Adaptive Genetic Algorithm for Fixed-Charge Transportation Problem

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Abstract—Competitive global markets oblige the firms to reduce their overall costs while maintaining the same customer service level and this can be achieved just through a precise and efficient management of their supply chain network. The Fixed Charge Transportation Problem (FCTP) which is a more comprehensive type of Transportation Problem (TP) has several applications from different aspects in this network. Since the problem is NP-hard and solving this problem with decisive methods and heuristics will be computationally time consuming and expensive, two Genetic Algorithm are applied for this problem and also two fuzzy logic controllers are developed to automatically tune two critical parameters (P_c and P_m) of one of these two GAs. Finally the results from the simple conventional GA and automatically tuned GA are compared together. This comparison demonstrated that the GA that is tuned with FLC reach the local optimum remarkably faster.

Keywords—Supply Chain Management (SCM), Logistics, Fixed Charge Transportation Problem (FCTP), Adaptive Genetic Algorithm, Fuzzy Logic Controller (FLC)

I. INTRODUCTION

IN nowadays globalized markets, since the competitiveness is dreadfully increasing, supply chain design has been gaining importance and attention [2]. Companies have to at least keep the same customer service level, while the market's competitiveness forces them to reduce their overall costs to maintain their profit margins.

A Supply Chain (SC) is a network of facilities and distribution centers that is responsible for procuring the materials, making intermediate and finished products out of those materials, and distributing the products to customers, and Supply Chain Management (SCM) is the strategy of integrating these functions together in order to facilitate the synchronization of all parts of this chain. A supply chain network consists of five major chains: R&D chain, purchasing chain, production chain, quality chain and logistic chain [20, 21].

Indubitably logistic chain is an important function of business and is evolving into strategic supply chain management. Logistics is often defined as the art of bringing the right amount of the right product to the right place at the right time and usually refers to supply chain problems [13].

Among logistics activities, transportation network design provides a remarkable potential to reduce the overall costs and also to improve the service level. The transportation problem (TP) is a well-known basic network problem which was originally proposed by Hitchcock [1]. This problem is a very complicated supply chain problem and is considered as NP-Hard problem [20, 21].

In order to make the transportation problem more practical, many research papers assume that a fixed cost is also incurred along with variable cost of each commodity. The Fixed Charge Transportation Problem (FCTP) is introduced in the origins of the Operations Research [17]. In a FCTP, a single commodity is shipped from origin (source, supply) locations to destination (sink, demand) locations. The objective is to find the combination of routes that minimizes the total variable and fixed costs while satisfying the supply and demand requirements of each origin and destination. While similar to the transportation problem, the FCTP is much more difficult to solve due to the presence of fixed costs which causes discontinuities in the objective function [19]. It has been shown that this problem is an NP-hard problem [17]. Since the problem is NP-hard, the computational time to obtain exact solution increases in a polynomial manner and very quickly becomes extremely difficult and long as the size of the problem increases.

Some old researches have turned to heuristic algorithms for solving FCTP because the methods are constrained by limits on computer time. Adlakha and Kowalski [3] proposed a simple heuristic algorithm for solving a small FCTP, which was more time consuming than the algorithms for solving a regular transportation problem. Glover, Amini, and Kochenberger [18], developed a parametric ghost image processing for this problem. Many researches consider the problem as mixed integer programming and solve it with the solutions such as the branch-and-bound and the cutting plane methods, but these methods are generally inefficient and computationally expensive [4], as

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they did not take advantage of special network structure of FCTP.

Different encoding schemes and representations are used for the Fixed-Charge Transportation Problems. Gottlieb, and Paulmann [16] developed and compared two GAs, one with permutation representation and another with matrix representation and at last they come to conclusion that GA with matrix representation yield better results. Gen, Li, and Ida [5], utilized the Prüfer number encoding to propose the spanning tree-based genetic algorithms for fixed-charge transportation problems. Su, and Zhan [15] developed a Genetic Algorithm with the sorted set of edge(S-ES) encoding scheme.

Jo, Li, and Gen [7] proposed a new feasibility criteria and repairing procedure for spanning-tree based chromosomes in GA. However Hajiaghaei-keshteli, Molla-Alizadeh-Zavardehi, and Tavakkoli-moghaddam [12], designed the chromosomes just with feasible solutions, which saved the computational time of repairing procedure in previous works. They also utilized the Taguchi experimental design to apply a robust calibration and ensure the best performance of the GA.

Since adjusting the crossover and mutation ratios has a great effect on GA's performance, a Fuzzy Logic Controller is also applied to automatic fine tuning of these ratios, according to Wang, Wang, and Hu [8].

II. MATHEMATICAL MODEL

The FCTP can be considered as a distribution problem in which there are m sources (suppliers, warehouses or factories) and n destinations (customers or demand points). Shipping the commodities are possible from each of the m sources to any of the n destinations at a shipping cost per unit c_{ij} (unit cost for shipping from source i to destination j) plus a fixed cost f_{ij}, adopted as route's founding cost. Each source $i = 1, 2, \ldots, m$ has a_i units of supply, and each customer $j = 1, 2, \ldots, n$ has a demand of b_j units. The objective is to determine which routes are to be established and the size of the shipment on those routes, so that the total cost of satisfying the supply constraints, in order to meet the demands, is minimized. Standard FCTP formulation is shown as follows:

Minimize $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} (c_{ij}.x_{ij} + f_{ij}.y_{ij}),$

Subject to:

$$\sum_{i=1}^{m} x_{ij} \geq b_j \qquad\qquad j = 1,...,n$$

$$\sum_{j=1}^{n} x_{ij} \leq a_i \qquad\qquad i = 1,...,m$$

$$\forall i, j \qquad x_{ij} \geq 0$$

$$y_{ij} = 0 \quad if \quad x_{ij} = 0$$
$$y_{ij} = 1 \quad if \quad x_{ij} > 0$$

where x_{ij} is the unknown variable to be shipped on the route (i, j) from supplier i to customer j, c_{ij} is the shipping cost per unit from supplier i to customer j. a_i is the number of units available at supplier i, and b_j is the number of units demanded at costumer j. The transportation cost for shipping per unit from supplier i to customer j is $c_{ij} \times x_{ij}$. f_{ij} is the fixed cost related to route (i, j). The transportation problem assumed to be balanced in this paper, as the unbalanced problems could simply be converted to balanced one, through adding a dummy supplier or a dummy customer.

III. GENETIC ALGORITHM

As the problem proved to be NP-hard [17], and due to taking too much time to consider all the possible combinations, utilizing the conventional methods can only be applied for the small size problems with a few number of variables. In this case, a genetic algorithm is developed for this problem.

The GA searches a problem space with a population of chromosomes, each of which represents an encoded solution. A fitness value is assigned to each chromosome according to its performance in which the more desirable the chromosome, the higher the fitness value becomes. The population evolves by a set of operators until some stopping criterion is met. A typical iteration of a GA, a generation, proceeds as follows. The best chromosomes of the current population are copied directly to the next generation (reproduction). A selection mechanism chooses chromosomes of the current population in such a way that the chromosome with the higher fitness value has a greater probability of being selected (roulette wheel). The selected chromosomes mate and generate new offspring (crossover). After the mating process, each offspring might mutate by another mechanism called mutation. The new population is then evaluated again, and the whole process is repeated [14].

A. Representation

The spanning-tree based representation is used for genetic chromosomes since it would be appropriate for transportation problems. Gen and Cheng [6], introduced the use of the Prüfer number which can equally and uniquely represent all the possible trees in a network graph, for solving various network problems.

In this paper, the Prüfer number is created from randomly generated $m + n - 2$ digits in range $[1, m+ n]$. The Prüfer number would be considered feasible only if the number of connected arcs to suppliers' side is equal to the number of connected arcs to customers' side, which is formulated as follows:

$$\sum_{i=1}^{m} (L_i + 1) = \sum_{i=m+1}^{m+n} (L_i + 1)$$

Previously all the researches consider this criterion just in repairing phase, but Hajiaghaei-keshteli, Molla-Alizadeh-Zavardehi, and Tavakkoli-moghaddam [12], put an end to

it. They save the time by eliminating the need for recognizing the infeasible solutions and repairing them by generating only feasible chromosomes. They convert the above formula to the following formulas:

$$\sum_{i=1}^{m} L_i = n-1$$

and

$$\sum_{i=m+1}^{m+n} L_i = m-1$$

A string of n-1 digits from set of suppliers and a string of m-1 digits from set of customers are generated randomly, and finally the two produced strings are combined together at random to design a feasible chromosome.

The uniqueness of the Prüfer number for a special shipment strategy can be determined through decoding it. The same decoding procedure as [12] is utilized in this paper:

Step 1: Let P(T) be the original Prüfer number and let P'(T) be the set of all the nodes that are not part of P(T) and designed as eligible for consideration.
Step 2: Repeat the following process – (2.1) – (2.5) – until no digits are left in P(T).
 2.1 Let i be the lowest numbered eligible node in P'(T). Let j be the leftmost digit of P(T).
 2.2 If i and j are not in the same set O or D, add the edge (i, j) to tree T. Otherwise, select the next digit k from P(T) that is not included in the same set with i, exchange j with k, and add the edge (i,k) to the tree T.
 2.3 Remove j (or k) from P(T) and i from P'(T). If j (or k) does not occur anywhere in the remaining part of P(T), put it into P'(T).
 2.4 Assign the available amount of units to x_{ij} = min{a_i,b_j} (or x_{ik} = min{a_i, b_k}) to the edge (i, j) or (i,k)) where $i \in O$ and $j, k \in D$.
 2.5 Update availability $a_i = a_i - x_{ij}$ and $b_j = b_j - x_{ij}$ (or $b_k = b_k - x_{ik}$).
Step 3: If no digits remain in P(T) then there are exactly two nodes, i and j, still eligible in P'(T) for consideration. Add edge (i, j) to tree T and form a tree with $m+n-2$ edges.
Step 4: If there are no available units to assign, then stop. Otherwise, there are y plants with $a > 0$ units, and z costumers with $b > 0$ demands yet. One of these states occurs:
 I.   If $y = 1$ and $z = 1$, Add the edge between the plant and the customer to the tree and assign the available amount to the edge.
 II. If $y > 1$ and $z = 1$, Add the edge between the plants and the customer to the tree and assign the available amount to the edge.
 III. If $y = 1$ and $z > 1$, Add the edge between the plant and the customers to the tree and assign the available amount to the edge.
 IV. If $y > 1$ and $z > 1$, Consider them as a new transportation model with y plants and z customers, then generate Prüfer number, and Repeat step 1 to 4.

If a cycle exists; remove the edge that is assigned zero flow. A new spanning tree is formed with $m+n-2$ edges.

B. Initialization

Each generated chromosome is considered as an individual solution to the problem. In the first generation chromosomes are generated as many as population size. The random method is applied for generating the initial population.

C. Selection mechanism

As the total transportation cost including variable and fixed costs should be minimized in this problem, better solutions are those results with lower objective functions. The higher fitness value considered the better chromosome, so the applied function is formulated as follows:

$$\text{Fitness Value} = \frac{1}{\text{Objective Function}}$$

Since the Roulette-Wheel selection mechanism is deployed, the chromosomes with higher fitness values have more chance to be selected.

D. Genetic operators

Reproduction
The p_r% of the chromosomes with higher fitness values are transferred to the next generation (elite strategy).

Crossover
Crossover combines the two selected chromosomes' features in order to create two better offsprings. The remaining (p_c=1-p_r%) of the chromosomes in next generation going to be generated from crossover operation. The one-point crossover is used in the algorithm in such a way that the feasibility criterion has been met. It means that after selecting the point from which the parents are separated into two different parts, the first parts are directly copied to the associated offspring, but the second part is inherited from the opposite parent, regarding the two previously mentioned feasibility criteria.

Mutation
The mutation operator is an important process of any successful GA that reorganizes the structure of the genes so that the algorithm can escape from searching just in local optimum area. It can also be regarded as a simple local search technique.
Performing the crossover operation, offspring are going to be mutated with the probability of p_m. It means that a random number in range [0,1] is generated for each of the offsprings. If this number was less than p_m then the

mutation operation is going to be performed. The inversion mutation is utilized in this paper. In this operator two genes are randomly selected from an offspring and their positions are inverted. Since the chromosomes are feasible after crossover, the mutated offsprings are also going to be feasible.

IV. FUZZY LOGIC CONTROLLER

Being one of the most popular applications of Lotfi Zadeh's Fuzzy Set Theory [11], Fuzzy Logic Controllers (FLCs) are demonstrated initially by Mamdani [10] in 1974. FLCs are knowledge-based controllers that are usually derived from a knowledge acquisition process or are automatically synthesized from a self-organizing control architecture [9].

Creating a balance between exploration and exploitation plays a significant role in GA's performance. Therefore, regulating the GA parameters such as population size, number of generations, crossover probability, and mutation probability is one of the crucial subjects for providing this balance.

Two FLCs are used for automatically tuning the p_c and p_m based on the changes in the average fitness of the population [8]. Let $\Delta f(t)$ be the difference between average fitness function of the t_{th} and t-1_{th} generations and $\Delta f(t-1)$ be the difference between t-1_{th} and t-2_{th} generations, then the crossover and mutation ratios for the next generation is done using the following IF–THEN concept:

- If $|\Delta f(t) - \Delta f(t-1)| < \varepsilon$, is a small positive number near to zero), then rapidly increase the pc and pm for the next generation.
- If $\Delta f(t) - \Delta f(t-1) < -\varepsilon$ then decrease the pc and pm for the next generation.
- If $\Delta f(t) - \Delta f(t-1) > \varepsilon$ then pc and pm for the next generation.

The inputs for both FLCs are $\Delta f(t)$ and $\Delta f(t-1)$. The output for one FLC is the change in crossover ratio, $\Delta c(t)$, and for another one is the change in mutation ratio, $\Delta m(t)$. The membership functions of two inputs are illustrated in fig. 1, the membership function of $\Delta c(t)$ is illustrated in fig. 2, and the membership function of $\Delta m(t)$ is illustrated in fig. 3.
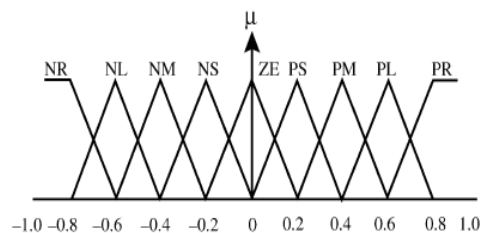

Fig. 1. The membership function for both inputs: $\Delta f(t-1)$ and $\Delta f(t)$
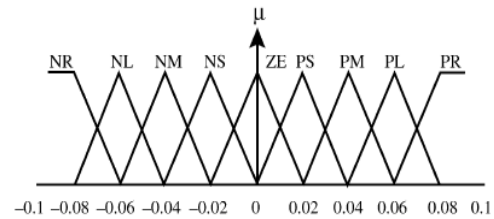

Fig. 2. The membership function for the output $\Delta c(t)$
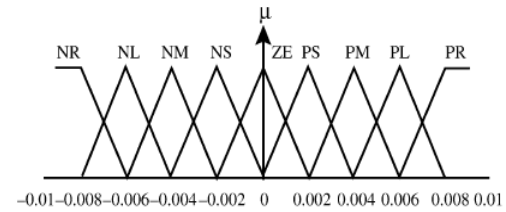

Fig. 3. The membership function for the output $\Delta m(t)$

NR, negative larger; NL, negative large; NM, negative medium; NS, negative small; ZE, zero; PS, positive small; PM, positive medium; PL, positive large; PR, positive larger.

After obtaining the inputs, $\Delta f(t)$ and $\Delta f(t-1)$, from GA, the FLC sets the inputs to their associated membership functions. According to available rules in fuzzy decision table, which is shown in table.1, a fuzzy value is extracted from each of the nine membership functions, and at last these function values are converted to one crisp output for each FLC.

Table 1   Fuzzy Decision Table

		$\Delta f(t-1)$								
		NR	NL	NM	NS	ZE	PS	PM	PL	PR
	NR	NR	NL	NL	NM	NM	NS	NS	ZE	ZE
	NL	NL	NL	NM	NM	NS	NS	ZE	ZE	PS
	NM	NL	NM	NM	NS	NS	ZE	ZE	PS	PS
	NS	NM	NM	NS	NS	ZE	ZE	PS	PS	PM
$\Delta f(t)$	ZE	NM	NS	NS	ZE	PM	PS	PS	PM	PM
	PS	NS	NS	ZE	ZE	PS	PS	PM	PM	PL
	PM	NS	ZE	ZE	PS	PS	PM	PM	PL	PL
	PL	ZE	ZE	PS	PS	PM	PM	PL	PL	PR
	PR	ZE	PS	PS	PM	PM	PL	PL	PR	PR

After achieving the outputs, the crossover probability and mutation probability for the next generation, t+1, is calculated through following equations:

$$Pc(t + 1) = Pc(t) + \Delta c(t)$$

$$Pm(t + 1) = Pm(t) + \Delta m(t)$$

where $P_c(t)$ is the crossover ratio at generation t, and $P_m(t)$ is the mutation ratio at generation t.

V. NUMERICAL EXPERIMENTS

In order to investigate the proposed algorithm's effectiveness and efficiency, its results are compared with the results obtained from simple GA (without FLC).

The problem sizes, stopping criterion, and population size are the same as Jo, Li, and Gen [7]. The problem sizes, total supplies (or total demands) and the ranges of the fixed costs which are generated randomly are demonstrated in table 2. The unit variable costs are integer values in the range 1 to 8 in both test problems which are shown in tables 3 and 4.

Table 2. Total supply (demand) and ranges of fixed cost

Problem size (m×n)	Total supply	Range of fixed cost
4×5	275	[50,100]
5×10	1485	[150,500]

Each problem size is run 10 times. Other genetic algorithm parameters that are common in both test problems are, *population-size*= 100, P_c= 0.5, and P_m= 0.3. The maximum number of generation in this test problem is 500.

Table 3. Unit variable cost and fixed costs in 4×5 problem

Plants	Costumers									
	Shipping costs c_{ij}					Fixed costs d_{ij}				
	1	2	3	4	5	1	2	3	4	5
1	8	4	3	5	8	60	88	95	76	97
2	3	6	4	8	5	51	72	65	87	76
3	8	4	5	3	4	67	89	99	89	100
4	4	6	8	3	3	86	84	70	92	88

The local optimal solution with the objective function of 1484 was found in both algorithms, but with a major difference. Running each algorithm ten times, the average generation number for getting to this local optima was 43 for the GA-FLC, while it was 72 for the simple GA. This is illustrated in fig. 4.
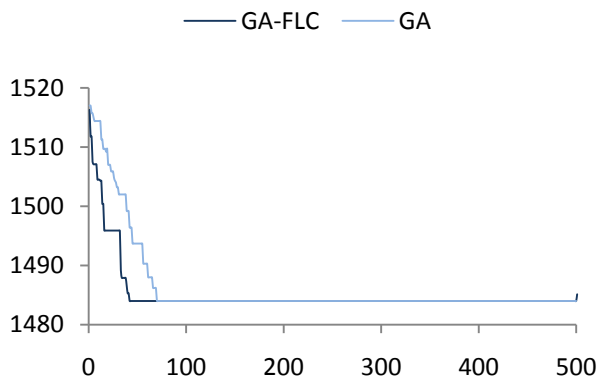

Fig. 4. Comparing GA-FLC and GA in test problem 1

For the test problem 2, which has the size 5×10, the maximum number of generations is 1000.

Running both algorithms for test problem 2, the comparative result is represented in fig. 5. As it can be perceived from this figure, GA-FLC has more expeditious steps towards the local optima. Additionally, the simple GA represents 6442 as the average of best objective functions after running for 1000 generations; while the GA-FLC performs better and exhibits 6406 as its final outcome.
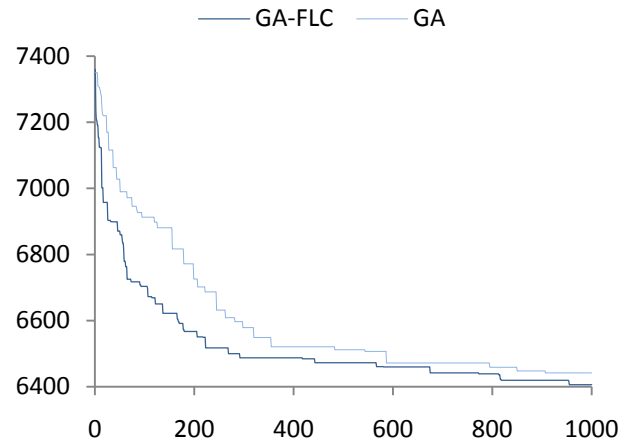

Fig. 5. Comparing GA-FLC and GA in test problem 2

VI. CONCLUSION AND FUTURE WORKS

This paper considered a transportation network between two parties in a supply chain, *i.e* supplier and customer, and tried to reduce the overall transportation costs to its minimum point, regarding both variable (per shipping product) and fixed (per route) costs, in such a way that all supply constraints, and also all demand constraints should be satisfied.

The spanning-tree based genetic algorithm was employed to solve the non-linear fixed charge transportation problem, which is considered to be NP-hard and couldn't be solved by traditional methods. For obtaining better results from GA, a Fuzzy Logic Controller is also employed to automatically tune the crossover probability and mutation probability of the algorithm. Comparing the simple GA and the GA with FLC, the latter showed an outstanding speed towards finding the local optimum.

Since FCTP is a kind of network model that can be used in several various fields such as transportation, computer science, manufacturing, decision support systems, scheduling, and *etc.,* there are great potential opportunities for developing further research in this field, for instance tuning more critical parameters from a genetic algorithm dynamically, such as population size or maximum number of generations, or creating a heuristic to find an appropriate initial population, or even adding a local search technique like simulated annealing to genetic algorithm.

Table 4. Unit variable cost and fixed costs in 5×10 problem

| | Costumers | | | | | | | | | | | | | | | | | | | |
| | Shipping costs c_{ij} | | | | | | | | | | Fixed costs d_{ij} | | | | | | | | | |
Plants	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
1	8	4	3	5	2	1	3	5	2	6	160	488	295	376	297	360	199	292	481	162
2	3	3	4	8	5	3	5	1	4	5	451	172	265	487	176	260	280	300	354	201
3	7	4	5	3	4	2	4	3	7	3	167	250	499	189	340	216	177	495	170	414
4	1	2	8	1	3	1	4	6	8	2	386	184	370	292	188	206	340	205	465	273
5	4	5	6	3	3	4	2	1	2	1	156	244	460	382	270	180	235	355	276	190

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