiosity (DC), and (3) Their effective combination. Based on the manner of comprehensively managing the trade-off between exploitation and exploration in the multi-swarm's heuristics and enforcement of group decision-making, the proposed MPSOIW $_{\alpha}$ /DC could be expected to greatly improve the search performance of the plain MPSOIW.

The MPSOIW $_{\alpha}$ /DC is an analogue of approach of multiple particle swarm optimization with diverive curiosity [16], which has been successfully applied to the plain multiple particle swarm optimizers (MPSO) and multiple canonical particle swarm optimizers (MCPSO) [18]. Nevertheless, the creation and actualization of the proposed method are not only to enhance the search ability and efficiency of the plain MPSOIW, but also to expand the applied range of cooperative PSO. This is just our motivation, and study purpose further to develop the approach of the curiosity-driven multi-swarm.

2 Basic Algorithms

For convenience to the following description, let the search space be N -dimensional, $\Omega \in \mathbb{R}^N$, the number of particles in a swarm be P , the position of the i -th particle be $\vec{x}^i = (x_1^i, x_2^i, \dots, x_N^i)^T$, and its velocity be $\vec{v}^i = (v_1^i, v_2^i, \dots, v_N^i)^T$, respectively.

The PSO: In the beginning of the PSO [3, 7] search, the particle's position and velocity are generated in random, then they are updated by

$$\begin{cases} \vec{x}_{k+1}^i = \vec{x}_k^i + \vec{v}_{k+1}^i \\ \vec{v}_{k+1}^i = c_0 \vec{v}_k^i + c_1 \vec{r}_1 \otimes (\vec{p}_k^i - \vec{x}_k^i) + c_2 \vec{r}_2 \otimes (\vec{q}_k - \vec{x}_k^i) \end{cases} \quad (1)$$

where c_0 is an inertial coefficient, c_1 is a coefficient for individual confidence, c_2 is a coefficient for swarm confidence. $\vec{r}_1, \vec{r}_2 \in \mathbb{R}^N$ are two random vectors in which each element is uniformly distributed over $[0, 1]$, and \otimes is an element-wise operator for vector multiplication. $\vec{p}_k^i (= \arg \max_{j=1, \dots, k} \{g(\vec{x}_j^i)\})$, where $g(\cdot)$ is the criterion value of the i -th particle at time-step k , is the local best position of the i -th particle up to now, and $\vec{q}_k (= \arg \max_{i=1, 2, \dots}$

$\{g(\vec{p}_k^i)\}$ is the global best position found by the whole particle swarm.

The PSOIW: For improving the convergence of the PSO, Shi et al. modified the update rule of particle's velocity by constant reduction of the inertia weight over time-step [4, 12] as follows.

$$\vec{v}_{k+1}^i = w(k)\vec{v}_k^i + w_1\vec{r}_1 \otimes (\vec{p}_k^i - \vec{x}_k^i) + w_2\vec{r}_2 \otimes (\vec{q}_k - \vec{x}_k^i) \quad (2)$$

where $w(k)$ is a variable inertia weight which is linearly reduced from a starting value, w_s , to a terminal value, w_e , with the increment of time-step k .

$$w(k) = w_s + \frac{w_e - w_s}{K} \times k \quad (3)$$

where K is the maximum number of iteration for implementing the PSOIW. In the original PSOIW, the boundary values, w_s and w_e , are adopted to 0.9 and 0.4, respectively, and $w_1 = w_2 = 2.0$ are used.

3 The MPSOIW α /DC

Figure 1 illustrates a flowchart of the MPSOIW α /DC. Concretely, the plural PSOIW s are executed in parallel, and the local random search (LRS) [17] is implemented to find the most suitable solution based on the result of each PSOIW. The continuous action of the PSOIW and LRS constitutes a hybrid search (i.e. memetic algorithm) [10]. It seems to be close to the HGAPSO [6] in search effect, which implements a plain GA and the PSO by a mixed operation for improving the adaptation to treat with various blended distribution problems.

The best solution, $\vec{q}_k^b (= \arg \max_{s=1, \dots, S} \{g(\vec{q}_k^s)\})$, where S is the total number of the plural PSOIW s , is determined with maximum selection from each best solution found by each hybrid search at time-step k . Subsequently, the best solution \vec{q}_k^b is put in a solution set of the multi-swarm for information processing.

The internal indicator [14, 15, 16] is to monitor whether the status of the best solution \vec{q}_k^b continues to change or not at all time-step for exhibiting the mechanism of diversive curiosity of the multi-swarm. It is defined as follows.

$$y_k(L, \varepsilon) = \max\left(\varepsilon - \sum_{l=1}^L \frac{|g(\vec{q}_k^b) - g(\vec{q}_{k-l}^b)|}{L}, 0\right) \quad (4)$$

As two adjustable parameters of the internal indicator, L is duration of judgment, and $\varepsilon (> 0)$ is the tolerance coefficient (sensitivity).

While the value of the output y_k is zero, this means that the multi-swarm is exploring the surroundings of the solution \vec{q}_k^b for "cognition". Accordingly, the control signal d_k is set to 0. If once the value of the output y_k become

positive, it indicates that the multi-swarm has lost interest, i.e. feeling boredom, to search the area around the best solution \vec{q}_k^b for "motivation". Therefore, the control signal d_k is set to 1. It is obvious that the function of the internal indicator accomplishes Loewenstein's assumption [8] for distinguishing and detecting the above two behavior patterns, "cognition" and "motivation", in search for interpreting the mechanism of diversive curiosity in psychology [1].

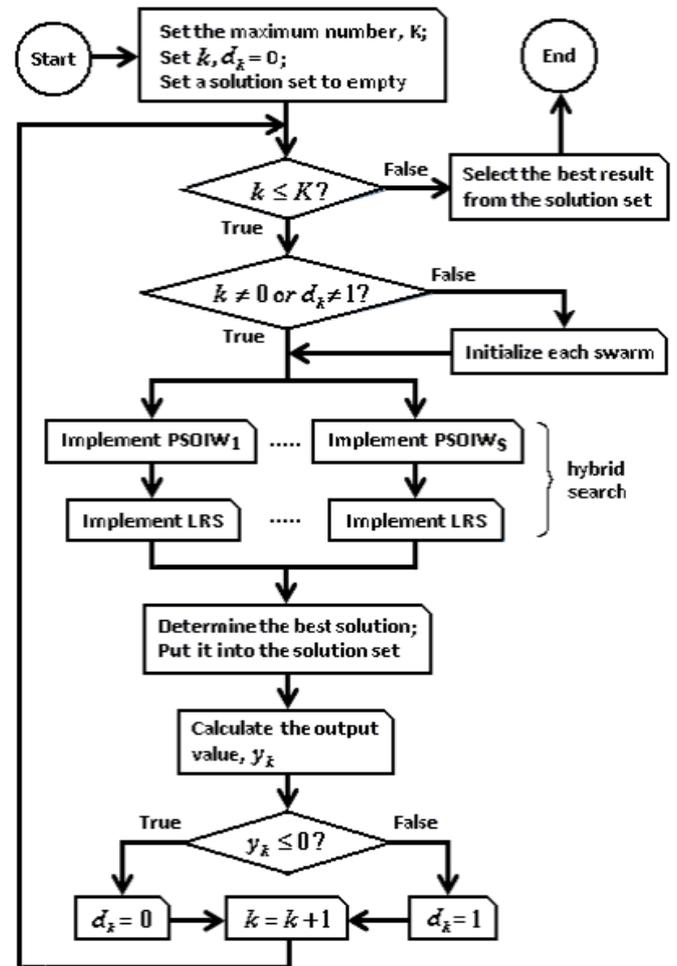


Figure 1: A flowchart of the MPSOIW α /DC

Based on the control signal $d_k = 1$, each particle swarm will be active by reinitialization in Figure 1 to further find other unknown solutions. Accordingly, boredom behavior of the multi-swarm in optimization is overcome by the reliable way of alleviating stagnation. Of course, the implementation style is not an isolated one, it also can be performed by other operation ways in practice.

The LRS: It is implemented as follows.

step-1: Let \vec{q}_k^s be a solution found by the s -th particle swarm at time-step k , and set $\vec{q}_{now}^s = \vec{q}_k^s$. Give the terminating condition, J (the total number of the LRS run), and set $j = 1$.

Table 1: Functions and criteria to the given suite of benchmark problems. The search space for each benchmark problem is limited to $\Omega \in (-5.12, 5.12)^N$.

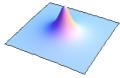
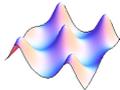
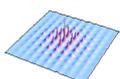
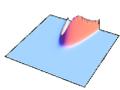
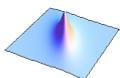
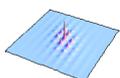
Problem	Function	Criterion (N=2)	
<i>Sphere</i>	$f_{Sp}(\vec{x}) = \sum_{d=1}^N x_d^2$	$g_{Sp}(\vec{x}) = \frac{1}{f_{Sp}(\vec{x}) + 1}$	
<i>Griewank</i>	$f_{Gr}(\vec{x}) = \frac{1}{4000} \sum_{d=1}^N x_d^2 - \prod_{d=1}^N \cos\left(\frac{x_d}{\sqrt{d}}\right) + 1$	$g_{Gr}(\vec{x}) = \frac{1}{f_{Gr}(\vec{x}) + 1}$	
<i>Rastrigin</i>	$f_{Ra}(\vec{x}) = \sum_{d=1}^N (x_d^2 - 10 \cos(2\pi x_d) + 10)$	$g_{Ra}(\vec{x}) = \frac{1}{f_{Ra}(\vec{x}) + 1}$	
<i>Rosenbrock</i>	$f_{Ro}(\vec{x}) = \sum_{d=1}^{N-1} [100(x_{d+1} - x_d^2)^2 + (1 - x_d)^2]$	$g_{Ro}(\vec{x}) = \frac{1}{f_{Ro}(\vec{x}) + 1}$	
<i>Schwefel</i>	$f_{Sw}(\vec{x}) = \sum_{d=1}^N \left(\sum_{j=1}^d x_j\right)^2$	$g_{Sw}(\vec{x}) = \frac{1}{f_{Sw}(\vec{x}) + 1}$	
<i>Hybrid</i>	$f_{Hy}(\vec{x}) = f_{Ra}(\vec{x}) + 2f_{Sw}(\vec{x}) + \frac{1}{12}f_{Gr}(\vec{x}) + \frac{1}{20}f_{Sp}(\vec{x})$	$g_{Hy}(\vec{x}) = \frac{1}{f_{Hy}(\vec{x}) + 1}$	

Table 2: The major parameters used in the MPSOIW α /DC.

Parameter	Value	Parameter	Value
the number of individuals, M	10	the number of LRS run, J	10
the number of generation, G	20	the range of LRS, σ_N^2	0.05
the number of particles, P	10	the duration of judgment, L	10 ~ 90
the number of particle swarms, S	3	the tolerance coefficient, ε	$10^{-6} \sim 10^{-2}$
the number of iterations, K	400		

step-2: Generate a random data, $\vec{d}_j \in \mathbb{R}^N \sim N(0, \sigma_N^2)$ (where σ_N is a small positive value given by user, which determines the small limited space). Check whether $\vec{q}_k^s + \vec{d}_j \in \Omega$ is satisfied or not. If $\vec{q}_k^s + \vec{d}_j \notin \Omega$ then adjust \vec{d}_j for moving $\vec{q}_k^s + \vec{d}_j$ to the nearest valid point within Ω . Set $\vec{q}_{new}^s = \vec{q}_k^s + \vec{d}_j$.

step-3: If $g(\vec{q}_{new}^s) > g(\vec{q}_{now}^s)$ then set $\vec{q}_{now}^s = \vec{q}_{new}^s$.

step-4: Set $j = j + 1$. If $j \leq J$ then go to the **step-2**.

step-5: Set $\vec{q}_k^s = \vec{q}_{now}^s$ to correct the solution found by the s -th particle swarm at time-step k . Stop the search.

4 Computer Experiments

To facilitate comparison and analysis of the performance indexes of the proposed method, we use a suite of the 5-dimensional (5D) benchmark problems [13] in Table 1. And Table 2 gives the major parameters employed for the next experiments.

Preliminaries: To achieve high-performance search, we use the method of meta-optimization, i.e. evolutionary particle swarm optimizer with inertia weight (EPSOIW) [19], to obtain an optimal PSOIW. Table 3 shows the resulting values of parameters in the estimated PSOIW to each given 5D benchmark problem with 20 trials.

Table 3: The resulting appropriate values of parameters in the PSOIW to each given 5D benchmark problem.

Problem	Parameters			
	\hat{w}_s	\hat{w}_e	\hat{w}_1	\hat{w}_2
<i>Sphere</i>	0.72±0.1	0.14±0.2	1.21±0.6	1.92±0.1
<i>Griewank</i>	0.77±0.1	0.23±0.2	1.26±0.6	0.28±0.1
<i>Rastrigin</i>	2.07±0.9	0.88±0.7	12.9±7.9	5.06±1.5
<i>Rosenbrock</i>	0.77±0.2	0.57±0.2	1.92±0.3	1.93±0.5
<i>Schwefel</i>	0.85±0.3	0.12±0.2	1.61±1.3	1.96±1.4
<i>Hybrid</i>	1.47±0.3	0.61±0.5	5.03±1.8	9.21±5.1

We observe that the average of the parameter values of

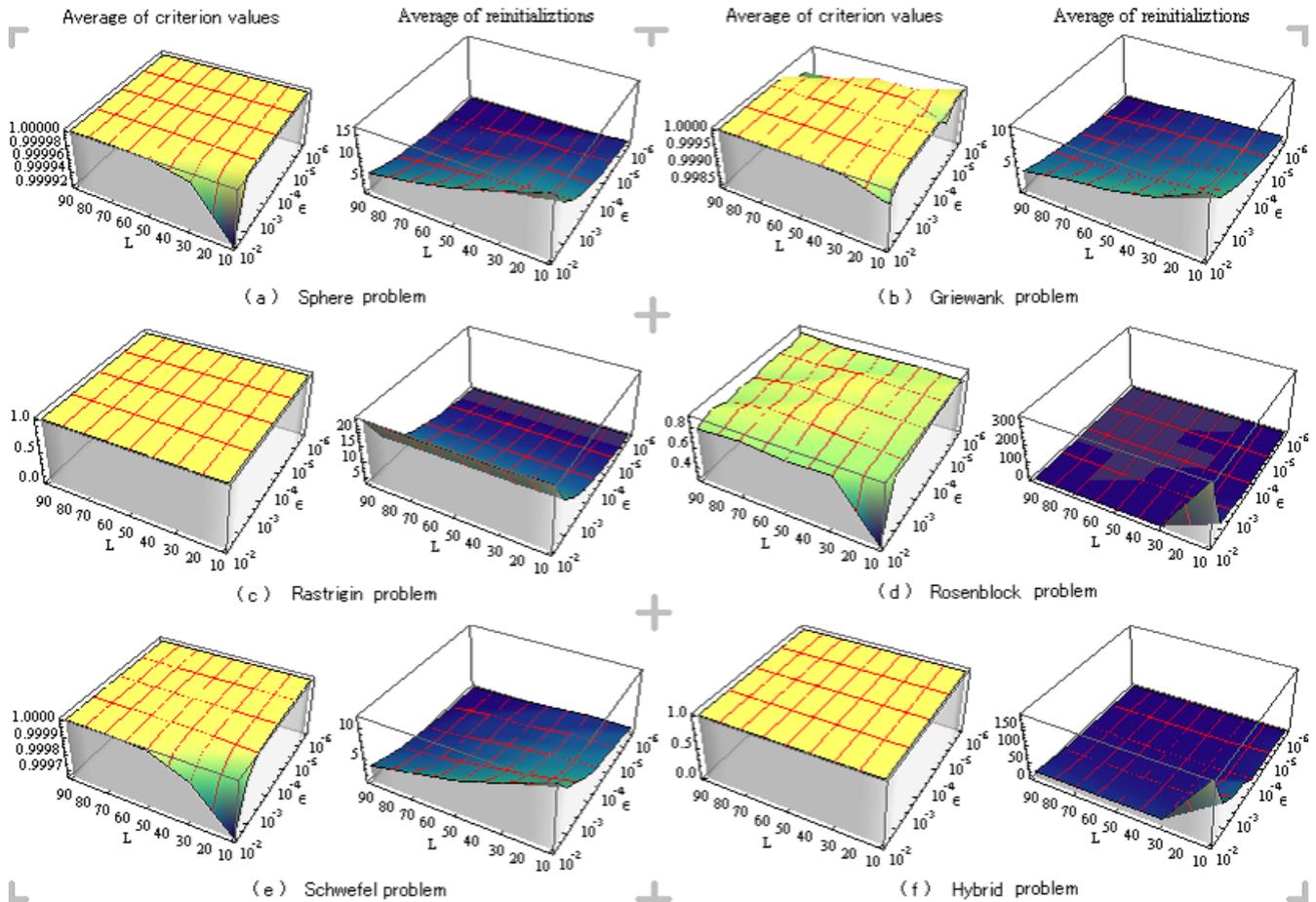


Figure 2: The distributions of average of criterion values and average of reinitialization frequencies with tuning the parameters, L and ε .

the estimated PSOIW are quite different from that of the original PSOIW. It indicates that different problem should be solved by different values of parameters obtained by the EPSOIW. These estimated PSOIW in Table 3 as the best optimizers, PSOIW*, are used in the MPSOIW α /DC for improving the search ability of the proposed method.

Results of the MPSOIW* α /DC: Figure 2 shows the resulting search performance of the MPSOIW* α /DC with 20 trials by tuning the parameters of the internal indicator, L and ε . The following characters of the MPSOIW* α /DC are observed.

- The average of reinitialization frequencies monotonously increases with increment of the tolerance parameter, ε , and decrement of the duration of judgment, L , for each benchmark problem.
- The average of criterion values do not change at all with tuning the parameters, L and ε , for the *Rastrigin* and *Hybrid* problems.
- To obtain superior search performance, the recommended range of parameters of the MPSOIW* α /DC: $L_{Sp}^* \in (10 \sim 90)$ and $\varepsilon_{Sp}^* \in (10^{-6} \sim 10^{-2})$ for the

Sphere problem; $L_{Gr}^* \in (10 \sim 90)$ and $\varepsilon_{Gr}^* \in (10^{-5} \sim 10^{-3})$ for the *Griewank* problem; $L_{Ra}^* \in (10 \sim 90)$ and $\varepsilon_{Ra}^* \in (10^{-6} \sim 10^{-2})$ for the *Rastrigin* problem; $L_{Ro}^* \in (30 \sim 70)$ and $\varepsilon_{Ro}^* \in (10^{-5} \sim 10^{-3})$ for the *Rosenbrock* problem; $L_{Sw}^* \in (10 \sim 90)$ and $\varepsilon_{Sw}^* \in (10^{-6} \sim 10^{-2})$ for the *Schwefel* problem; and $L_{Hy}^* \in (10 \sim 90)$ and $\varepsilon_{Hy}^* \in (10^{-6} \sim 10^{-2})$ for the *Hybrid* problem are available.

As to the *Rastrigin* and *Hybrid* problems, the resulting average of criterion values in Figure 2(c) and Figure 2(f) are mostly unchanged with tuning the parameters, L and ε . This phenomenon suggests that the optimized PSOIW* has powerful search ability well to deal with these multimodal problems.

We also observe that the average of reinitialization frequencies is over 300 times in the case of the parameters, i.e. $L=10$ and $\varepsilon = 10^{-2}$, for the *Rosenbrock* problem in Figure 2(d). Because the average of criterion values is the lowest than that in the other cases, it is considered that the search behavior of the multi-swarm seems to have entered “the zone of anxiety,” [2]. However, the average of reinitialization frequencies is close to 150 times

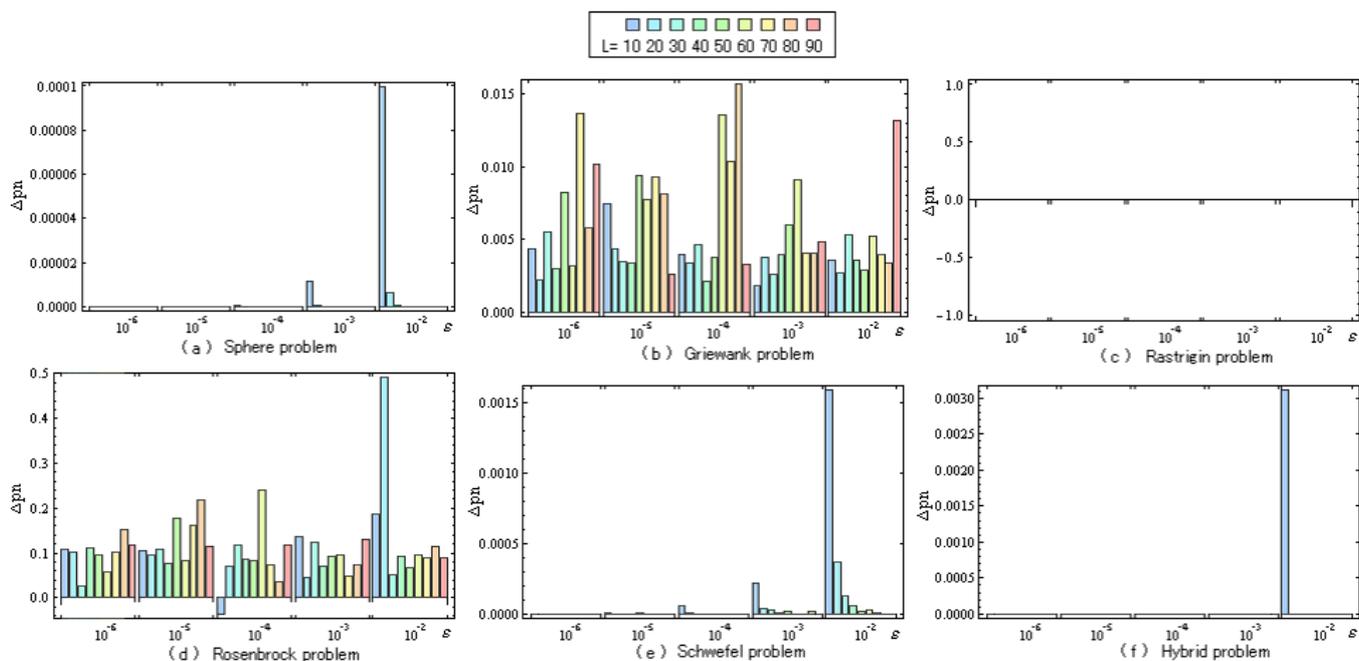


Figure 3: The performance comparison between the MPSOIW* α /DC and MPSOIW*/DC.

Table 4: The mean and standard deviation of criterion values in each method for each 5D benchmark problem with 20 trials. The values in bold signify the best result for each problem.

Problem	MPSOIW* α /DC	PSOIW*/DC	EPSOIW	PSOIW	OPSO	RGA/E
<i>Sphere</i>	1.000 \pm 0.000	0.999 \pm 0.001				
<i>Griewank</i>	1.000 \pm 0.000	1.000 \pm 0.000	0.984 \pm 0.006	0.850 \pm 0.119	0.944 \pm 0.043	0.945 \pm 0.078
<i>Rastrigin</i>	1.000 \pm 0.000	1.000 \pm 0.000	1.000 \pm 0.000	0.232 \pm 0.159	0.265 \pm 0.118	0.906 \pm 0.225
<i>Rosenbrock</i>	0.989 \pm 0.012	0.625 \pm 0.232	0.607 \pm 0.217	0.565 \pm 0.179	0.392 \pm 0.197	0.389 \pm 0.227
<i>Schwefel</i>	1.000 \pm 0.000	1.000 \pm 0.000	1.000 \pm 0.000	1.000 \pm 0.000	0.767 \pm 0.412	0.987 \pm 0.214
<i>Hybrid</i>	1.000 \pm 0.000	1.000 \pm 0.000	0.802 \pm 0.405	0.390 \pm 0.374	0.306 \pm 0.359	0.153 \pm 0.133

in the same case for the *Hybrid* problem in Figure 2(f), the situation of anxiety does not appear.

Effect of the LRS: Figure 3 shows the resulting performance difference between the MPSOIW* α /DC and MPSOIW*/DC corresponding to each given problem. Note that the difference is defined by $\Delta_{PN} = \bar{g}_P^* - \bar{g}_N^*$ (\bar{g}_N^* : the average of criterion values of the MPSOIW*/DC). Dissimilar to the preceding results, the search performance of the MPSOIW* α /DC is better than that of the MPSOIW*/DC in the most cases for each problem except the *Rastrigin* problem. This result clearly indicates that the LRS plays an essential role in drastically improving the search performance of the MPSOIW*/DC.

On the other hand, the effect of the LRS is not remarkable for the *Sphere*, *Schwefel*, and *Hybrid* problems. These results show that the effect of the LRS closely depends on the object of search, which related to how to set the parameter values for the running number, J , and the search range, σ_N^2 , and the inherent feature of the given benchmark problems. The details on discussion for the issue

are omitted here.

Comparison with Other Methods: For further illuminating the effectiveness of the proposed method, Table 4 gives the obtained experimental results of implementing these methods with 20 trials. It is well shown that the search performance of the MPSOIW* α /DC is better than that by the PSOIW*/DC, EPSOIW, PSOIW, OPSO (optimized particle swarm optimization) [9], and RGA/E. The results sufficiently reflect that the merging of both multiple hybrid search and the mechanism of diversive curiosity takes the active role in handling these benchmark problems. In particular, A big increase, i.e. the average of criterion values by implementing the MPSOIW* α /DC steeply rises from 0.565 to 0.989, in search performance is achieved well for the *Rosenbrock* problem.

5 Conclusion

A new method of cooperative PSO – multiple particle swarm optimizers with inertia weight with diversive cu-

riosity, MPSOIW α /DC, has been proposed in this paper. Owing to the essential strategies of decentralization in search and concentration in evaluation and behavior control, it has good capability to improve search efficiency of the plain MPSOIW by alleviating stagnation in handling complex optimization problems.

Applications of the MPSOIW α /DC to a suite of the 5D benchmark problems well demonstrated its effectiveness. The obtained experimental results verified that unifying the both characteristics of multi-swarm search and the LRS is successful and effective. In comparison with the search performance of the PSOIW/DC, EPSOIW, PSOIW, OPSO, and RGA/E, it is obvious that the proposed method has an enormous latent capability in treating with the given benchmark problems and the outstanding powers of multi-swarm search.

Accordingly, the basis of the development study of cooperative PSO research in swarm intelligence and optimization is further expanded and consolidated. It is left for further study to apply the MPSOIW α /DC to practical problems in the real-world.

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