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On Problems With Closure Properties

Stefan Porschen

***Abstract*—We consider a specific class of combinatorial search or optimization problems where the search space gives rise to a closure operator and essentially the hulls are the only relevant subsets that must be checked in a brute force approach. We suggest that such a closure property can help to reduce time complexities. Moreover we propose two types of (structural) parameterization of instance classes based on the closure property and outline how it could be used to achieve FPT characterizations. In this setting, three example problems are considered: a covering problem from combinatorial geometry, a variant of the autarky problem in propositional logic, and a specific graph problem on finite forests.**

***Index Terms*—closure operator, FPT, combinatorial optimization, computational complexity.**

I. Introduction

In this paper we propose a structural perspective on a specific class of combinatorial problems over set systems. Namely those for which a closure operator can be associated to or defined on the search space. Two types of closure properties are introduced. A first where the objective is a class invariant regarding the equivalence relation naturally defined by the closure operator; in this case the search space is the power set of the preimage of the closure operator. On basis of this closure property it can be possible to reduce the computational time complexity of the problem, which is demonstrated for a rectangular covering problem in the plane.

Another closure property called of the *second kind* is defined s.t. the hulls are the only relevant subsets relevant for the problem; in this case the closure operator is required to be present in the search space itself.

The traditional parameterization of combinatorial search problems usually is defined through the cardinality of the solution [2]. If a combinatorial search problem admits a closure property of the second kind, we propose a structural parameterization approach. the parameter of an instance class is given as the maximum cardinality of all hulls of the single element sets of the search space. We also propose another parameterization based on the cardinality of the hulls itself. Such structural parameterizations might help to gain fixed-parameter tractable (FPT) [2] instance classes of such a problem.

We discuss several examples of problems for which a closure property can be found. The first is an optimization problem that takes as input a finite point set in the plane and aims at finding a minimal rectangular covering subject to a certain objective function. We state the closure property and report how it can be used to reduce the time complexity of a dynamic programming optimization algorithm. Second, we address the autarky problem in propositional logic. The input here is a conjunctive normal formula and the question is, whether there is a subformula which can be satisfied independent of the remaining formula. A closure property closely related to the autarky problem is detected and a guide to a FPT characterization is proposed. Finally, we focus on a certain graph problem originally stemming from a falsifiability problem in propositional logic [3]. It takes as input a finite forest of rooted trees [4] and a mapping f assigning leaves to vertices which are considered as roots of subtrees. Then one is asked to find one leaf of every tree, s.t. none of these leaves is containd in the subtree rooted at the f-image of any other of these leaves. This problem is NP-complete and belongs to the class FPT w.r.t. the number of trees in the forest as parameter [3]. Identifying a closure operator one can define a variant of this graph problem, for which an additional parameter is introduced defined as the maximal hull cardinality as above.

Department 4, HTW Berlin, D-10313 Berlin, Germany, e-mail: porschen@htw-berlin.de.

II. On Closures and Equivalences

A basic concept in this paper is the well-known closure operator defined on the power set 2^M of a given set M. For convenience, let us recall the defining properties of a closure operator:

Definition 1: Given a set M (here always finite), a *closure operator* is a map

$$\sigma : 2^M \to 2^M$$

with the following properties:
(1) *extensity*: $\forall S \subseteq M \Rightarrow S \subseteq \sigma(S)$,
(2) *monotony:* $\forall S_1, S_2 \subseteq M : S_1 \subseteq S_2 \Rightarrow \sigma(S_1) \subseteq \sigma(S_2)$
(3) *idempotence:* $\forall S \subseteq M \Rightarrow \sigma(\sigma(S)) = \sigma(S)$.
Easy examples are the identity map id on 2^M given by $\mathrm{id}(S) = S$, for every $S \in 2^M$, or the constant map $c_M(S) = M$ for every $S \in 2^M$. A more interesting example is given by the convex hull operator: Let $M \subseteq \mathbb{R}2$ be a finite set of points in the euclidean plane. Then $\sigma(S) := \mathrm{conv}(S) \cap M$, for every $S \in 2^M$, defines a closure operator, as is easy to see, where $\mathrm{conv}(S)$ denotes the convex hull of all points in $S \subseteq \mathbb{R}^2$.

For convenience, we denote the image of a closure operator, i.e., the set of all *hulls* by $\mathcal{H}_\sigma(M) := \sigma(2^M)$. If $|S| = k$ we call $\sigma(S) =: H(S)$ a k-hull, for $0 \leq k \leq n$, and we denote the collection of all k-hulls by $\mathcal{H}_k$. Any closure operator on 2^M clearly defines an equivalence relation on 2^M by $S_1 \sim S_2$ iff $\sigma(S_1) = \sigma(S_2)$. The next result tells us which conditions must be satisfied by an equivalence relation so that it gives rise to a closure operator as above.

Theorem 1: Let M be finite set, and let $\sim \subseteq 2^M \times 2^M$ be an equivalence relation on 2^M. For each class $[S]$, $S \subseteq M$, set $W_{[S]} := \bigcup [S]$. Then it holds that $\sim$ satisfies:
$(i) \forall S \in 2^M : W_{[S]} \in [S]$, and
$(ii) \forall S_1, S_2 \in 2^M : S_1 \subseteq S_2 \Rightarrow W_{[S_1]} \subseteq W_{[S_2]}$,
if and only if

$$\sigma : 2^M \ni S \mapsto \sigma(S) := W_{[S]} \in 2^M$$

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is a closure operator satisfying $(*) : \sigma(S_1) = \sigma(S_2) \Rightarrow S_1 \sim S_2$, for all $S_1, S_2 \in 2^M$.

PROOF. First, let σ as defined above be a closure operator with property $(*)$. Then it is easy to see that even $\sigma(S_1) = \sigma(S_2) \Leftrightarrow S_1 \sim S_2$ holds for all $S_1, S_2 \in 2^M$. Since, by definition we have $\sigma(S) = W_{[S]}$ we get by monotony $\sigma(S) = \sigma(W_{[S]})$ implying $W_{[S]} \in [S]$ which is (i). (ii) is a direct implication of the monotony of σ.

Conversely, let $\sim$ be an equivalence relation satisfying both (i) and (ii). From (ii) it directly follows that σ as stated above is a monotone set mapping. For arbitrary $S \in 2^M$, we further have $S \subseteq W_{[S]} = \sigma(S)$ implying extensity of σ. Property (i) means $S \sim W_{[S]}$. Therefore we have $S_1 \sim S_2 \Leftrightarrow W_{[S_1]} \in [S_2]$ and $W_{[S_2]} \in [S_1]$ iif $W_{[S_1]} = W_{[S_2]} \Leftrightarrow \sigma(S_1) = \sigma(S_2)$, for arbitrary $S_1, S_2 \in 2^M$ which specifically implies $(*)$. Finally, we have idempotence of σ, because (i) leads to $[W_{[S]}] = [S]$. Thus $\sigma(\sigma(S)) = \sigma(W_{[S]}) = \bigcup[W_{[S]}] = \bigcup[S] = \sigma(S)$, for every $S \in 2^M$. □

In the next section we propose a class of search or optimization problems exhibiting a closure operator.

III. The Closure Property and Structural Parameters

Consider a combinatorial search problem Π such that each of its input instances I is associated to a base set $M(I)$. Let $F(I) := 2^{M(I)}$ denote the power set of the base set $M(I)$. Sometimes, but not in every case, $F(I)$ can be identified with the search space of the problem at hand. Examples for base sets are:

- discrete point set in the plane in the instance of a discrete geometry problem,
- vertex set, or edge set in the instance of a graph problem,
- set of variables or expressions in the instance of a logic problem.

Definition 2: Let Π be a combinatorial problem such that each instance $I \in \Pi$ admits a closure operator σ_I on $F(I)$.
(1) We say that a problem Π has a *closure property of the first kind*, if the search space is $2^{F(I)}$ and the objective is a class invariant w.r.t. the equivalence relation defined by σ_I.
(2) We say that Π has a *closure property of the second kind* if $F(I)$ is the search space and for deciding whether $I \in \Pi$, resp. for solving the search variant, it suffices to test all hulls given by σ_I.

Clearly, if a problem has a closure porperty of any kind, specifically we have an equivalence relation $\sim_I$ on $F(I)$ defined by $\sigma(I)$. From Thm. 1 we know that the union of all elements in each equivalence class belongs to the class itself which thus is the supremum of all sets in the class w.r.t. the lattice defined by σ_I. Therefore these suprema (maximal hulls) give rise to a distinct family of class representatives.

The "traditional" parameterization of a problem Π, usually is defined via the cardinality of the solution [2]. As example take the vertex cover problem on a simple graph, for which a well-known FPT characterization exists in the following parameterized version:
Input: Graph $G = (V, E)$, $k \in \mathbb{N}$.
Problem: Find a vertex cover of cardinality at most k in G or report that none exists.

In the following we shall define a structural parameterization by the cardinality of the image of a closure operator. A useful observation is:

Lemma 1: If a closure operator σ on 2^M admits at most k 1-hulls meaning $|\mathcal{H}_\sigma(1)| = |\{H(x) := \sigma(\{x\}) : x \in M\}| \leq k \leq n$, where n is the number of elements in M, then $|\mathcal{H}_\sigma(M)| \leq 2^k$.

PROOF. It suffices to show that for arbitrary $\emptyset \neq S \in 2^M$ it holds that $\sigma(S) = \sigma(\bigcup_{x \in S} H(x))$. From this the Lemma follows because $\bigcup_{x \in S} H(x)$ can be composed of at most k 1-hulls. Hence $\sigma(S)$ corresponds to a subset of $\{1, 2, \ldots, k\}$ implying the Lemma.
To prove the preceding assertion, let $\emptyset \neq S \in 2^M$ be arbitrary. Then there is $1 \leq p \leq n$ s.t. $|S| = p$; let $S = \{x_{i_1}, \ldots, x_{i_p}\}$. Then clearly $H(x_{i_q}) \subseteq S$, $1 \leq q \leq p$, and therefore $\bigcup_{j=1}^{p} H(x_{i_j}) \subseteq \sigma(S)$ implying $\sigma(\bigcup_{q=1}^{p} H(x_{i_q})) \subseteq \sigma(S)$ because σ is monotone and idempotent. On the other hand, we clearly have $x_{i_q} \in H(x_{i_q}), 1 \leq q \leq p$, thus $S \subseteq \bigcup_{q=1}^{p} H(x_{i_q})$ and by monotony we obtain $\sigma(S) \subseteq \sigma(\bigcup_{q=1}^{p} H(x_{i_q}))$ yielding the assertion and the Lemma. □

Let Π be a problem with closure property, and $k \in \mathbb{N}$. Then the instance subclass parameterized by k is given as

$$\Pi_k := \{I \in \Pi : \exists j \leq |M(I)| : \sum_{i=1}^{j} |\mathcal{H}_I(j)| \leq O(f(k))\}$$

where $f : \mathbb{N} \to \mathbb{N}$ is an arbitrary function and $\mathcal{H}_I(j) = \{\sigma_I(S) : S \subseteq M(I), |S| = j\}$. We call this the *parameterization by the number of hulls*.

Observe that, according to La. 1, we specifically have that $I \in \Pi_k$ if σ_I yields at most k 1-hulls.

Theorem 2: Let Π_k, $k \in \mathbb{N}$, have a closure property of the second kind, and assume that for each $I \in \Pi$ we can check the membership in Π_k in polynomial time. Moreover assume that the subproblem corresponding to each hull is computable in polynomial time (or at least in FPT time w.r.t. k). Then Π_k is in FPT w.r.t. k.

PROOF. The closure property of Π of the second kind implies that we only have to check all hulls instead of whole $F(I)$. Therefore if the subproblem corresponding to each hull can be tested in polynomial time or in FPT-time w.r.t. k we are done according to Lemma 1. □

Besides the parameterization by the number of hulls, another one may be useful, namely that by the maximal size of all hulls: Let Π be a problem s.t. every instance admits a closure operator. Note that we do not necessarily require a closure property as defined above. For $k \in \mathbb{N}$ fixed, we define the instance class Π'_k by

$$\Pi'_k := \{I \in \Pi : \max_{S \in 2^{M(I)}} |\sigma_I(S)| \leq k\}$$

i.e., the collection of all Π-instances I such that the cardinality of every hull is bounded by parameter k.

IV. Some Example Problems

This section is devoted to illustrate that problems with closure properties can occur in several areas. We start with a rectangular covering optimization problem which is identified to have a closure property of the first kind. Another problem lies in the area of propositonal logic, and turns out to have a closure property in the second kind. Finally a a graph problem is discussed for which the maximal hull parameter

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is, so that a kernelization and a bound of kernel form [2] can be achieved [9].

A. Rectangular Coverings of Point Sets

We consider the following optimization problem.
Given: Finite set M of points arbitrarily distributed in the euclidean plane. Objective function w on rectangular patches.
Problem: Find a covering R of M by rectangles such that $w(R) := \sum_{r\in R} w(r)$ is minimized.

Observe that a covering R is a selection of subsets $S \subseteq M$ each of which is covered optimally. So the search space, actually, is 2^{2^M} which by the closure property is reduced to $2^{\mathcal{R}(M)}$.

The corresponding closure property of this problem relies on the concept of *base points* $b(S) = \{z_d(S), z_u(S)\}$ of every $S \in 2^M$ which are defined through:
$x_d(S) := \min_{z\in S} x(z)$, $y_d(S) := \min_{z\in S} y(z)$ and $x_u(S) := \max_{z\in S} x(z)$, $y_u(S) := \max_{z\in S} y(z)$. The base points immediately yield the upper right and lower left diagonal points of the smallest rectangle $r(S)$ enclosing S.

There is an equivalence relation on 2^M defined by $S_1 \sim S_2$ iff $b(S_1) = b(S_2)$, for $S_1, S_2 \in 2^M$ with classes $[S]$.
Defining

$$\sigma : 2^M \ni S \mapsto \sigma(S) := r(S) \cap M \in 2^M$$

and $\mathcal{R}(M) := \{S \subseteq M : \sigma(S) = S\}$, we have:

Theorem 3 ([10]): $\sigma : 2^M \to 2^M$ is a closure operator; and there is a bijection between $\mathcal{R}(M)$ and $2^M/\sim$ defined by $S \mapsto [S], S \in \mathcal{R}(M)$. Moreover $\mathcal{R}(M)$ is of polynomial size in the variable $|M|$ and can be also be computed in polynomial time.
Obviously the objective w_I is a class invariant because $w_I(r(T))$ has the same value on every $S \in [S]$. Hence we face a problem with closure property of the first kind. Using this closure property the time complexity of a dynamic programming approach for solving the covering problem can be decreased. Concretely, one can show the following results [10]: There exists a dynamic programming algorithm solving this problem of time complexity $O(n^2 3^n)$, where n is the size of M. On behalf of the closure property in $F(I) = 2^{M(I)}$ this time complexity can be reduced to $O(n^6 2^n)$. Due to a more subtle closure property investigated in [7] one can establish in certain situations the slightly better bound of $O(n^4 2^n)$.

By the result above we can compute $\mathcal{R}(M)$ in polynomial time $O(p(|M|))$ where M is the set of input points, and p is a polynomial. Unfortunately, it is an open problem, of how to check the subproblems corresponding to each hull in polynomial time, or even in FPT-time. Therefore the question whether one can achieve a FPT-characterization of this problem w.r.t. the parameterization by image cardinality is left for future work.

B. Autarkies in CNF formulas

In this section we consider the propositional satisfiability problem (SAT) on conjunctive normal form (CNF) formulas [1]. It is convenient to regard a CNF formula C as a set of its clauses $C = \{c_1, \ldots, c_m\}$. By $V(C)$ we denote the set of all propositional variables occuring in C. The concept of autarky in the context of CNF-SAT was introduced in [6]. Roughly speaking, an *autark* set of variables can be removed from a CNF formula without affecting its satisfiability status. More precisely, given CNF formula C, we call a subset $U \subseteq V(C)$ an *autark set (of variables)*, iff there exists a (partial) truth assignment $\alpha : U \to \{0, 1\}$ satisfying the subformula $C(U)$ defined by $C(U) := \{c \in C : V(c) \cap U \neq \varnothing\}$. Removing $C(U)$ from C therefore preserves the satisfiability status of the resulting formula.

Consider the decision problem AUT:
Input: $C \in$CNF
Question: $\exists$ an autark set $U \subseteq V(C)$?

It is not hard to see that AUT is NP-complete. However, a basic open question is whether AUT is fixed-parameter tractable regarding the traditional parameterization, namely w.r.t. the parameter k defining the maximum cardinality of an autark set in the input formula. In the following we propose an alternative approach based on a parameterization defined by a closure property: Given $C \in$ CNF, then for every $U \subseteq V(C)$ we define the set $\sigma_C(U) \subseteq V(C)$ as

$$\sigma_C(U) := V(C) - V(C - C(U))$$

We call $\sigma_C(U)$ the *autarky closure or autarky hull* of U (introduced as *variable hull* in [8]).

Lemma 2: Given $C \in$ CNF, then $\sigma_C : 2^{V(C)} \to 2^{V(C)}$ as defined above is a (finite) closure operator.
PROOF. Extensity obviously holds true for σ_C. Let $U_1, U_2 \subseteq V(C)$ with $U_1 \subseteq U_2$, then $C(U_1) \subseteq C(U_2)$, hence $V(C - C(U_2)) \subseteq V(C - C(U_1))$. Now suppose there is $x \in \sigma_C(U_1)$ and $x \notin \sigma_C(U_2)$, then by definition $x \in V(C - C(U_2))$ and therefore $x \in V(C - C(U_1))$ contradicting the assumption, thus (ii) holds. Finally, let $W := \sigma_C(U)$, for $U \in 2^{V(C)}$. We have $C(W) = C(U)$ since no variable of W occurs outside $C(U)$, yielding $\sigma_C(W) = W$ which is idempotence. $\square$

Next, we have:

Lemma 3: Given $C \in$ CNF.
1.) For $U_1, U_2 \subset V(C)$, $U_1 \sim U_2 :\Leftrightarrow C(U_1) = C(U_2)$ defines an equivalence relation on $2^{V(C)}$ with classes $[U]$.
2.) The quotient space $2^{V(C)}/\sim$ is in 1:1-correspondence to $\{\sigma_C(U) : U \in 2^{V(C)}\}$.
PROOF. The first part is obvious. For proving the second part we claim that for each $U_1, U_2 \in 2^{V(C)}$ holds

$$\sigma_C(U_1) = \sigma_C(U_2) \quad \Leftrightarrow \quad U_1 \sim U_2$$

from which 2.) obviously follows. Now $U_1 \sim U_2$ means $C(U_1) = C(U_2)$ implying $\sigma_C(U_1) = \sigma_C(U_2)$. For the reverse direction we observe that $C(\sigma_C(U)) = C(U)$, for each $U \subseteq V(C)$. Therefore, $\sigma_C(U_1) = \sigma_C(U_2)$ implies $C(U_1) = C(\sigma_C(U_1)) = C(\sigma_C(U_2)) = C(U_2)$ thus $U_1 \sim U_2$. $\square$

The next result justifies the notion autarky closure:

Lemma 4: For $C \in$ CNF and $U \subseteq V(C)$, we have that $\sigma_C(U)$ is autark if U is autark.
PROOF. Suppose U is autark, but $\sigma_C(U)$ is not autark. Because $U \sim \sigma_C(U)$, we have $C(U) = C(\sigma_C(U))$, hence any truth assignment $\alpha : U \to \{0, 1\}$ satisfying $C(U)$ also satisfies $C(\sigma_C(U))$, thus $\sigma_C(U)$ is an autark set. $\square$

Observe that the last result tells us that instead of checking all subsets of $V(C)$ for autarky, it suffices to check the hulls only. Indeed, there can be left no autark set, because, if U is autark also $\sigma_C(U)$ is. And supposing no autarky hull W is

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autark, then there is no autark set at all, because otherwise its hull must have been checked positive for autarky. Therefore, we have a problem with a closure property of the second kind.

An autarky hull is called *free* (cf. [8]) if it does not contain any subhull.

Lemma 5 ([8]): All free hulls of C are 1-hulls. There exists at most $|V(C)|$ free hulls of C.

Free hulls have the computational property:

Lemma 6 ([8]): A free hull $U \subset V(C)$ can be checked for autarky in linear time.

Let $k \in \mathbb{N}$ be fixed, and suppose we were able to identify the instance class $\mathcal{C}(1,k) \subseteq$ CNF defined through the requirement that every $C \in \mathcal{C}(1,k)$ exhibits at most k 1-hulls. Then by Thm. 2 we would be able to achieve an FPT characterization for autarky testing in this class if one could check each 1-hull for autarky in FPT-time (not just the free 1-hulls). It is a future work task to investigate this question.

C. A variant of the shadow problem

Next we consider a specific problem from algorithmic graph theory, for basic notions on that topic cf. e.g. [4]. The problem is called the *shadow (independent set) problem* (SIS), and represents a falsifiability problem from propositional logic in terms of graph theory. SIS is given as follows:

Input: Finite forest F and a function, $f : L(F) \to V(F)$ from the set of all leaves into the set of all vertices of F, called the *shadow map*

Question: Does exist a set of $|F|$ many leaves, exactly one from each tree (called *transversal*) that are *mutually shadow independent*?

In this context, let the *shadow of a leaf* ℓ be the set of all leaves in the subtree rooted at the vertex $f(\ell)$. Then two leaves ℓ_1, ℓ_2 are called *shadow-independent* iff ℓ_1 is no element of the shadow of ℓ_2 and vice versa; this notions transfers directly to every set of leaves. SIS stems from the falsifiability problem of pure implicational formulas in propositional logic and is NP-complete.

A parameterized version of SIS was defined in [3] where the parameter is the number of trees in the forest F: For fixed $k \in \mathbb{N}$ let $\mathrm{SIS}_k := \{(F, f) \in \mathrm{SIS}: |F| \leq k\}$. Also a first FPT characterization of the time complexity of SIS_k was achieved in [3]. A certain improvement of this complexity bound was achieved in [5]. However, for SIS_k so far no kernelization has been constructed explicitly.

Now, in [9] a problem variant is proposed for which an FPT-bound with kernelization was provided. In fact this bound is based on the maximum closure parameter for a specific closure property which can be identified as follows: Consider the set $\hat{L}$ of all leaves in the forest which occur in at least one shadow. Then a closure operator σ on $2^{\hat{L}}$ is defined as follows: For $S \subseteq \hat{L}$ s.t. all leaves of S lie in the same tree, let $\sigma(S)$ be the set of leaves in the largest shadow containing the whole set S; in [9] this largest tree is called envelope. If S is distributed over more than one tree. Then $\sigma(S)$ is defined as the union over all these trees of the leaves in every envelope containing the corresponding fragment of S in the tree.

On basis of this closure property one can define a parameter resting on the maximum hull cardinality bound:

$$s_f := \max_{S \in 2^{\hat{L}}} |\sigma(S)|$$

In fact this parameter corresponds to the largest shadow in F generated by f. The corresponding vector-parameter $\kappa := (k, s) \in \mathbb{N}^2$ is a pair, and one arrives at:

Definition 3:

$$\mathrm{SIS}_\kappa := \{(F, f, \kappa) : (F, f) \in \mathrm{SIS}, |F| \leq k, s_\sigma \leq s\}$$

As in SIS_k the parameter component k controls the forest F, whereas the second parameter component s controls the shadow map f

The next result from [9] states that the problem variant SIS_κ is in the class FPT w.r.t. κ and moreover has a bound of the kernel form:

Theorem 4 ([9]): Whether $(F, \sigma, \kappa = (k, s)) \in \mathrm{SIS}_\kappa$ can be decided in time $O(n^2 + [s\rho(k, s)]^3 3^k)$, where $n = |V(F)|$ and $\rho(k, s) = k^2(s + 1)$.

Therefore the maximal hull parameterization indeed enables one to construct a kernelization.

V. Concluding Remarks and Open Problems

We introduced the class of combinatorial problems having a closure property defined by the existence of a closure operator on the corresponding search space. We proposed two types of *structural* parameterization which possibly could help to gain an FPT complexity characterization for such problems which are NP-hard. A guideline of how this could be achieved was outlined. Moreover we discussed three example problems and examined their closure property as well as it could be used to decrease time bounds or to obtain FPT improvements, respectively.

There are left several open questions for future work: Does there exist a connection between the "traditional" parameterization of problems with closure porperty and the structural one? And/or, does it make sense to consider vector parameterizations: traditional and structural parameters simultaneously? Can one find other relevant examples for problems with closure properties.

Finally, for the two example problems discussed first in this paper, the question of FPT-characterization based on the image cardinality of the corresponding closure operator remained open and should be addressed by future work.

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