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Characteristics of Multimode Fibers at High Frequency Region using Rayleigh Distribution

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***Abstract* – Multimode fiber (MF) has recently been considered as a high data rate medium for signals transmitted at its high frequency region. This high data rate transmission is made possible since there are many many passbands, which can be adopted as channels for sending many subcarrier signals. These passbands are located at different frequencies and vary from fiber to fiber. These different frequencies are determined from the delays of guided modes in the fiber. The passbands at high frequency region of multimode fiber are studied in this paper. Rayleigh distribution is used to statistically model the delays of guided modes. The characteristics in terms of the average magnitude response and the average bandwidth of these passbands are considered. It is found that the only factor that determines the average magnitude response of the passbands is the number of guided modes. Additionally, the average bandwidth of these passbands depends on the standard deviation of the delays.**

***Index Terms* – Multimode Fiber, Rayleigh Distribution, Uniform Distribution, Bandpass Bandwidth**

I. INTRODUCTION

The only medium that can carry a very high data rate signal with an acceptable bit-error-rate (BER) is optical fibers. There are two major types of optical fibers used in the transmission system; that is, single-mode fiber and multimode fiber [1]. These two are mainly different in terms of the number of guided modes supported by the fiber. One major disadvantage of multimode fiber is that it can carry quite low data rate signal comparing to the single-mode fiber. The capacity of multimode fiber is normally in the range of 300 to 500 MHz-km. It means that if a signal with a data rate of 10 Gbps is transmitted over a multimode fiber, the achievable distance has to be less than 50 m, which is not quite useful in practice. However, it has been found by many groups of researchers [2 -5] that multimode fibers can be carried a signal with much higher data rate if the signal is transmitted over the high frequency range of multimode fiber. The limit in terms of the data rate and the distance of multimode fiber at high frequency region has also been shown in [6].

Considering the frequency response of multimode fiber, it has been shown that the frequency response of multimode fiber at frequencies much higher than the 3-dB intermodal bandwidth is relatively flat with amplitudes of 6 to 10 dB below the amplitude at zero frequency [2, 4, 5]. The advantage of these high frequencies is that they can be used as a medium to transmit a high bit rate signal. In [5] it has been shown that a high bit rate signal (up to 1.45 Gbps) can be transmitted over a distance of 5 km using subcarrier multiplexing (SCM) with multimode fiber (MF) system. Since the frequency response of multimode fiber at these high frequencies is not totally flat, there are many nulls within this passband region. If some subcarriers are located at deep nulls, the received signals from those subcarriers will be degraded significantly. The effects of having some subcarriers located at deep nulls can be overcome by using a technique called "Diversity coding" [5]. Considering the multimode fiber model shown in [4], it was assumed that the delays of guided modes are uniformly distributed. This might not be a correct distribution in practice since the guided modes travel along the fiber with different paths and the probability that each mode reaches the receiving end should not be uniformly distributed. Rayleigh distribution is more suitable for being used as the statistical model and will be adopted in this paper.

The organization of this paper is as followed. In Section II, the multimode fiber model will be given. Comparisons between uniform and Rayleigh distributions in being the statistical model for the delays will be shown. In Section III, the average magnitude response of multimode fiber at high frequency region is studied. And, the average bandwidth of passband regions is discussed in Section IV. Finally, in Section V, the conclusion of the work in this paper is given.

II. MULTIMODE FIBER MODEL

As known commonly that there are many guided modes supported by the multimode fiber, the delays corresponding to these modes are then different as they reach the receiving end. The impulse response of the multimode fiber with N_{mode} guided modes can be considered as the combination of the delta functions corresponding to different delays ($t_{d,n}$) [4, 5]; that is,

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$$h_{fiber}(t) = \sum_{n=1}^{N_{\text{mode}}} \delta\left(t - t_{d,n}\right) \quad (1)$$

Taking Fourier transform of (1), the frequency response of the complex envelope of the multimode fiber with N_{mode} guided modes is achieved; that is,

$$H_{fiber}(f) = \sum_{n=1}^{N_{\text{mode}}} e^{-j2\pi f \cdot t_{d,n}} \quad (2)$$

From (2), in order to get the frequency response of the multimode fiber, the delays for all N_{mode} guided modes have to be found; that is, the modal index of each guided mode has to be found. However, for practical multimode fibers the number of guided modes has to be large to avoid modal noise. It has been shown [4, 5] that these time delays can be modeled to be independent realizations of a random variable, which is uniformly distributed about an average time delay ($t_{d,avg}$) with the maximum deviation of $t_{d,dev}$ as shown in (3).

$$f_{t_{d,n}}\left(t_{d,n}\right) = \begin{cases} \dfrac{1}{2t_{d,dev}} & ;\ t_{d,avg} - t_{d,dev} \le t_{d,n} \le t_{d,avg} + t_{d,dev} \\ 0 & ;\ \text{elsewhere.} \end{cases} \quad (3)$$

Note that $t_{d,avg}$ and $t_{d,dev}$ depend on the fiber length, the number of guided modes, and the refractive index profile of the fiber.

Applying the maximum time deviation of 5 ns, and number of guided modes to be 100 modes; the frequency response of the complex envelope of the multimode fiber in units of dB is shown Fig.1.

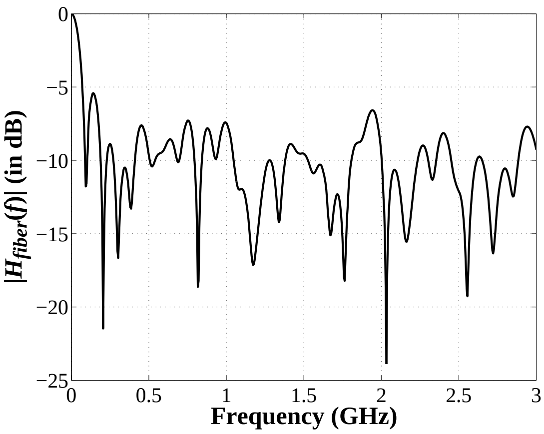

Fig. 1. Magnitude response of the complex envelope of the multimode fiber modeled with uniform distribution, N_{mode} = 100 and $t_{d,dev}$ = 5 ns.

From Fig.1, it is seen that if the signal is transmitted over the 3-dB intermodal bandwidth of this fiber, the achieved data rate will be approximately 100 Mbps since the 3-dB bandwidth is roughly 100 MHz. However, it is seen that there are many passbands at high frequencies, which can be used to carry many signals; for example, at 0.45 GHz, 0.65 GHz, 0.75 GHz, 1.0 GHz, and so on. Combining data rates from these passbands, the total data rate that can be achieved will be significantly higher than the data rate achieving only from the 3-dB intermodal bandwidth. Nevertheless, as discussed previously, the uniform distribution in (3) might not be a suitable distribution for being used. Next, the delay ($t_{d,n}$) is assumed to be Rayleigh distributed [7] with two important parameters; σ and $t_{d,start}$ (the starting time delay); as shown in (4).

$$f_{t_{d,n}}\left(t_{d,n}\right) = \begin{cases} \dfrac{\left(t_{d,n} - t_{d,start}\right)}{\sigma^2} e^{\left(\frac{-\left(t_{d,n} - t_{d,start}\right)^2}{2\sigma^2}\right)} & ;\ t_{d,n} \ge t_{d,start} \\ 0 & ;\ \text{otherwise} \end{cases} \quad (4)$$

From (4), the average time delay ($t_{d,avg}$) and the delay standard deviation (σ_d) can be achieved by

$$t_{d,avg} = t_{d,start} + \sigma\sqrt{\frac{\pi}{2}} \text{ and } \sigma_d = \sigma\sqrt{2 - \frac{\pi}{2}} \quad (5)$$

Applying the Rayleigh distribution to the delay, σ_d has to be determined. Here, the value of $t_{d,dev}$ used in Fig.1, which is from uniform distribution, is translated to the standard deviation of 2.89 ns. Hence, σ_d of 2.89 ns with 100 guided modes are applied to (4) and (2), respectively. The frequency response of the complex envelope of the multimode fiber using Rayleigh distribution is shown Fig.2.

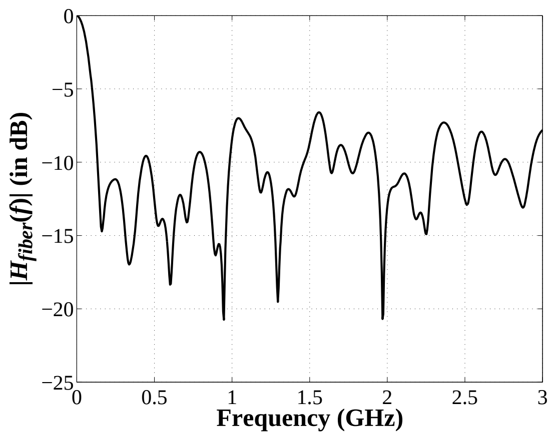

Fig. 2. Magnitude response of the complex envelope of the multimode fiber modeled with Rayleigh distribution, N_{mode} = 100 and σ_d = 2.89 ns.

Considering Fig.2, it is seen that there are also many available passbands at high frequency region. Comparing this figure to Fig.1, it is seen that they are only different in terms of the locations of passbands. This difference is from the fact that these two responses are from different sets of delays and different types of delay distributions. However, there are some similar

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features among these two figures; for example, the 3-dB intermodal bandwidth at low frequency and the passband bandwidth at high frequency are almost identical. Additionally, considering the average magnitude of the high frequency region, both responses have an average magnitude of -10 dB approximately.

Next, the average magnitude at high frequency and the average passband bandwidth will be discussed assuming that the delays are Rayleigh distributed. The results are from the simulation using previous discussion.

III. Average Magnitude Response

The average magnitude response of the multimode fiber at high frequency is determined, in this section. This average magnitude response is quite crucial in practical system since it tells us about how much optical power is required in order to compensate the attenuation. For example, as seen in Fig. 1 and 2 that the average magnitude is approximately -10 dB, the optical signal has to have at least 10 dB in addition to its normal operating power. However, the delay standard deviation used in these two figures is 2.89 ns with 100 guided modes. Varying these two factors should give some ideas about the average magnitude. To further study about the factors affecting the average magnitude, different magnitude responses each with different parameters; that is, the number of modes (N_{mode}) and delay standard deviation (σ_d) are determined.

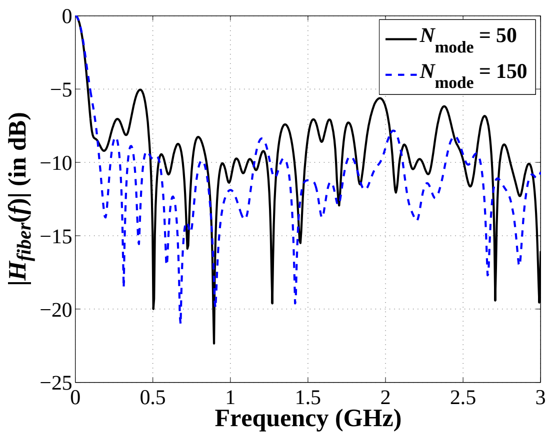

Fig. 3. Magnitude responses of the complex envelope of the multimode fiber modeled with Rayleigh distribution, N_{mode} = 50 (solid line) and 150 (dash line); and σ_d = 2.89 ns.

In Fig. 3, the magnitude responses of multimode fiber with 50 and 150 guided modes are shown in Fig.3. These two responses are from the same σ_d of 2.89 ns. With 50 guided modes, at high frequency, the average magnitude response of MF is clearly higher than that from the case of 150 guided modes. It can be roughly led to a conclusion that the average magnitude response of MF depends on the number of guided modes. However, the effect of the delay standard deviation has not been obtained. In Fig.4, the average magnitude response of MF at high frequency region is given and shown as a function on N_{mode} with different values of σ_d.

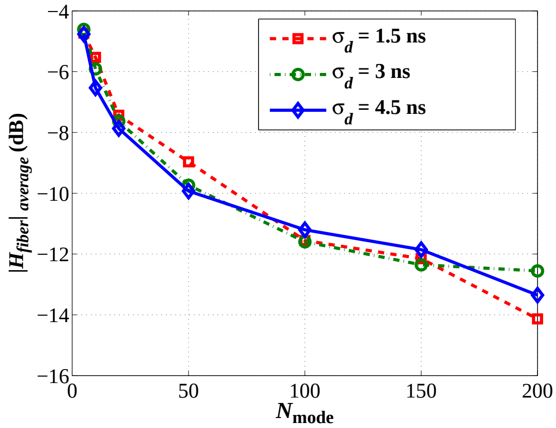

Fig. 4. Average magnitude response multimode fiber at high frequency region as a function of number of guided modes with σ_d = 1.5, 3.0, and 4.5 ns.

From Fig.4, it is clearly shown that there is no significant difference between three curves with different values of σ_d. It is also seen that the average magnitude response of MF at high frequency region depends solely on the number of guided modes; that is, as the number of guided modes is increased, the average magnitude response is lessened. For example, at N_{mode} = 50, 100, 150; the average magnitude responses are -9.5, -11.5, and -12 dB, respectively.

IV. Average Bandpass Bandwidth

In this section, another parameter namely the average bandpass bandwidth of MF at high frequency region is studied. From Fig. 2 it is seen that the bandwidth of these passbands is approximately 50 to 200 MHz depending on the chosen passband. However, the study on the average bandpass bandwidth has not been discussed. This average bandwidth is also another important parameter to be used in data transmission since it tells us how large the data rate can be carried by each passband. Considering Fig.2, it is seen that the number of guided modes and the delay standard deviation are fixed. In this section, the effects of these two factors on the average bandpass bandwidth are determined.

The effect of the delay standard deviation on the average bandpass bandwidth is studied and shown in Fig.5. It is shown that the average bandpass bandwidth from the case of σ_d = 1.5 ns is larger than that from the case of σ_d = 3.0 ns. Hence, the delay standard deviation must have an effect on the average bandpass bandwidth. The effect of number of guided modes cannot be obtained from this figure since the number of guided modes is fixed to be 100 modes.

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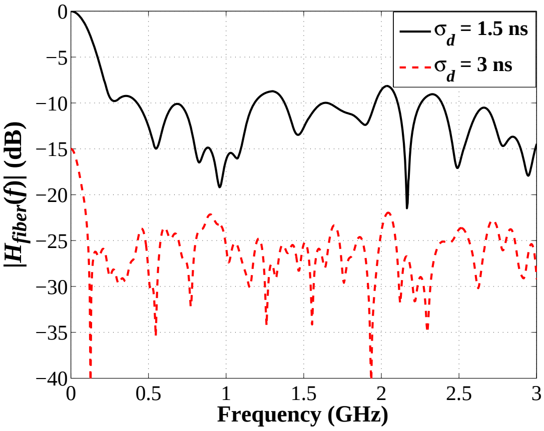

Fig. 5. Magnitude responses of the complex envelope of the multimode fiber modeled with Rayleigh distribution with N_{mode} = 100. The two curves correspond to different values of σ_d, with the lower curve displaced by 15 dB.

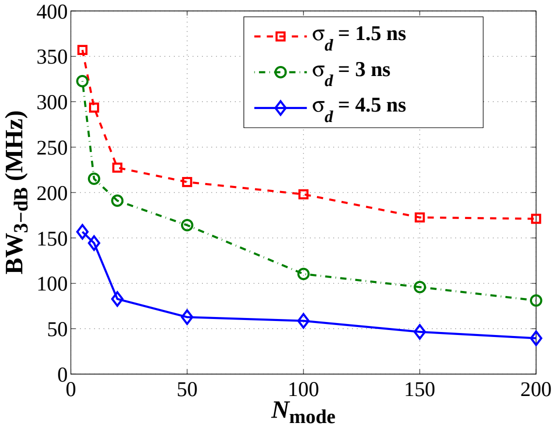

Fig. 6. Average bandpass bandwidth of multimode fiber at high frequency region as a function of number of guided modes with σ_d = 1.5, 3.0, and 4.5 ns, respectively.

In Fig. 6, the effect of guided modes on the average bandpass bandwidth is given. It should be noted that the average bandpass bandwidth is determined from the average of many 3-dB bandpass bandwidths. It is seen that the number of guided modes strongly affect the average bandpass bandwidth if the number of guided modes is small; that is, in the range of 50 guided modes or smaller. However, in practice, the number of guided modes in MF is much higher than 50 modes, hence, the effect from this range of number of guided modes can be disregarded. And, as the number of guided modes increases, it slightly affects the achieved bandpass bandwidth; for example, with σ_d = 4.5 ns, the average bandpass bandwidths for N_{mode} = 50, 100, and 150, are approximately 50 MHz.

Furthermore, from Fig. 6, at σ_d = 1.5, 3.0, and 4.5 ns, the average bandpass bandwidths are approximately 180, 100, and 50 MHz, respectively. It is seen that σ_d affects the average bandpass bandwidth.

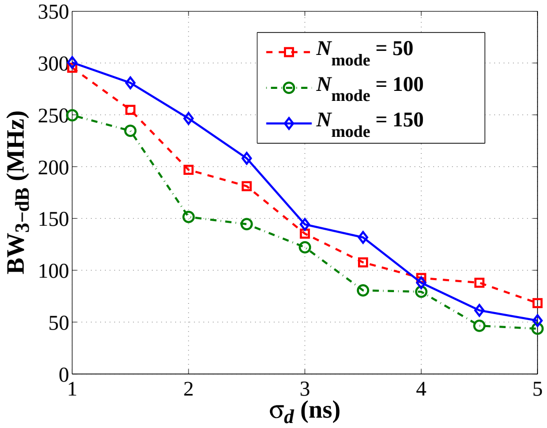

Fig. 7. Average bandpass bandwidth of multimode fiber at high frequency region as a function of delay standard deviation (σ_d) with N_{mode} = 50, 100, and 150 modes, respectively.

The effects of N_{mode} and σ_d on the average bandpass bandwidth can be viewed as shown in Fig.7. It is seen that the average bandpass bandwidth does not strongly depend on the number of guided modes but mainly depends on the value of σ_d. As the value of σ_d is increased, the average bandpass bandwidth is decreased. For example, at σ_d of 2, 3, and 4 ns, the average bandpass bandwidth are 200, 140, and 90 MHz, respectively. Note that in practice, the value σ_d of depends on the length of the multimode fiber. If the fiber length increases, the value of σ_d becomes higher resulting in a smaller bandpass bandwidth.

V. Conclusions

Rayleigh distribution has been discussed and compared to the previous study in which the uniform distribution was adopted. It is found that Rayleigh distribution is more suitable for being used as the delay distribution in multimode fiber. Adopting Rayleigh distribution to the delays, the average magnitude response and the average bandpass bandwidth at high frequency region of multimode fiber have been studied. It is found that the average magnitude response is inversely proportional to the number of guided modes. And, the only factor that determines the average bandpass bandwidth is the delay standard deviation. The output of this study can be utilized in the real system for estimating the loss (attenuation) from transmitting signals over high frequency region of multimode fiber. Additionally, relating the delay standard deviation to the fiber length, the data rate in which each passband can be carried can be obtained.

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