Efficiently Fuzzy Controlling with Dynamic Window in Elliptic Curve Cryptography Sensor Networks

Xu Huang, Dharmendra Sharma and Pritam Gajkumar Shah

***Abstract*—It is noted that the wireless sensor networks (WSN) based on the rapid progress of wireless communications and embedded micro electro mechanical systems technologies are becoming important part in our daily life. However, the security of the WSN becomes one of the major concerns in its applications. Even elliptic curve cryptography (ECC) prominently offers great potential benefits for WSN security there is still a lot of work needs to be done due to WSN has very restraint running conditions such as limited energy source, capability of computing, etc. Also it is well known that scalar multiplication operation in ECC accounts for about 80% of key calculation time on wireless sensor network motes. In our current paper we present an optimized dynamic window based on our previous research works. The whole quality of service (QoS) has been improved under this algorism in particularly the power consuming is more efficiently. The simulation results showed that the average calculation time, due to fuzzy conditions decreased from previous 26 to current 9 as a whole the calculation time, decreased by approximately 17% in comparison to our previous algorithms in an ECC wireless sensor network.**

***Index Terms*— Elliptic curve cryptography (ECC), scalar multiplication, non-adjacent form, slide window, fuzzy**

I. Introduction

THE high demand for various applications shows the fact that the rapid progress of wireless communications has become popular in our daily life. With the growth in very large scale integrated (VLSI) technology, embedded systems and micro electro mechanical systems (MEMS) has enabled production of inexpensive sensor nodes, which can transit data over a distances with free media and efficient use of power [1, 22, 23]. In the WSN systems, the sensor node will detect the interested information, processes it with the help of an in-built microcontroller and communicates results to a sink or base station. Normally the base station is a more powerful node, which can be linked to a central station via satellite or internet communication to form a network. There are many deployments for wireless sensor networks depending on various applications such as environmental monitoring e.g. volcano detection [2,3], distributed control systems [4], agricultural and farm management [5], detection of radioactive sources [6], and computing platform for tomorrows' internet [7]. Generally speaking, a typical WSN architecture can be shown in Figure 1.

Contrast to traditional networks, a wireless sensor network normally has many resource constraints [4] due to the limited size. As an example, the MICA2 mote consists of an 8 bit ATMega 128L microcontroller working on 7.3 MHz. As a result nodes of WSN have limited computational power. Radio transceiver of MICA motes can normally achieve maximum data rate of 250 Kbits/s, which restricts available communication resources. The flash memory that is available on the MICA mote is only 512 Kbyte. Apart from these limitations, the onboard battery is 3.3.V with 2A-Hr capacity. Therefore, the above restrictions with the current state of art protocols and algorithms are expensive for sensor networks due to their high communication overhead.

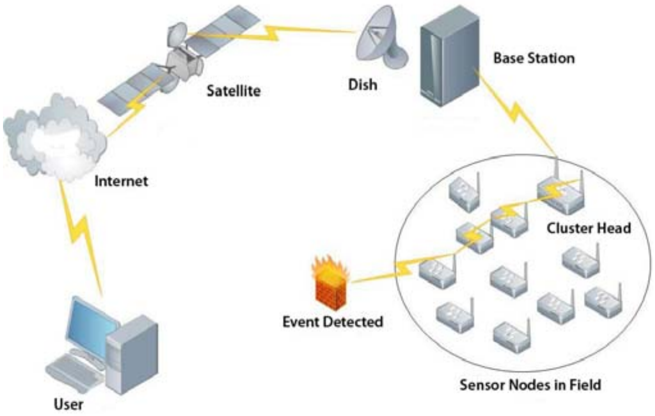

Figure 1: A Typical WSN architecture

Recalled that the Elliptic Curve Cryptography was first introduced by Neal Koblitz [9] and Victor Miller [10] independently in the early eighties. The advantage of ECC over other public key cryptography techniques such as RSA, Diffie-Hellman is that the best known algorithm for solving elliptic curve discrete logarithm problem (ECDLP) which is the underlying hard mathematical problem in ECC which will take the fully exponential time. On the other hand the best algorithm for solving RSA and Diffie-Hellman takes sub exponential time [11]. In summary, the ECC problem can only be solved in exponential time and, to date, there is a lack of sub exponential methods to attack ECC.

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An elliptic curve E over $GF(p)$ can be defined by $y^2 = x^3 + ax + b$ where $a, b \in GF(p)$ and $4a^3 + 27b^2 \neq 0$ in the $GF(p)$.

The point (x, y) on the curve satisfies the above equation and the point at infinity denoted by ∞ is said to be on the curve.

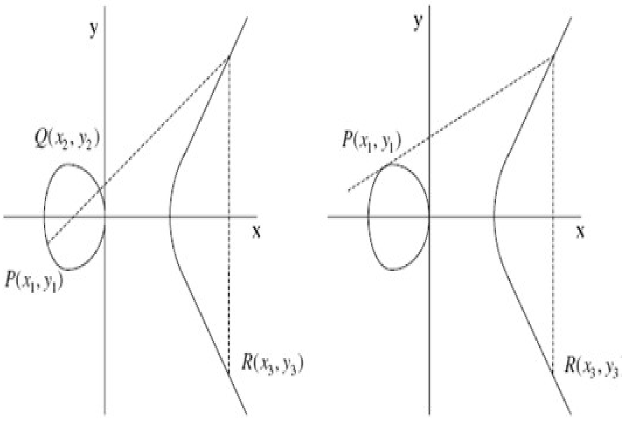
If there are two points on the curve namely, $P\ (x_1, y_1)$, $Q\ (x_2, y_2)$ and their sum is given by point $R(x_3, y_3)$ the algebraic formulas for point addition and point doubling are given by following equations:

We have: $x_3 = \lambda^2 - x_1 - x_2$

$$y_3 = \lambda(x_1 - x_3) - y_1$$

$$\lambda = \frac{y_2 - y_1}{x_2 - x_1}, \quad \text{if } P \neq Q$$

$$\lambda = \frac{3x^2 + a}{2y_1}, \quad \text{if } P = Q$$



(a) Addition: $P + Q = R$ (b) Doubling: $P+P = R$
Figure 2: Point addition and point doubling on elliptic curve.

Here, the addition, subtraction, multiplication and inverse are the arithmetic operations over $GF(p)$, which can be shown in Figure 2.

II. Elliptic Curve Diffie-Hellman Scheme (ECDH) Proposed for WSN

Before we get into our innovation method, we need to have a closer look at the popular legacy scheme for WSN. As per [13] the original Diffie-Hellman algorithm with RSA requires a key of 1024 bits to achieve sufficient security but *Diffie Hellman based on ECC* can achieve the same security level with only 160 bit key size.

The classical Elliptic Curve Diffie Hellman scheme operates as shown in the Fig. 3

Initially Alice and Bob agree on a particular curve with base point P. They generate their public keys by multiplying P with their private keys namely K_A and K_B. After sharing public keys, they generate a shared secret key by multiplying public keys by their private keys. The secret key is $R = K_A{*}Q_A = K_B{*}Q_B$. With the known values of Q_A, Q_B and P, it is computationally intractable for an eavesdropper to calculate K_A and K_B which are the private keys of Alice and Bob. As a result, adversaries cannot calculate R the shared secret key.

In ECC two heavily used operations are involved: scalar multiplication and modular reduction. Gura *et. al.* [14] showed that 85% of execution time is spent on scalar multiplication. Scalar Multiplication is the operation of multiplying point P on an elliptic curve E defined over a field $GF(p)$ with positive integer k which involves point addition and point doubling. Operational efficiency of kP is affected by the type of coordinate system used for point P on the elliptic curve and the algorithm used for recoding of integer k in scalar multiplication.

This research paper proposes an innovative algorithm based on one's complement for representation of integer k which accelerates the computation of scalar multiplication in wireless sensor networks.

The number of point doubling and point addition operations in scalar multiplication depends on the recoding of integer k. Expressing integer k in binary format highlight this dependency.

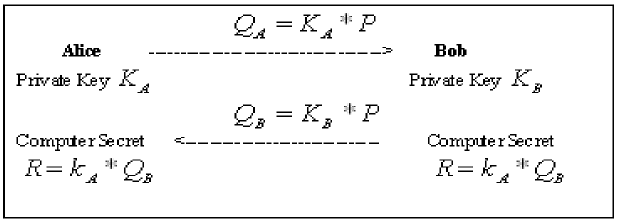


Figure 3: Diffie-Hellman protocol based on ECC

The number of zeros and number of ones in the binary form, their places and the total number of bits will affect the computational cost of scalar multiplications. The Hamming weight as represented by the number of non-zero elements, determines the number of point additions and bit length of integer K determines the number of point doublings operations in scalar multiplication.

One point addition when $P \neq Q$ requires one field inversion and three field multiplications [13]. Squaring is counted as regular multiplication. This cost is denoted by $1I + 3M$, where I denotes the cost of inversion and M denotes the cost of multiplication.

One point doubling when $P = Q$ requires $1I + 4M$ as we can neglect the cost of field additions as well as the cost of multiplications by small constant 2 and 3 in the above formulae.

Binary Method

Scalar multiplication is the computation of the form $Q = kP$, where P and Q are the elliptic curve points and k is positive integer. This is obtained by repeated elliptic curve point addition and doubling operations. In binary method the integer k is represented in binary form:

$$k = \sum_{j=0}^{l-1} K_j 2^j,\ K_j \in \{0,1\}$$

The cost of multiplication when using binary method depends on the number of non-zero elements and the length of the binary representation of k. If the representation has $k_{l-1} \neq 0$ then binary method require $(l - 1\ )$ point doublings and $(W{-}1)$ where l is the length of the binary expansion of k, and W is the Hamming weight of k (i.e., the number of non-zero elements in expansion of k). For example, if $k = 629 = (1001110101)_2$, it will require $(W{-}1) = 6 - 1 = 5$ point additions and $l - 1 = 10 - 1 = 9$ point doublings operations.

The binary method scans the bits of K either from left-to-right or right-to-left. The binary method for the computation of kP is given in the following *algorithm 1*, as shown below:

Algorithm 1: Left to right binary method for point multiplication

```
Input: A point $P \in E(F_q)$, an
        $k = \sum_{j=o}^{l-1} K_j 2^j, \ K_j \in \{0,1\}$
$l$ bits integer
Output: $Q = kP$
    1. $Q \leftarrow \infty$
    2. For $j = l$ □ 1 to 0 do:
        2.1 $Q \leftarrow 2Q$,
        2.2 if $k_j = 1$ the $Q \leftarrow Q + P$.
    3. Return Q.
```

Signed Digit Representation Method

The subtraction has virtually the same cost as addition in the elliptic curve group. The negative of point (x, y) is $(x, -y)$ for odd characters. This leads to scalar multiplication methods based on addition–subtraction chains, which help to reduce the number of curve operations. When integer k is represented with the following form, it is a *binary signed digit representation*.

$$k = \sum_{j=0}^{l} S_j 2^j, \ \ S_j \in \{1,0,-1\}$$

When a signed-digit representation has no adjacent non zero digits, i.e. $S_j S_{j+1} = 0$ for all $j \geq 0$ it is called a non-adjacent form (NAF).

The following *algorithm 2* computes the NAF of a positive integer given in binary representation.

Algorithm 2: Conversion from Binary to NAF

```
                $k = \sum_{j=0}^{l-1} K_j 2^j, \ K_j \in \{0,1\}$
Input: An integer
                $k = \sum_{j=0}^{l} S_j 2^j, \ \ S_j \in \{1,0,-1\}$
Output: NAF
    1. $C_0 \leftarrow 0$
    2. For $j = 0$ to $l$ do:
    3. $C_{j+1} \leftarrow [(K_j + K_{j+1} + C_j)/2]$
    4. $S_j \leftarrow K_j + C_j - 2C_{j+1}$
    5. Return $(S_l \ldots S_0)$
```

NAF usually has fewer non-zero digits than binary representations. The average hamming weight for NAF form is $(n - 1)/3.0$. So generally it requires $(n - 1)$ point doublings and $(n - 1)/3.0$ point additions. The binary method can be revised accordingly and is given another algorithm for NAF, and this modified method is called the *Addition Subtraction* method.

III. Dynamitic Window with Fuzzy Controller in ECC Proposed Algorithm Based

We are going to use the algorithm based on *subtraction by utilization of the 1's* complement is most common in binary arithmetic. The 1's complement of any binary number may be found by the following equation [19-22]:

$$C_1 = (2^a - 1) - N \tag{1}$$

where C_1 = 1's complement of the binary number, a = number of bits in N in terms of binary form, N = binary number

From a closer observation of the equation (1), it reveals the that any positive integer can be represented by using minimal non-zero bits in its 1's complement form provided that it has a minimum of 50% Hamming weight. The minimal non-zero bits in positive integer scalar are very important to reduce the number of intermediate operations of multiplication, squaring and inverse calculations used in elliptical curve cryptography as we have seen in previous sections.

The equation (1) can therefore be modified as per below:

$$N = (2^a - C_1 - 1) \tag{2}$$

For example, we may take N =1788 then it appears

N= $(11011111100)_2$ in its binary form

C_1= 1's Complement of the number of N= $(00100000011)_2$

a is in binary form so we have $a = 11$

After putting all the above values in the equation (2) we have:

$1788 = 2^{11} - 00100000011 - 1$, this can be reduced as below:

$$1788 = 100000000000 - 00100000011 - 1 \tag{3}$$

So we have

$1788 = 2048 - 256 - 2 - 1 - 1$

As is evident from equation (3), the Hamming weight of scalar N has reduced from 8 to 5 which will save 3 elliptic curve addition operations.

The above recoding method based on one's complement subtraction combined with sliding window method provides a more optimized result.

Let us compute [763] P (in other words $k = 763$) as an example, with a *sliding window algorithm* with K recoded in binary form and window sizes ranging from 2 to 10. It is observed that as the window size increases the number of pre-computations also increases geometrically. At the same time number of additions and doubling operations decrease. Now we present the details for the different window size to find out the optimal window size using the following example:

Window Size w = 2

$763 = (1011111011)_2$

No of precomputations = $2^w - 1 = 2^2 - 1 = [3]\,P$

$763 = \underline{10}\ \underline{11}\ \underline{11}\ \underline{10}\ \underline{11}$

The intermediate values of Q are

P, 2P 4P, 8P, 11P, 22P, 44P, 47P, 94P, 95P, 190P, 380P, 760P, 763P

Computational cost = 9 doublings, 4 additions, and 1 pre-computation.

We continue to derive the remaining calculations for Window Size *w* = 6, Window Size *w* = 7, Window Size *w* = 8, Window Size *w* = 9, and Window Size *w* = 10. The results for all calculations are presented in Table 1.

The effects of "doublings" and "additions" as shown in Table 1 are further considered in the next section.

Algorithm for sliding window scalar multiplication on elliptic curves.

```
$Q \leftarrow P_\infty$ and $i \leftarrow l - 1$
while $i \geq 0$ do
    if $n_i = 0$ then $Q \leftarrow [2]Q$ and $i \leftarrow i - 1$
    else
        $s \leftarrow \max(i - k + 1, 0)$
        while $n_s = 0$ do $s \leftarrow s + 1$
        for $h = 1$ to $i - s + 1$ do $Q \leftarrow [2]Q$
        $u \leftarrow (n_i \ldots\ldots n_s)_2$ $[n_i = n_s = 1$ and $i - s + 1 \leq k]$
        $Q \leftarrow Q \oplus [u]P$ [u is odd so that [u]P is precompute d]
        $i \leftarrow s - 1$
return Q
```

TABLE 1
WINDOW SIZE VS NO OF DOUBLINGS, ADDITIONS AND PRE COMPUTATIONS

Window Size	No of Doublings	No of Additions	No of Pre computations
2	9	4	1
3	7	3	3
4	6	2	7
5	5	1	15
6	4	1	31
7	3	1	61
8	3	1	127
9	1	1	251
10	0	0	501

It is interesting to note that Figure 4 shows the charts obtained from table 1(in order to see the relation clearly, we used so-called amplified chart because of the range is only limited 0 to 10 in window size). As we would like to know the further details about the relations between "Doubling" and "Addition", from Figure 4 it clearly shows that the "Pre-computation" and "doubling" are the dominated factors hence we need to focus on this factor in our discussion of window size.

IV. FUZZY CONTROLLER SYSTEM IN ECC

It is clear, from above description that there is a tradeoff between the computational cost and the window size as shown in Table 1. However, this tradeoff is underpinned by the balance between computing cost (or the RAM cost) and the pre-computing (or the ROM cost) of the node in the network.

It is also clear that, from above description that the variety of wireless network working states will make this control complex and calculations could be relatively more expensive.

Therefore, we propose a fuzzy dynamic control system, to provide dynamic control to ensure the optimum window size is obtained by tradeoff between pre-computation and computation cost.

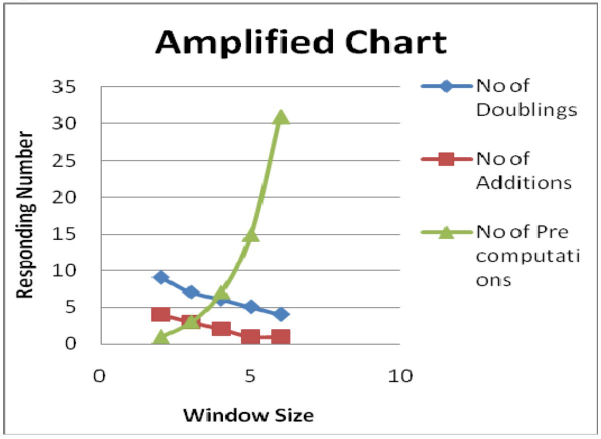


Fig. 4: amplified chart (focus on limited range) for the Table 1

The fuzzy decision problem introduced by Bellman and Zadeh has as a goal the maximization of the minimum value of the membership functions of the objectives to be optimized. Accordingly, the fuzzy optimization model can be represented as a multi-objective programming problem as follows [21]:

$$Max : \min\{\mu_s(D)\} \,\&\, \min\{\mu_l(U_l)\} \quad \forall s \in S \,\&\, \forall l \in L$$

$$\text{such that} \quad A_l \leq C_l \qquad \forall l \in L,$$

$$\sum\nolimits_{r \in R_P} x_{rs} = 1 \qquad \forall p \in P \,\&\, \forall s \in S,$$

$$x_{rs} = 0 \ \text{ or } \ 1 \qquad \forall r \in R \,\&\, \forall s \in S$$

In above equation, the objective is to maximize the minimum membership function of all delays, denoted by *D*, and the difference between the recommend value and the measured value, denoted by *U*.

The Fuzzy control system is extended from and shown in Figure 5. For accurate control, we designed a three inputs fuzzy controller. The first input is *storage room*, which has three statuses, showing storage room in one of the three, namely (a) low, (b) average, and (c) high. The second input is pre-computing working load (*PreComputing*) in one of three states, namely (a) low, (b) average, and (c) high. The third input is *Doubling*, expressing how much working load for the calculation "doubling" which has three cases, namely (a) low, (b) average, and (c) high. The output is one, called *WindowSize,* to express the next window size should be moved in which way, which has three states for the window sizes, namely (a) down, (b) stay, and (c) up.

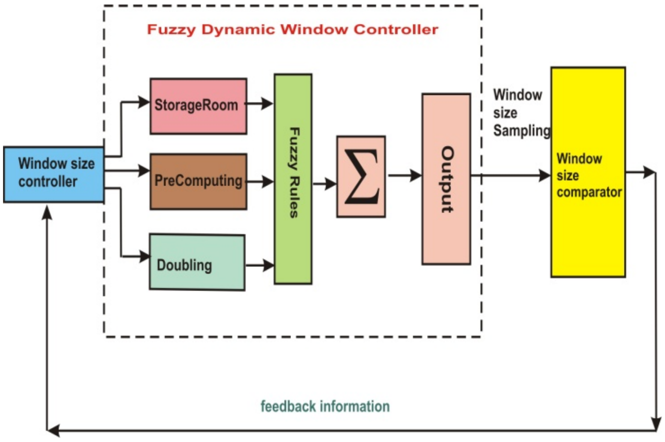


Figure 5: Three inputs fuzzy window control system

There are only 9 *Fuzzy Rules* listed as follows (weight

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are unit) due to StorageRoom in Figure 5 can be ignored due to the results of Figure 4 although it is a factor needs to be appeared in terms of this control system.

1. If (PreComputing is low) and (Doubling is low) then (WindowSize is Up)
2. If (PreComputing is low) and (Doubling is average) then (WindowSize is Up)
3. If (PreComputing is low) and (Doubling is high) then (WindowSize is stay)
4. If (PreComputing is average) and (Doubling is low) then (WindowSize is Up)
5. If (PreComputing is average) and (Doubling is average) then (WindowSize is Up)
6. If (PreComputing is average) and (Doubling is high) then (WindowSize is stay)
7. If (PreComputing is high) and (Doubling is low) then (WindowSize is Up)
8. If (PreComputing is high) and (Doubling is average) then (WindowSize is stay)
9. If (PreComputing is high) and (Doubling is high) then (WindowSize is stay)

The number at each fuzzy condition with a bracket is the weight number, currently it is unit. Later we shall change it with different number according to the running situations as described in the next. The three inputs with 9 fuzzy rules in *Mamdani model* running fuzzy controller part. The three inputs are *StorageRoom*, *PreComputing* and *Doubling*. The output is *WindowSize*. It is noted if we did not take the advantage of Figure 4, there will be at least 26 fuzzy rules need to be considered as shown in our previous paper [23]. This is because that the "*StorageRoom*" has low, average, high with other two parameters's combinations.

The output with *StorageRoom* and *PreComputing* is shown in Figure 6. The surface *StorageRoom* vs. *Doubling* is shown in Figure 7.

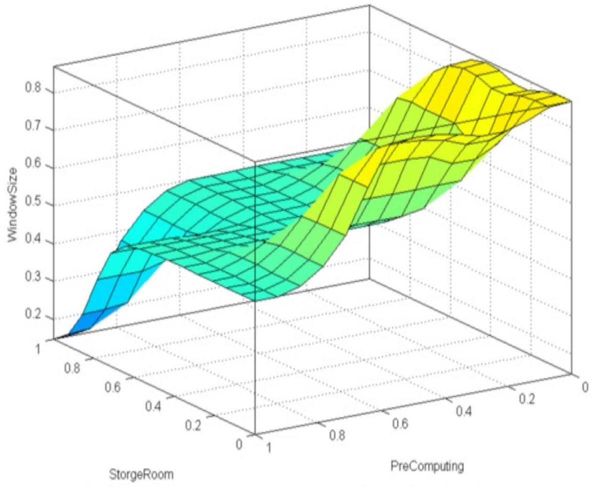

Figure 6: The output of the surface for the *StorageRoom* vs. *PreComputing*.

From above figures, it is clearly observed that in the low window size side, if the storage room is low the dominated function of "doubling" will play role as Figure 6 shown but if the window size is at the high side, the storage room will be fairly stay at the middle either for PreComputing or Doubling, which is the doubling will sharply increased when window size a little bit larger that also can be shown from Table 1.

From Figure 7 it is clearly to show when the storage room is getting big, it would be nice to have larger window size for the "doubling".

Now if we change the weight for above fuzzy rules as such the rules 1,5 10, 13, 14, 15, 16, 18, 20 ,21, 22, 23, 25, and 26 are set in 0.5 (the rest will keep the same) due to the major functions are controlled by the storage room, and doubling will rapidly increasing by the window size larger. The outputs will changed as the average storage room will increased 0.04% and the other two inputs are decreased by 0.02% the output become window staying a little wider side by 0.003%.

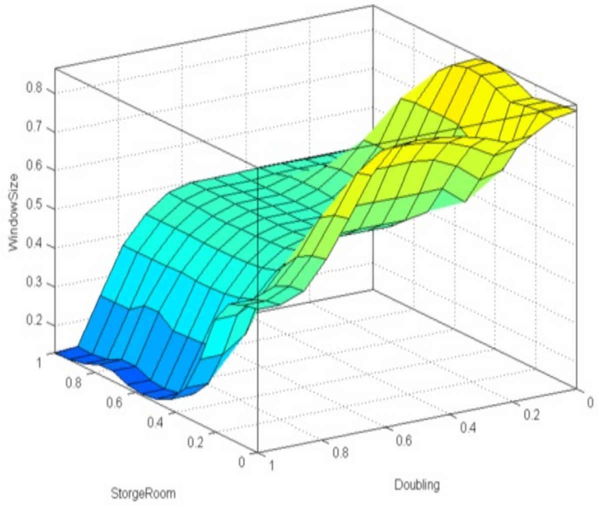

Figure 7: The output of the surface for the *StorageRoom* vs. *Doubling*.

It is clear that this fuzzy controller for the dynamic window is also involved a tradeoff between accuracy and control costs. For example the same system may go further for the second order parameters, not just check the changes about the input variables but also check the change tendencies of the variables, which will be discussed in another paper.

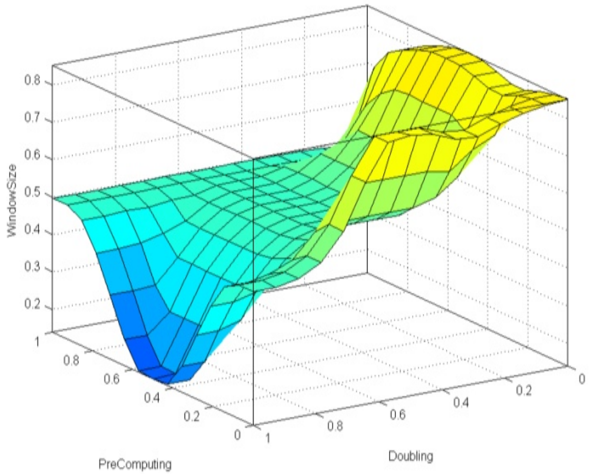

Figure 8: The output of the surface for the *StorageRoom* = constant (0.4) and *PrecComputing* vs. *Doubling*.

If we keep the *StorageRoom* constant, e.g. *StorageRoom* = 0.4, and the output of the "*PreComputing*" vs. "*Doubling*" is shown in Figure 8. Now in order to show the "StorageRoom" is not dominated factor as we discussed above, we took another value, "StorageRoom" = 0.8 and the output of the "*PreComputing*" vs. "*Doubling*" is shown in Figure 9.

It is observed that the output of the "*PreComputing*" vs. "*Doubling*" is not much difference between Figures 8 and 9.

From Figures 6 and 7 we can observe that the “dominated factor” is not the members of the “*StorageRoom*” in Figure 8 due to the fact that the *StorageRoom* goes to large than 40% (or 0.4) the factors *PreComputing* and *Doubling* are dominated in Fig. 6 and Fig. 7 separately. The *StorageRoom* cannot play a role in the process of the dynamic window, which is underpinned by the fact that has been shown in Figures 4. The simulations of the example described in above were implemented. Also it is noted that with equation (2), the computational cost has been reduced from 3 additions as in the binary method to only 1 addition in one’s complement subtraction form. The number of pre-computations has remained the same.

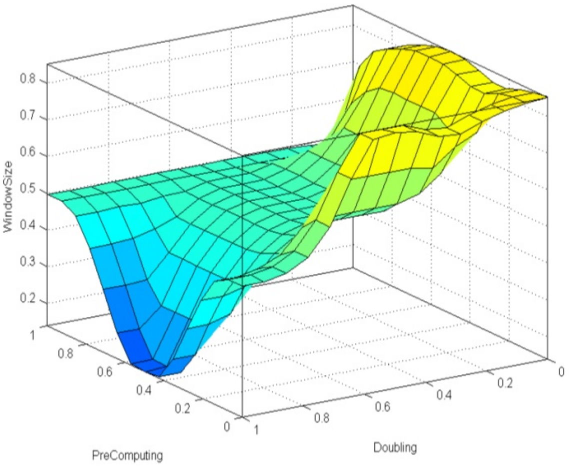

Figure 9: The output of the surface for the *StorageRoom* = constant (0.8) and *PreComputing* vs. *Doubling*.

In our simulations, the proposed method together with a fuzzy window size controller makes the ECC calculation in the current algorism is about 17% more efficient than the methods in [23] with the same QoS level.

V. Conclusion

The positive integer in point multiplication may be recorded with *one’s complement subtraction* to reduce the computational cost involved in this heavy mathematical operation for wireless sensor network platforms. As the NAF method involves modular inversion operation to get the NAF of binary number, the one’s complement subtraction can provide a very simple way of recoding the integer. There is always a decision between pre-computing and computing, the former is related to the storage and the latter is associated with computing capability and capacity. The window size may be the subject of trade-off between the available RAM and ROM at a particular instance on a sensor node, which can be controlled by fuzzy controller. The final simulation in a sensor wireless network shows that about 17% more efficient than our previous method [23] can be obtained with an ECC sensor network.

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