

M_s Based Approach for Simple Robust PI Controller Tuning Design

R. Vilanova, V. Alfaro, O. Arrieta

Abstract—This paper addresses the problem of providing simple tuning rules for a Two-Degree-of-Freedom (2-DoF) PI controller (PI_2) with robustness considerations. In order to deal with the well known performance/robustness tradeoff, an analysis is conducted first that allows the determination of the lowest closed-loop time constant that guarantees a desired robustness. Simple tuning rules are generated by considering specific values for the Maximum Sensitivity value. These tuning rules, provide all the controller parameters parameterized in terms of the open-loop normalized dead-time allowing the user to select a High/Medium/Low robust closed-loop control system.

Index Terms—PI control, Robustness, Process Control

I. INTRODUCTION

Most of the single-loop controllers used in practice are found under the form of a PI/PID controller. Their success is mainly due to its simple structure and meaning of the corresponding three parameters. This fact makes PID control easier to understand by the control engineers than other most advanced control techniques. This fact has motivated a continuous research effort to find alternative tuning and design approaches to improve PI/PID based control system's performance.

Recently, tuning methods based on optimization approaches with the aim of ensuring good stability robustness have received attention in the literature [1], [2]. Also, great advances on optimal methods based on stabilizing PID solutions have been achieved [3], [4]. However these methods, although effective, use to rely on somewhat complex numerical optimization procedures and do not provide tuning rules. Instead, the tuning of the controller is defined as the solution of the optimization problem.

Among the different approaches, the direct or analytical synthesis constitutes a quite straightforward approach to PID controller tuning. With this respect, the usual approach is to specify the desired closed-loop transfer function and to solve analytically for the feedback controller. In cases where the process model is of simple structure, the resulting controller has the PI/PID structure. Most of the analytically developed tuning rules are related with the servo-control problem while the consideration of the load-disturbance specifications has received not so much attention. However it is well known that if we optimize the closed-loop transfer function for

a step-response specification, the performance with respect to load-disturbance attenuation can be very poor [5]. This is indeed the situation, for example, for IMC controllers that are designed in order to attain a desired set-point to output transfer function presenting a sluggish response to the disturbance [6].

The need to deal with both kind of properties and the recognition that a control system is, inherently, a system with Two Degrees-of-Freedom (2-DoF) - two closed-loop transfer functions can be adjusted independently -, motivated the introduction of 2-DoF PI/PID controllers [7]. The point is that, with a few exceptions such as the AMIGO [8] and Kappa-Tau; $\kappa - \tau$; [9] methods, no analytical expressions are provided for all controller parameters (feedback and reference part) and, at the same time, ensure a certain robustness degree for the resulting closed-loop. To provide simple tuning expressions and, at the same time, guarantee some degree of robustness are the main contributions of the paper.

This second degree of freedom is found on the presented literature as well as in commercial PID controllers under the form of the well known set-point weighting factor (usually called β) that ranges within $0 \leq \beta \leq 1.0$, being the main purpose of this parameter to avoid excessive proportional control action when a reference change takes place. Therefore the use of *just a fraction* of the reference.

As the design is based on a load-disturbance specification, in order to improve the resulting step-response performance, the available second degree of freedom under the form of a set-point weighting factor will be fully included into the design. While in [10] just some ad-hoc values are used that show that better step response can be obtained, in this work a selection rule is provided on the basis of a desired set-point to output transfer function. Therefore providing the a full tuning for a 2-DoF PI controller.

Even the presented procedure can be applied with any desired robustness level, maybe in practice the designer would like to use the robustness parameter on a more qualitative way, having, for example, three choices depending on the desired degree of robustness: (low, medium, high). This is to say the use of a controller with a minimum acceptable robustness level (that would be represented by $M_s = 2.0$), a robust controller (that would be represented by $M_s = 1.6$) or a highly robust controller (that would be represented by $M_s = 1.4$). With this consideration on hand, the previous corresponding values of M_s are introduced into the previously got general expressions and the resulting relations further simplified in order to get simple robust autotuning rules according to the specified robustness degree.

The organization of the paper is as follows. Next section introduces the framework and notation related to 2-DoF PID controllers as well as how the analytical load-disturbance

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R. Vilanova and O. Arrieta are with the Departament de Telecomunicació i d'Enginyeria de Sistemes, Escola d'Enginyeria, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain.

V.M. Alfaro is with the Departamento de Automática, Escuela de Ingeniería Eléctrica, Universidad de Costa Rica, P.O. Box 11501-2060 UCR San José, Costa Rica.

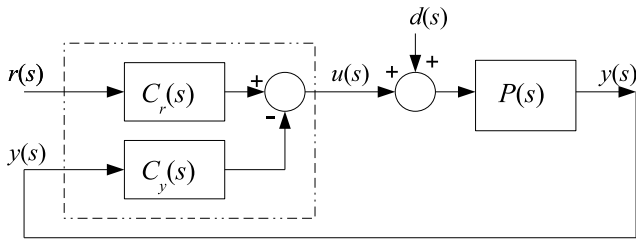


Fig. 1. 2-DoF Control System.

based design problem is formulated. Section 3 presents the development of the robust approach to PI design. Section 4 is devoted to the obtention of simple direct tuning rules for the most usual robustness levels. Section 5 presents comparative simulation examples and, finally, on Section 6 conclusions are conducted as well as an outline of continuing research.

II. PROBLEM FORMULATION

Consider the *Two-Degree-of-Freedom* (2-DoF) feedback control system of Fig. 1 where $P(s)$ is the controlled process transfer function, $C_r(s)$ the *set-point controller* transfer function, $C_y(s)$ the *feedback controller* transfer function, and $r(s)$ the set-point, $d(s)$ the load-disturbance, and $y(s)$ the controlled variable. The output of the 2-DoF controller is given by

$$u(s) = C_r(s)r(s) - C_y(s)y(s) \quad (1)$$

For a PI_2 controller [11] it is

$$u(s) = \underbrace{K_c \left(\beta + \frac{1}{T_i s} \right)}_{C_r(s)} r(s) - \underbrace{K_c \left(1 + \frac{1}{T_i s} \right)}_{C_y(s)} y(s) \quad (2)$$

where K_c is the controller gain, T_i the integral time constant, and β the set-point weighting factor ($0 \leq \beta \leq 1$).

The closed-loop control system response to a change in any of its inputs, will be given by

$$y(s) = \underbrace{\frac{C_r(s)P(s)}{1 + C_y(s)P(s)}}_{M_{yr}(s)} r(s) + \underbrace{\frac{P(s)}{1 + C_y(s)P(s)}}_{M_{yd}(s)} d(s) \quad (3)$$

where $M_{yr}(s)$ is the transfer function from set-point to process variable: the *servo-control* closed-loop transfer function or complementary sensitivity function $T(s)$; and $M_{yd}(s)$ is the one from load-disturbance to process variable: the *regulatory control* closed-loop transfer function or disturbance sensitivity function $S_d(s)$.

If $\beta = 1$, all parameters of $C_r(s)$ are identical to the ones of $C_y(s)$. In such situation, it is impossible to specify the dynamic performance of the control system to set-point changes, independently of the performance to load-disturbances changes. Otherwise, if the contrary, $\beta < 1$, given a controlled process $P(s)$, the feedback controller $C_y(s)$ can be selected to achieve a target performance for the regulatory control $M_{yd}(s)$, and then use the set-point weighting factor in the set-point controller $C_r(s)$, to modify the servo-control performance $M_{yr}(s)$.

The proposed *Analytic Robust Tuning of Two-Degree-of-Freedom PI controllers (ART₂)* [12], [13], is aimed at producing a control system that responds fast and without oscillations to a step load-disturbance, with a maximum sensitivity lower than a specified value; in order to assure robustness; and which will also show a fast non-oscillating response to a set-point step change, not requiring strong or excessive control effort variations (*smooth* control).

A. Outline of Controller Design Procedure

The first step in the Two-Degree-of-Freedom controller synthesis consists of obtaining the feedback controller $C_y(s)$, required to achieve a target $M_{yd}^t(s)$ regulatory closed-loop transfer function. From 3 once the controlled process is given and the target regulatory transfer function, $M_{yd}^t(s)$, specified the required feedback controller can be synthesized. The resulting feedback controller design equation is

$$C_y(s) = \frac{P(s) - M_{yd}^t(s)}{P(s)M_{yd}^t(s)} = \frac{1}{M_{yd}^t(s)} - \frac{1}{P(s)} \quad (4)$$

Once, as a first step, the feedback controller $C_y(s)$, is obtained from 4, on a second step, the set-point controller $C_r(s)$ free parameter (β) can be used in order to modify the servo control closed-loop transfer function $M_{yr}(s)$.

III. TUNING RULES FOR 2-DOF PI CONTROL

Consider the First-Order-Plus-Dead-Time (FOPDT) controlled process given by

$$P(s) = \frac{K_p e^{-Ls}}{Ts + 1} \quad (5)$$

where K_p is the process gain, T the time-constant, and L its dead-time. From here and after, $\tau_o = L/T$ will be referred as the controlled process *normalized dead-time*. In this work process models with normalized dead-time $\tau_o \leq 2$ are considered. Processes with long dead-time will need some kind of dead-time compensation scheme (a Smith predictor, for example).

For the FOPDT process the specified regulatory control target closed-loop transfer function is chosen as

$$M_{yd}^t(s) = \frac{Ks e^{-Ls}}{(\tau_c T s + 1)^2} \quad (6)$$

and the closed-loop target function selected for the servo-control as

$$M_{yr}^t(s) = \frac{e^{-Ls}}{\tau_c T s + 1} \quad (7)$$

where τ_c will be the *dimensionless design parameter*. It is the ratio of the closed-loop control system time constant to the controlled process time constant.

The specified target closed-loop transfer functions 6 and 7 will provide non-oscillating responses to step changes in both, the set-point and the load-disturbance, with an adjustable speed.

A. Controller Parameters

In order to synthesize the 2-DoF PI controller for the FOPDT process it is necessary to use a rational function in s as an approximation of the controlled process dead-time. This approximation will affect the closed-loop response characteristics. Using the Maclaurin first order series for the dead-time

$$e^{-Ls} \approx 1 - Ls \quad (8)$$

and 5 and 6 in 4, the PI_2 controller tuning equations are obtained as

$$\kappa_c = K_c K_p = \frac{2\tau_c - \tau_c^2 + \tau_o}{(\tau_c + \tau_o)^2} \quad (9)$$

$$\tau_i = \frac{T_i}{T} = \frac{2\tau_c - \tau_c^2 + \tau_o}{1 + \tau_o} \quad (10)$$

where κ_c and τ_i are the controller *normalized parameters*.

In order to assure that the controller parameters 9 and 10 have positive values, the design parameter τ_c must be selected within the range

$$0 < \tau_c \leq 1 + \sqrt{1 + \tau_o} \quad (11)$$

The resulting regulatory control closed-loop transfer function is

$$M_{yd}(s) = \frac{T_i s e^{-Ls}}{K_c (\tau_c T s + 1)^2} \quad (12)$$

B. Set-point Weighting Factor

As the closed-loop transfer functions are related by $M_{yr}(s) = C_{yr}(s)M_{yd}(s)$, by using controller $C_r(s)$, $M_{yr}(s)$ can be written as

$$M_{yr}(s) = \frac{K_c (\beta T_i s + 1)}{T_i s} M_{yd}(s) \quad (13)$$

Introducing in 13 the regulatory control closed-loop transfer function 12 and also the controller parameters 9 and 10, the servo-control transfer function then becomes

$$M_{yr}(s) = \frac{(\beta T_i s + 1) e^{-Ls}}{(\tau_c T s + 1)^2} \quad (14)$$

As the servo-control target transfer function was specified in 7, from 7, 13 and 14 in order to obtain a non-oscillatory response, an adequate selection of the set-point weighting factor would be $\beta = \tau_c T / T_i$, and then

$$\beta = \frac{\tau_c T}{T_i}, \quad 0 < \tau_c \leq 1 \quad (15)$$

outside this range

$$\beta = 1, \quad 1 < \tau_c < 1 + \sqrt{1 + \tau_o} \quad (16)$$

This weighting factor also has influence in the controller output when the set-point changes. Effectively, the instantaneous change on the control signal caused by a sudden change in the reference signal of magnitude Δr is given by

$$\Delta u_r = K_c \beta \Delta e = K_c \beta \Delta r \quad (17)$$

therefore, when very fast regulatory control responses are desired, high controller gain values are required, and the controller instantaneous output change when the set-point changes may be high. Then the controller output will be limited to be not greater than the total change on the set-point and then the set-point weighting factor selection criteria becomes

$$\beta = \min \left\{ \frac{1}{K_c}, \frac{\tau_c T}{T_i}, 1 \right\} \quad (18)$$

C. Control System Robustness

The maximum sensitivity

$$M_s = \max_{\omega} |S(j\omega)| = \max_{\omega} \left| \frac{1}{1 + C_y(j\omega)P(j\omega)} \right| \quad (19)$$

will be used as an indication of the closed-loop control system robustness.

A robustness analysis has been performed. This analysis shows that the control system maximum sensitivity M_s depends of the model normalized dead-time τ_o and the design parameter τ_c .

In order to avoid the loss of robustness when a very low τ_c is used, it is necessary to establish a lower limit to this design parameter. This relative loss of stability is greater when the normalized model dead-time τ_o is high.

The design parameter lower limit for a given robustness level can be expressed in parameterized form as

$$\tau_{cmin} = k_1(M_s) + k_2(M_s)\tau_o \quad (20)$$

where the k_1 and k_2 are show in Table I.

TABLE I
EQUATION 20 CONSTANTS

M_s	1.2	1.4	1.6	1.8	2.0
k_1	0.4836	0.4152	0.3441	0.3254	0.3042
k_2	1.8982	0.9198	0.6659	0.4853	0.3822

The design parameter equations (20) can be expressed as a single equation as

$$\tau_{cmin} = k_{11}(M_s) + \left[\frac{k_{21}(M_s)}{k_{22}(M_s)} \right] \tau_o \quad (21)$$

$$k_{11}(M_s) = 1.384 - 1.063M_s + 0.262M_s^2$$

$$k_{21}(M_s) = -1.915 + 1.415M_s - 0.077M_s^2$$

$$k_{22}(M_s) = 4.382 - 7.396M_s + 3.0M_s^2$$

Also it can be seen that; as usual; as the system becomes slower its robustness increases but if very slow responses are specified the system robustness starts to decrease, therefore the upper limit of the design parameters τ_c also needs to be constrained By combining the design parameter performance and robustness constraints it may be selected within the range

$$\max(0.50, \tau_{cmin}) \leq \tau_c \leq 1.50 + 0.3\tau_o \quad (22)$$

where τ_{cmin} is given by (21).

IV. SIMPLIFIED AUTOTUNING RULES FOR 2-DOF PI CONTROL

To provide the possibility of specify any possible desired robustness level within the range $M_s \in [1.2 - 2.0]$ is of great interest as this provides a complete view of the robustness-performance tradeoff and a quantified measure of how restrictive a robustness level can be depending on the process normalized dead-time. However, from a more practical point of view, the following question arises: When a desired $M_s = 1.57$ will be specified? With this respect, as the M_s value is being recognized as a *de facto* standard measure of robustness, an M_s value of 2.0 is recognized as the minimum acceptable robustness level. This could be considered a *low* degree of robustness. According to a similar measure, and in order to make the analysis simpler, a *medium* degree of robustness is associated here with $M_s = 1.6$ while a *high* degree of robustness will correspond to $M_s = 1.4$. This broad classification allows a qualitative specification of the control system robustness.

According to this principle, the above mentioned three values of M_s are used here to generate the corresponding estimate for the lowest allowable closed-loop time-constant with 20 and introduce such time -constant value into the PI parameter equations (9), (10) and (15). The resulting controller parameters will be, in this case, expressed just in terms of the process normalized dead-time τ_o as:

- High-Robustness Tuning ($M_s = 1.4$)

$$\begin{aligned} \kappa_c &= \frac{-0.23\tau_o + 0.64}{\tau_o + 0.16} \\ \tau_i &= \frac{-0.85\tau_o^2 + 2.1\tau_o + 0.65}{\tau_o + 1} \\ \beta &= \frac{0.9\tau_o + 0.4}{\tau_i} \end{aligned} \tag{23}$$

- Medium-Robustness Tuning ($M_s = 1.6$)

$$\begin{aligned} \kappa_c &= \frac{-0.17\tau_o + 0.74}{\tau_o + 0.16} \\ \tau_i &= \frac{-0.44\tau_o^2 + 1.85\tau_o + 0.6}{\tau_o + 1} \\ \beta &= \frac{0.66\tau_o + 0.35}{\tau_i} \end{aligned} \tag{24}$$

- Low-Robustness Tuning ($M_s = 2.0$)

$$\begin{aligned} \kappa_c &= \frac{-0.1\tau_o + 0.86}{\tau_o + 0.15} \\ \tau_i &= \frac{1.12\tau_o + 0.16}{\tau_o + 0.37} \\ \beta &= \frac{0.39\tau_o + 0.3}{\tau_i} \end{aligned} \tag{25}$$

Fig. 2 shows the generated values for a grid of $\tau_o \in [0.1 - 2.0]$ as well as the regression curves that gives rise to the above formulae for the normalized gain (κ_c) and integral time (τ_i) as well as for the set-point weighting factor β .

V. EXAMPLES

Consider the FOPDT controlled process

$$P_1(s) = \frac{e^{-0.5s}}{s + 1} \tag{26}$$

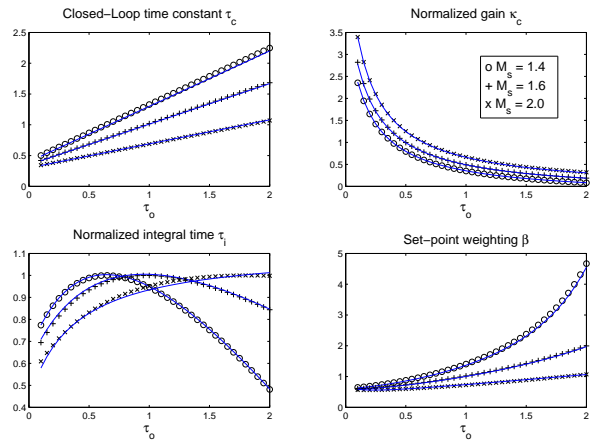


Fig. 2. PI Normalized Parameters for Low, Medium and High Robustness.

By using the full design equations, the controller parameters and achieved robustness for different values of the desired closed-loop time constant τ_c are given in Table II.

TABLE II
EXAMPLE 1 - ART₂ PI PARAMETERS

τ_c	K_c	T_i	β	M_s
0.6	1.11	0.98	0.67	1.75
0.8	0.86	0.97	0.82	1.49
1.2	0.51	0.97	1.0	1.26
1.4	0.37	0.89	1.0	1.20

As can be seen from Table II, to increase the control system robustness is necessary to decrease its speed. The designer may tackle the design problem in the inverse way, specifying the control system minimum robustness. Using the process normalize dead-time ($\tau_o = 0.5$ for this example) and equations 20 and 22 the recommended lower limit for the design parameter to obtain a specified minimum robustness are estimated and listed in Table III.

TABLE III
DESIGN PARAMETER MINIMUM VALUES

M_s^d	2.0	1.8	1.6	1.4	1.2
τ_{cmin}	0.5	0.567	0.677	0.875	1.433

In order to evaluate the performance of the simple tuning rules, the corresponding values of M_s^d are taken. The controller parameters for the complete and autotuning relations are shown in Table IV.

TABLE IV
EXAMPLE 1 - PI PARAMETERS; COMPLETE AND AUTOTUNING

M_s^d	Complete Tuning			Autotuning		
	K_c	T_i	β	K_c	T_i	β
1.4	0.7914	0.9789	0.8688	0.7955	0.9917	0.8571
1.6	0.9958	0.7346	0.6864	0.9924	0.9333	0.7286
2.0	1.2547	0.8312	0.5978	1.2462	0.8276	0.5981

Fig. 3 shows the closed-loop time responses for the different controller values. As it can be seen, output responses and control values for the tuning got using the complete expressions and those got from the simple autotuning ones cannot be distinguished. Therefore the performance obtained is completely equivalent to that of the full tuning rules.

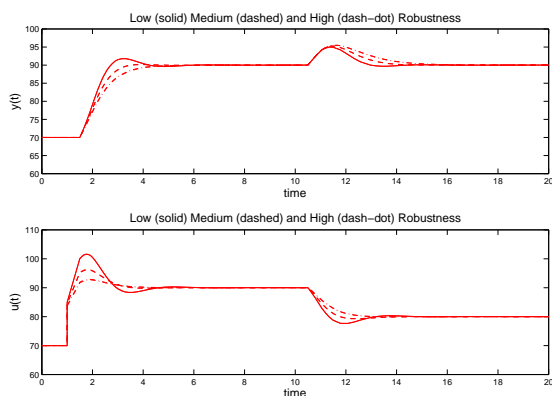


Fig. 3. Example 1 - System responses for the three robustness levels and comparing the complete and simple autotuning rules.

VI. CONCLUSIONS

An approach for automatic tuning of robust PI 2-DoF controller has been proposed. The method is analytically based; therefore called Analytical Robust Tuning (ART_2); and starts from a First-Order-Plus-Dead-Time controlled process model to obtain a control system that responds fast and without oscillations to a step load-disturbance, with a maximum sensitivity lower than a specified value; in order to assure robustness; and which will also show a fast non-oscillating response to a set-point step change, not requiring strong or excessive control effort variations (*smooth* control).

Given a prescribed robustness level expressed in terms of the Maximum Sensitivity value (M_s), the lowest allowable closed-loop time constant is determined. On that basis, the disturbance to output transfer function is matched and, on a second step, the control system performance to a set-point modified by an adequate selection of the Two-Degree-of-Freedom controller set-point weighting factor β . The use of $\beta \leq 1$ values allows to decrease the servo-control response maximum overshoot when very fast responses have been specified for the regulatory control. However, values larger than 1 may be generated if the system response is too slow. The resulting tuning can take any desired value for M_s as the design parameter and generate, in a parameterized way, the three controller parameters (K_c , T_i and β).

On the basis of the general approach, three different robustness levels are defined corresponding to the Maximum Sensitivity values of: $M_s = 1.4$, $M_s = 1.6$ and $M_s = 2.0$. Simple tuning rules are generated by considering these M_s values. The resulting autotuning rules provide all the controller parameters parameterized in terms of the model normalized dead-time allowing the user to select for a High/Medium/Low Robust closed-loop system. The proposed autotuning expressions are therefore compared with other well known tuning rules also conceived with the same robustness spirit, showing the proposed approach is able to guarantee the same robustness level with an improvement of the system time performance.

Current research is conducted on the extension of the approach to a 2-DoF PID and to introduce alternative ways of designing the disturbance attenuation characteristics.

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