

Quasi Sliding Mode Control of Generalized Lorenz Chaotic Systems

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Abstract—This paper presents a systematic design procedure to stabilize generalized Lorenz chaotic systems based on sliding mode control. In contrast to the previous works of sliding mode control, the concept of quasi sliding mode control is firstly introduced such that continuous control input is obtained to avoid chattering phenomenon. Furthermore, under the proposed control law, the system states can be stabilized and driven into an arbitrary and predictable neighborhood of zero. This method can also be easily extended to a general class of chaotic systems.

Index Terms-- Quasi sliding mode control, chaos, generalized Lorenz chaotic systems, chattering phenomenon

I. INTRODUCTION

Over the last two decades, since the pioneering work of Ott, Grebogi and Yorke [1], chaos control has become one of interesting issues in nonlinear systems. A chaotic system is a very special nonlinear dynamical system and it possesses several properties such as the sensitivity to initial conditions, as well as an irregular, unpredictable behavior and thereby confines the precise operation of physical systems, such as mechanical systems, biological systems and power converters, etc. Therefore, various effective methods have been proposed over the past decades to achieve the control and stabilization of chaotic systems, such as optimal control [2], the sliding method control [3-8], state feedback control [9, 10] and the backstepping design technique [11, 12], etc.. In a robust control system, sliding mode control (SMC) is frequently adopted because SMC can offer many inherent advantages, such as fast response, good transient performance and insensitive to variation in plant parameters or external disturbances. However, in the conventional SMC systems [3-8], ideal sliding mode only exists with infinite frequency switching operation. From practical point of view, thus control input is impossible to implement and will cause the undesired chattering phenomenon.

Motivated by the aforesaid, this study aims to present a control scheme to suppress chaos for generalized Lorenz chaotic systems based on SMC. In contrast to the previous works of SMC, the concept of quasi sliding mode control (QSMC) is first introduced such that a continuous control input is obtained to avoid chattering phenomenon as frequently in the conventional sliding mode control systems. Furthermore, under the proposed control law, the system

states can be stabilized and driven into an arbitrary and predictable neighborhood of zero. Finally, an illustrative example is presented to demonstrate the effectiveness of the proposed QSMC scheme.

II. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

In this section, we consider the chaos suppression of a generalized Lorenz chaotic system (GLCS) via a QSMC.

II.1 Generalized Lorenz chaotic systems

We consider the following GLCS:

$$\begin{aligned} \dot{x}_1(t) &= (10 + \frac{25}{29}k) \cdot [x_2(t) - x_1(t)] \\ \dot{x}_2(t) &= (28 - \frac{35}{29}k)x_1(t) + (k-1)x_2(t) - x_1(t)x_3(t) \\ \dot{x}_3(t) &= (-\frac{8}{3} - \frac{1}{87}k)x_3(t) + x_1(t)x_2(t) \end{aligned} \quad (1)$$

$$[x_1(0) \ x_2(0) \ x_3(0)]^T = [x_{10} \ x_{20} \ x_{30}]^T$$

where $x(t) = [x_1(t) \ x_2(t) \ x_3(t)]^T \in R^3$ is state vector, $[x_{10} \ x_{20} \ x_{30}]^T$ is the initial value vector, and k is the system parameter with $0 \leq k < 1$. The dynamics of this system has been extensively studied in [13] for a space range of the amplitude of the term k and displays chaotic behavior for each $0 \leq k < 1$. Figures 1(a)-(d) show the chaotic motion of system (1) for $k = 0.2$ with initial condition of $[x_{10} \ x_{20} \ x_{30}]^T = [1 \ 1 \ 0.5]^T$. In the following, we will consider the chaos suppression of the GLCS and give an explicit and simple procedure to establish a QSMC to achieve the control goal.

II.2 Problem formulation

Consider the GLCS as shown in (1), to control the system effectively, we introduce a control input u to the differential equation of state x_2 . By adding this input, the equation of the controlled system can be expressed by

$$\begin{aligned} \dot{x}_1(t) &= (10 + \frac{25}{29}k) \cdot [x_2(t) - x_1(t)] \\ \dot{x}_2(t) &= (28 - \frac{35}{29}k)x_1(t) + (k-1)x_2(t) - x_1(t)x_3(t) + u \\ \dot{x}_3(t) &= (-\frac{8}{3} - \frac{1}{87}k)x_3(t) + x_1(t)x_2(t) \end{aligned} \quad (2)$$

The goal of this study is to design a QSMC such that the resulting state of system can be driven to predictable and desired bounds, i.e.

$$\lim_{t \rightarrow \infty} |x_i| \leq \gamma_i, i=1,2,3 \quad (3)$$

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where $\gamma_i \geq 0$ are predictable constants depending on the parameter chosen in the QSMC, which will be stated later.

In consequence, to achieve this control goal for the GLCS, there exist two major phases. First, it needs to select an appropriate switching surface for the system such that the quasi sliding motion on the manifold can result in $\lim_{t \rightarrow \infty} |e_i| \leq \gamma_i, i = 1, 2, 3$. Second, it needs to determine a QSMC such that the existence of the quasi sliding manifold can be guaranteed.

III. SWITCHING SURFACE DESIGN AND DEFINITION OF QUASI SLIDING MANIFOLD

To complete the above two phases, a switching surface is defined as follows:

$$s(t) = x_2(t) + cx_1(t) \quad (4)$$

where $s \in R$ and $c > -1$ is a chosen constant. Therefore, the following dynamics of $x_1(t)$ in (2) can be obtained as

$$\dot{x}_1(t) = -\lambda_1 x_1(t) + (10 + \frac{25}{29}k)s(t) \quad (5)$$

where $\lambda_1 = (10 + \frac{25}{29}k)(1+c)$.

Before continuing to discuss the response of state x_1 , we give the definition of quasi sliding manifold as follows.

Definition 1: The system is said to be in the quasi sliding manifold if there exists $\delta_Q > 0$ and $t_Q > 0$ such that any solution $x(\cdot)$ of controlled system (2) satisfies $|s(t)| \leq \delta_Q$, for all $t \geq t_Q$.

Solving the differential equation (5) for x_1 when $t \geq t_Q$ results in

$$x_1(t) = e^{-\lambda_1(t-t_Q)} x_1(t_Q) + (10 + \frac{25}{29}k) \int_{t_Q}^t e^{-\lambda_1(t-\tau)} s(\tau) d\tau \quad (6)$$

As shown in Definition 1, when the system enters the quasi sliding manifold, one has $|s(t)| \leq \delta_Q$ for $t \geq t_Q$. Furthermore, since $c > -1$ is determined to result in $\lambda_1 > 0$, the bound for state x_1 can be obtained as

$$\begin{aligned} |x_1(t)| &= \left| e^{-\lambda_1(t-t_Q)} x_1(t_Q) + (10 + \frac{25}{29}k) \int_{t_Q}^t e^{-\lambda_1(t-\tau)} s(\tau) d\tau \right| \\ &\leq e^{-\lambda_1(t-t_Q)} |x_1(t_Q)| + (10 + \frac{25}{29}k) \delta_Q e^{-\lambda_1 t} \int_{t_Q}^t e^{\lambda_1 \tau} d\tau \\ &= e^{-\lambda_1(t-t_Q)} |x_1(t_Q)| + (10 + \frac{25}{29}k) \delta_Q \frac{1 - e^{-\lambda_1(t-t_Q)}}{\lambda_1} \end{aligned} \quad (7)$$

Eqn. (7) with $\lambda_1 > 0$ ensures that

$$\lim_{t \rightarrow \infty} |x_1(t)| \leq \gamma_1 = (10 + \frac{25}{29}k) \frac{\delta_Q}{\lambda_1} = \frac{\delta_Q}{1+c} \quad (8)$$

Furthermore, by (4), the bound for $x_2(t), t \rightarrow \infty$, can be also obtained as

$$\begin{aligned} \lim_{t \rightarrow \infty} |x_2(t)| &= \lim_{t \rightarrow \infty} |s(t) - cx_1(t)| \\ &\leq \lim_{t \rightarrow \infty} |s(t)| + \lim_{t \rightarrow \infty} |c| |x_1(t)| \leq \gamma_2 = (1 + \frac{|c|}{1+c}) \delta_Q \end{aligned} \quad (9)$$

Meanwhile, after $|x_i| \leq \gamma_i, i = 1, 2$, solving the differential eqn. (2) for error state x_3 results in

$$\lim_{t \rightarrow \infty} |x_3(t)| \leq \gamma_3 = \frac{\gamma_1 \gamma_2}{(\frac{8}{3} + \frac{1}{87}k)} \quad (10)$$

Obviously, the bounds of $\gamma_i, i = 1, 2, 3$ are relative to δ_Q . Therefore, to control the system with a smaller value of δ_Q is important and the solution will be given in the following section.

IV. QSMC DESIGN FOR THE QUASI SLIDING MANIFOLD

Having established an appropriate switching surface and estimating the bounds of the states of system, this section aims to design an QSMC to drive the dynamics (2) into the quasi sliding manifold, $|s(t)| \leq \delta_Q$. To ensure the occurrence of the quasi sliding manifold, the continuous QSMC is proposed as

$$u(t) = -w\eta \frac{s}{|s| + \delta} \quad (11)$$

where $w > 1, \delta > 0$ and

$$\eta = \left| (28 - \frac{35}{29}k)x_1 + (k-1)x_2 - x_1 x_3 + c(10 + \frac{25}{29}k)(x_2 - x_1) \right|$$

The proposed control scheme above will guarantee the quasi sliding mode motion for the system (2), and is proven in the following theorem.

Theorem 1: Consider the system (2), if this system is controlled by $u(t)$ in (11). Then the system trajectory converges to the quasi sliding manifold, $|s(t)| \leq \delta_Q = \frac{w\delta}{w-1}$.

Proof of Theorem 1: Let the Lyapunov function of the system be $V = \frac{1}{2}s^2$, then taking the derivative of V and introducing (4) (11), one has

$$\begin{aligned} \dot{V} &= s\dot{s} \\ &= s(\dot{x}_2 + c\dot{x}_1) \\ &= s((28 - \frac{35}{29}k)x_1 + (k-1)x_2 - x_1 x_3 \\ &\quad + u + c(10 + \frac{25}{29}k)(x_2 - x_1)) \end{aligned} \quad (12)$$

$$\begin{aligned} &\leq \eta |s| - w\eta \frac{s^2}{|s| + \delta} \\ &\leq \eta |s| - w\eta (|s| - \frac{|s|\delta}{|s| + \delta}) \end{aligned}$$

Since $\frac{|s|\delta}{|s| + \delta} \leq \delta$, we have

$$\begin{aligned} \dot{V} &= s\dot{s} \\ &\leq \eta |s| - w\eta \delta \\ &\leq (1-w)\eta (|s| - \frac{w\delta}{w-1}) \end{aligned} \quad (13)$$

Since $w > 1$ has been chosen in the controller (11), (13) implies that $\dot{V} < 0$ whenever $|s(t)| > \delta_0 = \frac{w\delta}{w-1}$. That is to say that $|s|$ will converges to the region of $|s(t)| \leq \delta_0 = \frac{w\delta}{w-1}$.

Thus the proof is achieved completely.

Remark 1: Since the QSMC in (11) is continuous, chattering is eliminated.

Remark 2: In fact, δ is a design parameter, in QSMC (11) therefore, one can select a sufficient small value of δ to make δ_0 as well as $\gamma_i, i=1,2,3$ arbitrarily bounded in the neighborhood of zero.

Remark 3: From the above analysis, a procedure for the robust control of chaos in the GLCS is proposed as follows.

Step 1: Select $c > -1$ to ensure a stable quasi sliding manifold.

Step 2: Obtain the switching function s from eq. (4) and select the control parameters in eqn. (11).

Step 3: Calculate the predictable error bounds $\gamma_i, i=1,2,3$ by eqs.(8)-(11) to estimate the performance of control.

Step4: QSMC from eqn. (11).

V. NUMERICAL EXAMPLE

In this section, simulation results are presented to demonstrate and verify the effectiveness of the proposed QSMC scheme. The system parameter is chosen as $k = 0.2$. The initial states are $x_1(0) = 1, x_2(0) = 1, x_3(0) = 0.5$. According to step 1 in Remark 3, we select $c = 1 > -1$ to result in a bounded quasi sliding manifold. Therefore, the switching function s is

$$s(t) = x_2 + x_1 \quad (14)$$

and the QSMC can be obtained as

$$u(t) = -w\eta \frac{s}{|s| + \delta} \quad (15)$$

with $w = 2 > 1, \delta = 0.05$.

By eqns. (8)-(10), we can predict that $|s(t)| \leq \delta_0 = 0.1$ and the states are bounded by $\gamma_1 = 0.025, \gamma_2 = 0.175$, and $\gamma_3 = 0.0016$. The simulation results are shown in Figures 2-4. Figure 2 and Figure 3 present, respectively, the corresponding $s(t)$ and state responses. The continuous QSMC control is shown in Figure 4. For the first five seconds, the system is uncontrolled and the trajectories are chaotic and the QSMC (15) become active at $t = 5$ second and the system state can be bounded by $\gamma_i, i=1,2,3$ calculated above, as we predict. In particular, it is worthy of note that the chattering does not appear due to the continuous control input as shown in Figure 4.

VI. CONCLUSIONS

In this paper, a quasi sliding mode control scheme for generalized Lorenz chaotic systems is studied. The concept of

quasi sliding mode control has been introduced firstly to avoid chattering phenomenon as frequently in the conventional sliding mode control systems. Numerical simulations have verified the effectiveness of the proposed method.

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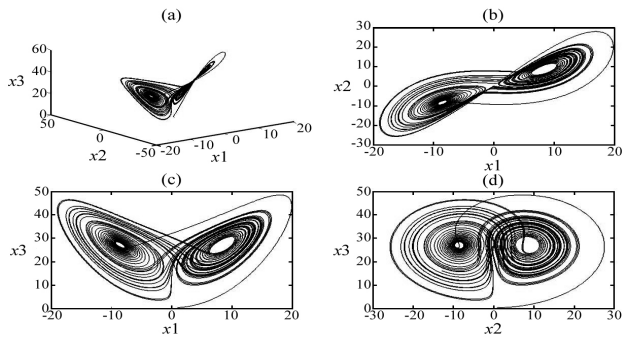


Fig. 1. (a) Trajectories of GLCS (b) Trajectories projected on the $x_1 - x_2$ plane (c) Trajectories projected on the $x_1 - x_3$ plane (d) Trajectories projected on the $x_2 - x_3$ plane.

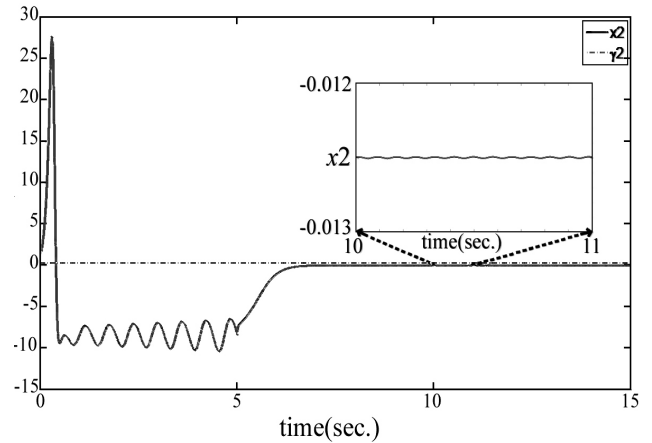


Fig. 3. (b). The state response of the controlled system.

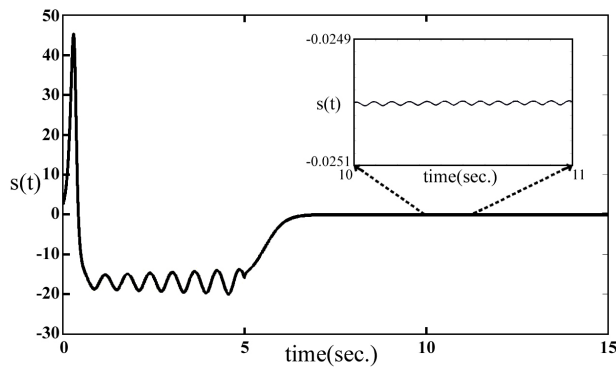


Fig. 2. The time response of switching function $s(t)$

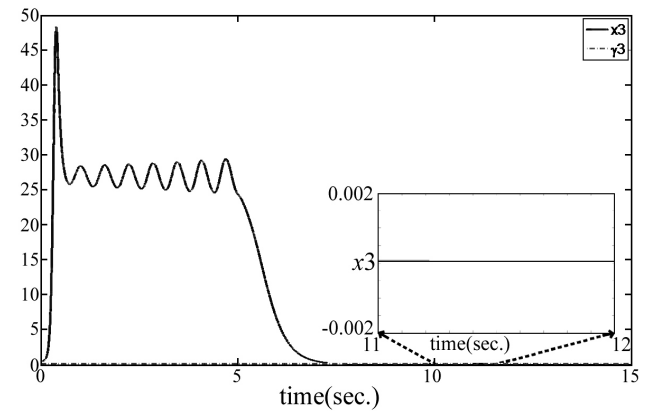


Fig. 3. (c). The state response of the controlled system.

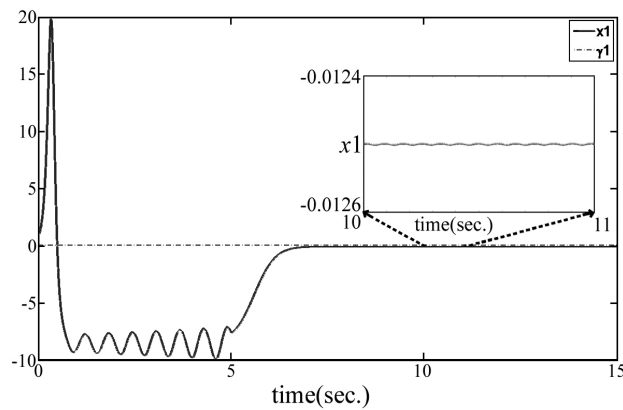


Fig. 3. (a). The state response of the controlled system.

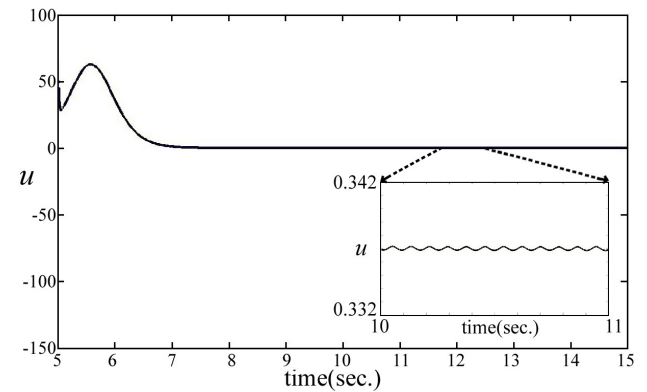


Fig. 4. . The time response of continuous QSMC (15).