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Weighted Scale-Free Random Graph Model

Chen Xinyi

Abstract—**Systems as diverse as genetic networks or the World Wide Web are best described as networks with complex topology. A common property of many large networks is that the vertex connectivities follow a scale-free power-law distribution. This feature was found to be a consequence of three generic mechanisms: (i) networks expand continuously by attaching addition of new vertices, (ii) new vertex with different number edges of weighted selected connectioned to different vertices in the system, and (iii) new vertices attach preferentially to sites that are already well connected. A model based on these three ingredients reproduces the observed stationary scale-free distributions, which indicate that the development of large networks is governed by robust self-organizing phenomena that go beyond the particulars of the individual systems.**

Index Terms—-scale-free model, vertex, weighted random networks, degree distribution

I. Introduction

THE inability of contemporary science to describe systems composed of nonidentical elements that have diverse and nonlocal interactions currently limits advances in many disciplines, ranging from molecular biology to computer science [1]. The difficulty of describing these systems lies partly in their topology. Many of them form rather complex networks whose vertices are the elements of the system and whose edges represent the interactions between them. For example, living systems form a huge genetic network whose vertices are proteins and genes, the chemical interactions between them representing edges [2]. At different organizational levels, a large network is formed by the nervous system, whose vertices are the nerve cells, connected by axons [3]. But equally, complex networks also occur in social science, where vertices are individuals or organizations and the edges are the social interactions between them [4], or in the World Wide Web(WWW), whose vertices are HTML documents connected by links pointing from one page to anther [5,6]. Because of their large size and the complexity of their interactions, the topology of these networks is largely unknown.

In[7], Barabási and Albert (BA) showed that it is possible to grow a network with a power law degree distribution by using a preferential growth mechanism: starting with a small number(m_0) of vertices, at every time step the system grows by attaching a new vertex with m ($\leq m_0$) edges links to m different "old" vertices that are already presented in the system; the attachment is preferential because the probability that a new vertex will connect to vertex i, with degree k_i, is $\Pi(k_i)=k_i/\sum_j k_j$. After t time steps, the model leads to a random network with $t+m_0$ vertices and mt edges. This network evolves into a scale-invariant state with the probability that a vertex has k edges, following a power law with an exponent γ=2.9±0.1. Because the power law observed for real networks describes systems of rather different sizes at different stages of their development, it is expected that a correct model should provide a distribution whose main features are independent of time. Indeed, $P(k)$ is independent of time (and subsequently independent of the system size m_0+t), indicating that despite its continuous growth, the system organizes itself into a scale-free stationary state.

The BA model generates networks with the power law exponent γ= 2.9±0.1. The model has been used to model the Internet. But real network is not included in the model. Firstly, there are not vertices with degree one (when $m\neq1$) in the model, and in the degree distribution of the real network is not a strict power law as it has more vertices with degree two than vertices with degree one [8] ($P(2)$= 38%> $P(1)$ = 26%, see Table I). Secondly grows of real network is not that new vertex with same number of edges link to different "old" vertices already preferential in the system.

TABLE I
NETWORK PARAMETERS

Number of vertices	Average degree	Degree distribution			Exponent of power-law
N	$<k>$	$P(k=1)$	$P(k=2)$	$P(k=3)$	γ
11122	5.4	26%	38%	14%	2.22

II. Main Results

In this paper, we have showed a model based on these three ingredients which naturally lead to observed scale-invariant distribution. To incorporate the growing character of the network, starting from a small number ($m_0\geq3$) of vertices, at every time step we added a new vertex with n(n is the positive integer in max{0.26-$P(n=1)$, 0.38-$P(n=2)$, 0.14-$P(n=3)$,0.22-$P(n>3)$}, where $P(k)$ is degree distribution of the present system, and when $n>3$, n is a random number selected from 4 to $m(m_0)$) edges that links the new vertex to n different vertices which already presented in the system. To incorporate preferential attachment, we assumed that the probability Π that a new vertex will be connected to vertex i depends on the connectivity k_i of that vertex, so that $\Pi(k_i)=k_i/\sum_j k_j$. After t time steps, the model leaded to a random network with $t+m_0$ vertices and these edges are between t and mt. Thus, by [7], this network is evolving into a scale-invariant state with the probability that a vertex has k edges, following a power law with an exponent γ_{model}=3(Fig. 1). Because the power law

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observed for real networks describes systems of rather different sizes at different stages of their developments, it is expected that a correct model should provide a distribution whose main features are independent of time. Indeed, as Fig. 1 demonstrates, $P(k)$ is independent of time (and subsequently independent of the system size

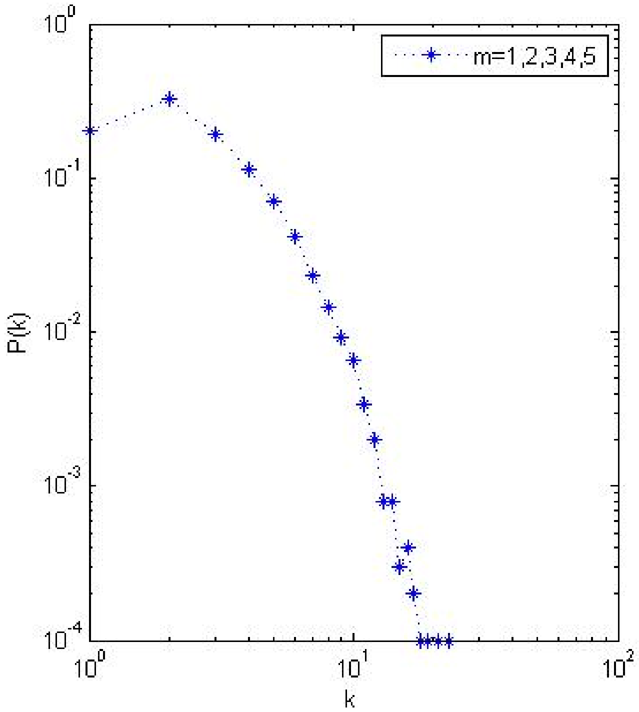

Fig. 1. The power-low connectivity distribution at t=10000, in the case of m_0=5, m=5.

m_0+t), indicating that despite of its continuous growth, the system organizes itself into a scale-free stationary state.

The development of the power-law scaling in the model indicates that growth and preferential attachment of randomly selected edges have played an important role in network development. Because of the preferential attachment, a vertex that acquires more connections than another one will increase its connectivity at a higher rate; thus, an initial difference in the connectivity between two vertices will increase further as the network grows. The rate at which a vertex acquires edges is $\partial k_i/\partial t= k_i/2t$, where $k_i(t)$ is between $(t/t_i)^{0.5}$ and $3(t/t_i)^{0.5}$, t_i is time at which vertex i is added to the system. Then, a scaling property that could be directly tested ones time-resolved data on network connectivity becomes available. Thus older (with smaller t_i) vertices increase their connectivity at the expense of the younger (with larger t_i) ones, leading over time to some vertices that are highly connected. A "rich-get-richer" phenomenon can be easily detected in real networks. Furthermore, this property can be used to calculate γ analytically. The probability that a vertex i has a connectivity smaller than k, $P[k_i(t)<k]$, can be presented as $P(t_i > m^2t/k^2)$, for fixed m edges that link the new vertex to m different vertices in the system. Assuming that we add the vertices to the system at equal time intervals, we obtain $P(t_i > m^2t/k^2)=1- P(t_i \le m^2t/k^2)=1-m^2t/k^2(t+m_0)$. The probability density $P(k)$ can be obtained from $P(k)=\partial P[k_i(t)<k]/\partial k$, which over long time periods leads to the stationary solution

$$\frac{2}{k^3} \le P(k) \le \frac{2m^2}{k^3}$$

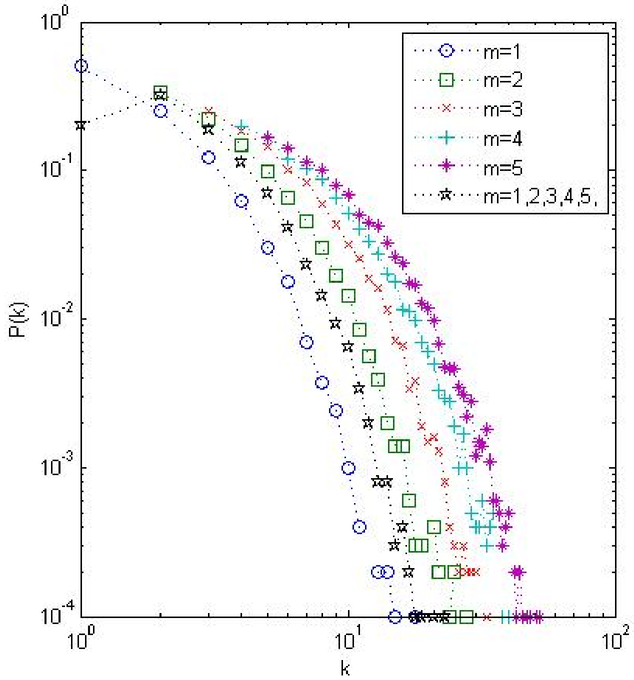

Fig. 2. The exponential connectivity distribution for model at t=10000.

giving γ=3, independent of 3. Thus degree distribution $P(k)$ of the model between $2/k^3$ and $2m^2/k^3$(Fig.2 and Fig. 3).

From numerical simulations, we found that the interactive growth also satisfied the following two characteristics observed in the Internet measurements. (1) Majority of new

TABLE II
MODEL PARAMETERS

Number of vertices	Average degree	Degree distribution			Exponent of power-law
N	$<k>$	$P(k=1)$	$P(k=2)$	$P(k=3)$	γ
10000	2	20.18%	32.18%	18.99%	3

vertices are added to the system by attaching them to one or two old vertices ($m \le 2$). (2) Degree distribution of the autonomous systems graph is not a strict power-law as it has more vertices with degree two than vertices with degree one ($P(2)$ = 32.18%>P(1) = 20.18%, in the case m=5, see Table II, and $P(2)$ = 31.55%>P(1) = 19.56%, in the case m=6, see Table III).(3) In fact, $P(k)$ of the model is between $2/k^3$ and $8/k^3$(see Table II and Table III). Therefore, the model is closer to the actual Internet network. In many cases, the results are difficult to prove, and need techniques which can not use in the theory of classical random graphs. We have learned through empirical studies, models, and analytic approaches that real networks are far from being random, but display generic organizing principles shared by rather different systems. Therefore, we expected that extending BA model, whenever appropriate, would lead to a much more realistic description of several real systems. Consequently, this paper would be a deduction of BA model.

The scale-invariant state observed in all systems where detail data has been available to us is a generic property of many complex networks, with applicability reaching far beyond the quoted examples.

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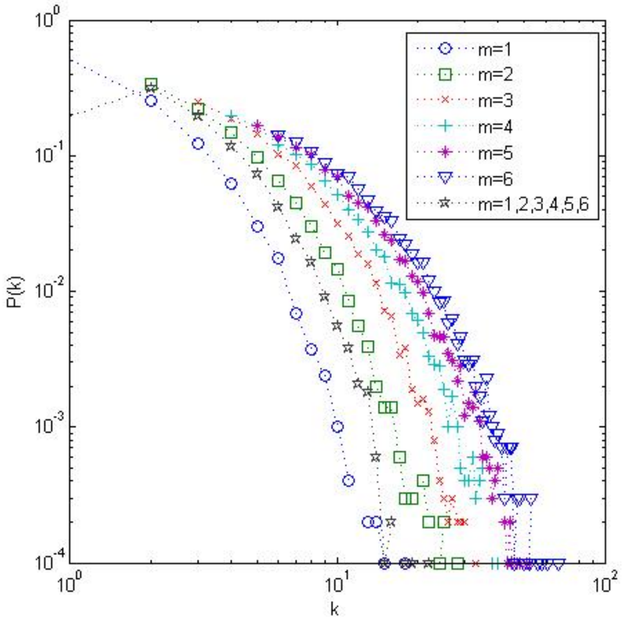

Fig. 3. The exponential connectivity distribution for model at t=10000.

TABLE III
MODEL PARAMETERS

Number of vertices	Average degree	Degree distribution			Exponent of power-law
N	<*k*>	*P*(*k*=1)	*P*(*k*=2)	*P*(*k*=3)	γ
10000	3	19.56%	31.55%	19.39%	3

These mechanisms also could explain the origin of the social and economic disparities governing competitive systems, considering that the scale-free in homogeneities are the inevitable consequence of self-organization, just like the local decisions made by the individual vertices.

However, the validation of the model was not conducted with measurement data based on the BGP-tables, but the traceroute-derived AS graph, which is regarded as a more realistic and reliable measurement of the Internet[9].

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