

# Structure Specified Robust $H_\infty$ Loop Shaping Control of a MIMO Electro-hydraulic Servo System using Particle Swarm Optimization

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**Abstract**— A fixed-structure controller based on robust  $H_\infty$  loop shaping control is proposed in this paper. It can be used to guarantee the robust performance under a structure-specified controller. In this proposed technique, Particle Swarm Optimization (PSO) is applied in the design controller, and the inverse of infinity norm from disturbances to states is formulated as the objective function in searching the optimal controller. Simulation results of MIMO electro-hydraulic servo system show that the proposed controller has simpler structure than that of the conventional  $H_\infty$  loop shaping controller and its stability margin is near the  $H_\infty$  loop shaping controllers.

**Index Terms**— fixed-structure robust  $H_\infty$  loop shaping, Particle Swarm Optimization, MIMO electro-hydraulic servo system

## I. INTRODUCTION

The Electro-hydraulic servo systems are well known. Electro-hydraulic actuator is an attractive choice for being used in both industrial and non-industrial applications because of the following fast dynamic response, high power to inertia ratio and control accuracy. Controlling of these systems is important because of their highly nonlinear. In recent years, the robust control has received much attention that can be guarantee for system under conditions of uncertainty, parameter changes, and disturbances. However, the robust controllers are difficulty in practical applications. The simple controller such as PI, PID controller is today's most commonly used control in servo systems. This problem extends the gap between the theoretical and practical approaches.

To solve this problem, the design of a fixed-structure robust controller has been proposed and has become an interesting area of research because of its simple structure and practicable controller order. In [1], a robust  $H_\infty$  optimal control problem with a structure specified controller was solved by using genetic algorithm (GA). As concluded in [1], GA is a simple and capable method to design a fixed-structure  $H_\infty$  optimal controller. B.S. Chen. *et. al.* [2] proposed a PID design algorithm for mixed  $H_2/H_\infty$  control. In their paper, PID control parameters were tuned in the stability domain to achieve mixed  $H_2/H_\infty$  optimal control. A

similar idea was proposed in [3] by using the intelligent GA to solve the mixed  $H_2/H_\infty$  optimal control problem. However, the fixed structure controller based on  $H_\infty$  optimal control designed in [1-3] are difficulty for both the uncertainty of the model and the performance are essentially chosen weights.

Alternatively, a fixed structure  $H_\infty$  loop shaping controller is proposed by [4-5]. A. Umut Genc in 2000 [4] adopted the concept of state space approach and BMI optimization. As shown in this research, specifying the initial solution has a huge effect on the optimal solution because of the local minima problem. S. Patra *et.al.* in [5] designs an output feedback robust controller that has the same structure as the pre-compensator weight which is normally designed by PI. Though the fixed structure  $H_\infty$  loop shaping control techniques mentioned above are easy to select weighting function, they requires only two specified weights, pre- and post-compensator weights [6], for shaping the nominal plant so that the desired open loop shape is achieved. Fortunately, the selection of such weights is based on the concept of classical loop shaping, which is a well known technique in controller design.

However, the resulting controller in [4-5] is always ineffective and the problem of local minima often occurs in the design. To solve these problems, searching algorithms such as genetic algorithm, particle swarm optimization technique, tabu-search, etc., can be employed. In this paper, we proposed a new design technique by the PSO based fixed-structure robust  $H_\infty$  loop shaping control. PSO is employed to find the parameters of the controllers. The structure of controller in the proposed technique is selectable; in this paper, the fixed-structure robust PI controller is designed. Simulation results show that a controller designed by the proposed approach has a good performance and robustness as well as a simple structure.

The remainder of this paper is organized as follows. Section II covers the MIMO Electro-hydraulic Servo system modeling. In section III, conventional  $H_\infty$  loop shaping and the proposed technique are discussed as well as PSO algorithm. The design examples and results are demonstrated in section IV. And in section V the paper is summarized.

## II. MODELING

MIMO electro-hydraulic servo system is shown in Fig. 1 which consists of a position control system and a force control system [7]. The position control system which is used to control the actuator movement and the force control system which is used to supply a required force to the

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system load. The objective of the electro-hydraulic servo system is to satisfy the requirements such as zero steady state errors in motion of the actuator and force output.

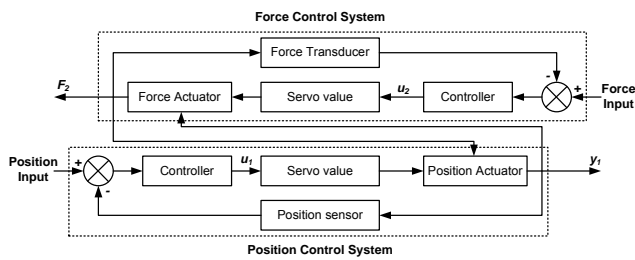


Fig.1. MIMO electro-hydraulic servo system

The state-space of the MIMO electro-hydraulic servo system is

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \quad (1)$$

The dynamic model of this system is an MIMO system which has 2 outputs ( $y$ ),  $F_2$  – force of the system and  $y_1$  – position of the actuator, and 2 inputs ( $u$ ),  $u_1$  – input servo value of the position control system, and  $u_2$  – input servo value of the force control system.

### III. CONVENTIONAL $H_\infty$ LOOP SHAPING CONTROL AND PROPOSED TECHNIQUE

#### A. Conventional $H_\infty$ Loop Shaping Control

$H_\infty$  loop shaping control [6] is an efficient method to design a robust controller. This approach requires only a desired open loop shape in frequency domain which is specified by the weighting functions, pre-compensator ( $W_1$ ) and post-compensator ( $W_2$ ). The uncertainty model of the system is formulated as normalized co-prime factors that divide the shaped plant ( $G_s$ ) into nominator factor ( $N_s$ ) and denominator factor ( $M_s$ ) which is shown in Fig.2. Consequently, the shaped plant can be written as:

$$G_s = W_1 G W_2 \quad (2)$$

$$G_s = (N_s + \Delta_{N_s})(M_s + \Delta_{M_s})^{-1} \quad (3)$$

where  $\Delta_{N_s}$  and  $\Delta_{M_s}$  are the uncertainty transfer functions in the nominator and denominator factors, respectively.  $\|\Delta_{N_s}, \Delta_{M_s}\|_\infty \leq \varepsilon$ , where  $\varepsilon$  is the stability margin. The determination of the normalized co-prime and the solving of the  $H_\infty$  loop shaping control can be seen from [8].

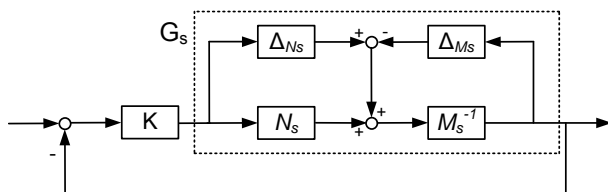


Fig.2. Co-prime factor robust stabilization problem.

In this approach, the pre-compensator ( $W_1$ ) and post-compensator ( $W_2$ ) weights for achieving the desired loop shape are defined then optimal stability margin ( $\varepsilon_{opt}$ ) is solved by the following equation.

$$\gamma_{opt} = \varepsilon_{opt}^{-1} = \inf_{stab K} \left\| \begin{bmatrix} I \\ K \end{bmatrix} (I + G_s K)^{-1} M_s^{-1} \right\|_\infty \quad (4)$$

If the ( $\varepsilon_{opt}$ ) is too low, then go to select new weighting function. Select the stability margin ( $\varepsilon < \varepsilon_{opt}$ ) and then synthesize the controller,  $K_\infty$ , by solving the following inequality.

$$\begin{aligned} \|T_{zw}\|_\infty &= \left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (I + G_s K_\infty)^{-1} M_s^{-1} \right\|_\infty \\ &= \left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (I + G_s K_\infty)^{-1} \begin{bmatrix} I & G_s \end{bmatrix} \right\|_\infty \leq \varepsilon^{-1} \end{aligned} \quad (5)$$

The feedback controller ( $K$ ) is

$$K = W_1 K_\infty W_2 \quad (6)$$

#### B. Proposed Technique

The proposed technique begin with determining the structure of the controller ( $K(p)$ ). The parameter,  $p$ , of the controller is changeable. Then, PSO is used to find the right parameter,  $p$ . In robust problem, the stability margin ( $\varepsilon$ ) is single index to indicate performance of the designed controller which is obtained as follows.

$$\|T_{zw}\|_\infty^{-1} = \varepsilon = \left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (I - G_s K_\infty)^{-1} \begin{bmatrix} I & G_s \end{bmatrix} \right\|_\infty^{-1} \quad (7)$$

Where  $K_\infty$  can be found by  $K_\infty = W_1^{-1} K(p) W_2^{-1}$ . Suppose that  $W_1$  and  $W_2$  can be inverted. Generally,  $W_2$  is chosen to be equal to identity matrix  $I$ . Therefore, objective function can be written in this form:

$$\begin{aligned} \text{Objective function} &= \varepsilon = \|T_{zw}\|_\infty^{-1} \\ &= \left\| \begin{bmatrix} I \\ W_1^{-1} K(p) \end{bmatrix} (I - G_s W_1^{-1} K(p))^{-1} \begin{bmatrix} I & G_s \end{bmatrix} \right\|_\infty^{-1} \end{aligned} \quad (8)$$

For this design of controller, the controller  $K(p)$  will be designed to minimize the infinity norm from disturbance to state ( $\|T_{zw}\|_\infty$ ) or maximize ( $\varepsilon$ ) by PSO method. This method is capable in solving many engineering problems. Fig. 3 shows the swarm's movement which is the basic idea of PSO. As seen in this figure, a bird represents the particle and the position of each particle represents the candidate solution. Moreover, it is requires only upper, lower bounds of solution and PSO parameters such as the population of swam( $n$ ), lower and higher boundary ( $p_{min}$ ,  $p_{max}$ ) of the problem, minimum and maximum velocity of particles ( $v_{min}$ ,  $v_{max}$ ), minimum and maximum iteration( $i_{max}$ ). PSO is an iterative algorithm. In each iteration, the value of fitness ( $f_s$ ) of each population in the  $i^{th}$  generation is calculated. Then, choose the population that gives the highest fitness value to use as the answer of the generation. The inertia weight ( $Q$ ), value of velocity ( $v$ ) and position ( $p$ ) of each population in the present generation ( $i$ ) are updated by using this equation (9), (10) and (11), respectively.

$$Q = Q_{max} - \left( \frac{Q_{max} - Q_{min}}{i_{max}} \right) i \quad (9)$$

$$v_{i+1} = Qv_i + \alpha_1[\gamma_{1i}(P_b - p_i)] + \alpha_2[\gamma_{2i}(U_b - p_i)] \quad (10)$$

$$p_{i+1} = p_i + v_{i+1} \quad (11)$$

Where  $\alpha_1, \alpha_2$  are acceleration coefficients  
 $\gamma_{1i}, \gamma_{2i}$  are any random number in (0→1) range



Fig. 3. The movement of a swarm.

Based on the PSO technique, in this problem, sets of controller parameters  $p$  is formulated as a particle and the fitness can be written as:

$$Objective\ function = \left\{ \left\| \begin{bmatrix} I \\ W_1^{-1}K(p) \end{bmatrix} (I - G_s W_1^{-1}K(p))^{-1} [I \quad G_s] \right\|_{\infty}^2 \right. \\ \left. \begin{matrix} 0.0001 \\ \dots \end{matrix} \right\} \quad \dots(12)$$

From (13); fitness value will be defined to equal a constant with very little value. The controller from particle makes the system unsteady. The flow chart of the design of the proposed technique can be shown in Fig.4.

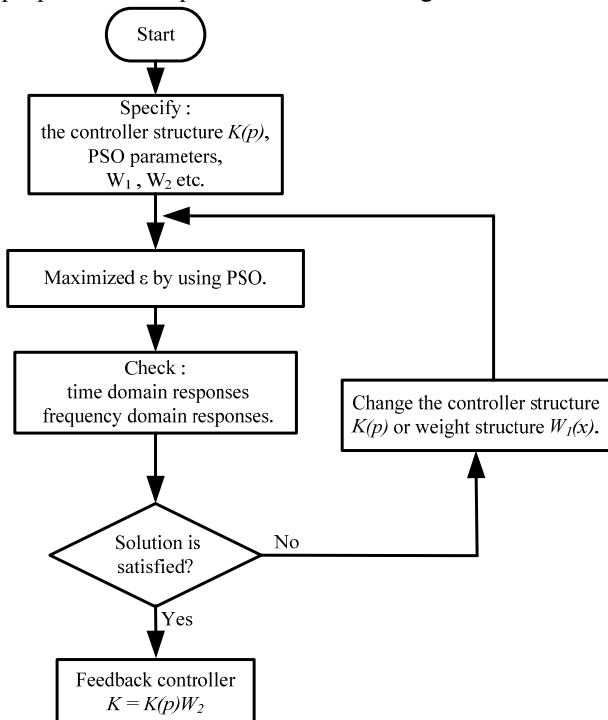


Fig. 4. Flow chart of the proposed design procedure.

#### IV. SIMULATION RESULTS

The state-space of a nominal plant can be seen in [7]. The state vector of this plant consists of the four variables which

are supply pressure in force control system, supply pressure in position control system, position of the actuator and velocity of the actuator. The details of this plant are given in appendix A. In this paper, the pre- and post-compensator weights are chosen as:

$$W_1 = \begin{bmatrix} \frac{0.8s+60}{s+0.001} & 0 \\ 0 & \frac{0.8s+60}{s+0.001} \end{bmatrix}, \quad W_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (13)$$

In this paper, the structure of controller is selected as:

$$K(p) = \begin{bmatrix} \frac{p_1s+p_2}{s+0.001} & \frac{p_3s+p_4}{s+0.001} \\ \frac{p_5s+p_6}{s+0.001} & \frac{p_7s+p_8}{s+0.001} \end{bmatrix} \quad (14)$$

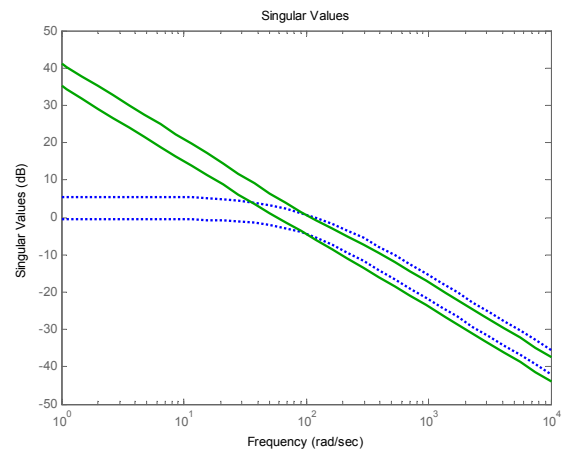


Fig.5. Singular values (---plant), (— Shaped plant) of MIMO electro-hydraulic servo system.

Singular values of MIMO electro-hydraulic servo system and desired loop shape are plotted in Fig.5. As seen in this figure, the bandwidth and performance are significantly improved by the compensator weights. The shaped plant has large gains at low frequencies for performance and small gains at high frequencies for noise attenuation. With these weighting functions, the robust requirement is satisfied.

By using (13), the optimal stability margin of the shaped plant is found to be 0.7034. This value indicates that the selected weights are compatible with robust stability requirement in the problem. To design the conventional  $H_{\infty}$  loop shaping controller, stability margin 0.6682 is selected. As a result, the final controller (full order  $H_{\infty}$  loop shaping controller) is 8<sup>th</sup> order and complicated.

In the optimization problem, the upper and lower bounds of control parameters and PSO parameters are given in Table 1.

Table 1 PSO parameters and controller parameters range.

Parameter	value
minimum velocities	0
maximum velocities	0.2
acceleration coefficients	2.1

minimum inertia weights	0.6
maximum inertia weights	0.9
maximum iteration	60
population size	500
$p1-s$	$[-60, 60]$

After running the PSO for 60 iterations when running PSO for 33 iterations, the optimal solution is obtained as:

$$K(p) = \begin{bmatrix} \frac{-0.5355s - 34.503}{s + 0.001} & \frac{-0.1304s - 2.5367}{s + 0.001} \\ \frac{-0.0258s - 2.897}{s + 0.001} & \frac{-0.3682s - 36.641}{s + 0.001} \end{bmatrix} \quad (15)$$

Fig.6 shows the fitness or stability margin ( $\varepsilon$ ) of the controller in each generation. The best answer evolved by PSO has a stability margin of 0.4780.

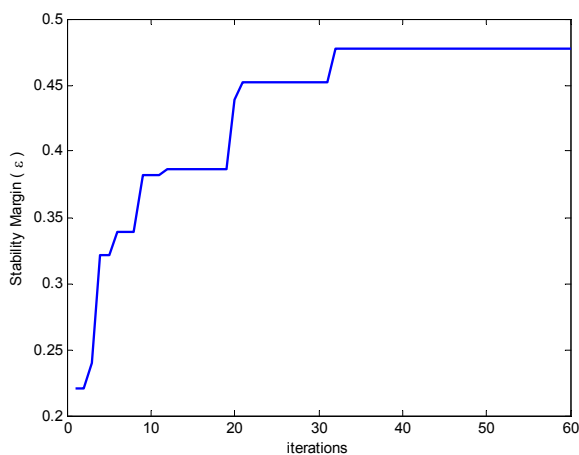
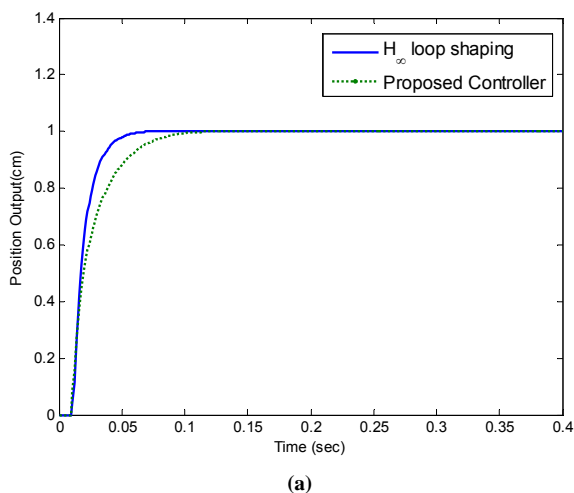
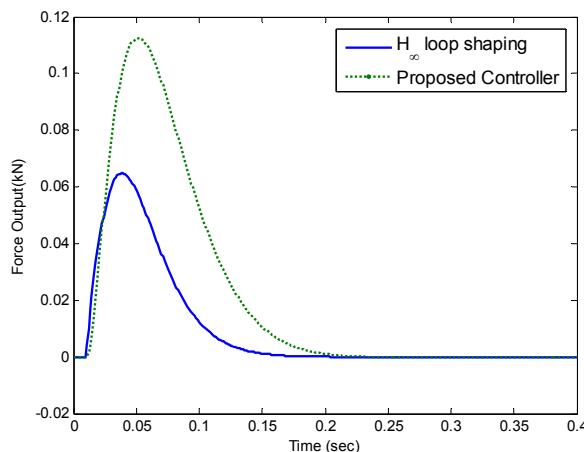


Fig.6. Stability margin ( $\varepsilon$ ) versus iteration.



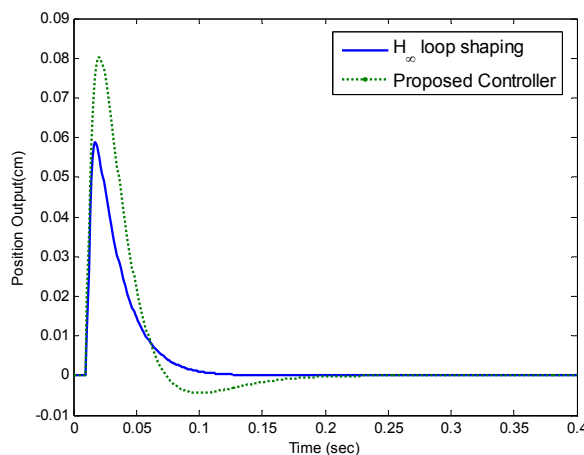
(a)



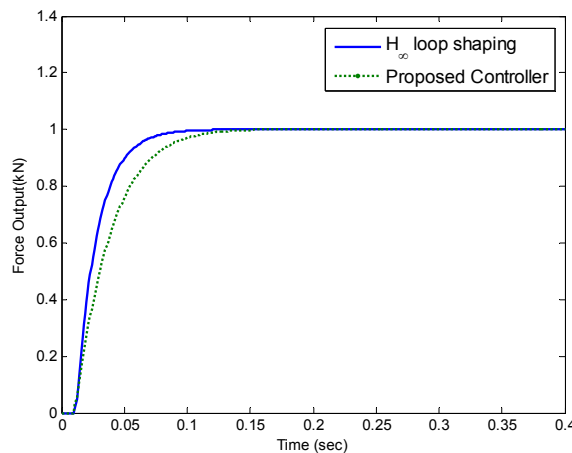
(b)

Fig.7. Output response of the system both when the unit step is entered to position command.

Fig.7 shows the response of the output of the system in 2 channels (input servo value of the position control system and input servo value of the force control system). When the unit step is fed into the position command, it can be found that the proposed controller performs well. Its response is close to the  $H_\infty$  loop shaping controller, with no overshoot. Fig.8 shows the responses of the system when unit step is fed into force command.



(a)



(b)

Fig.8. Output response of the system both when the unit step is entered to force command.

## V. CONCLUSION

This paper presents a new technique for designing a fixed structure robust  $H_\infty$  loop shaping controller which proposed technique can be applied a robust controller for a MIMO electro-hydraulic servo system. In the proposed can select structure of controller. Based on the notion of classical  $H_\infty$  loop shaping, stability margin ( $\epsilon$ ) is used to indicate robustness and performance of the proposed controller. This parameter is defined as the objective function of searching the optimal solution by PSO method. In this technique make easy because PSO simplifies the method. Simulation results demonstrate that the proposed technique is adjustable and reasonable.

## APPENDIX

In this paper, the linearized model of the MIMO electro-hydraulic servo system is taken from [7], that is

$$A = \begin{bmatrix} -65.58 & 59.98 & 18.02 & -14.15 \\ 25.98 & -121.5 & 0 & 25.98 \\ 12.45 & 12.38 & -84.65 & 0 \\ 0.99 & -0.91 & -21.53 & -61.52 \end{bmatrix}$$

$$B = \begin{bmatrix} 17.79 & 0 \\ 158.1 & 0.13 \\ -4.68 & 79.8 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1.07 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

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