Harmonic Elimination of Space Vector Modulated Three Phase Inverter

Nisha G. K., *Member, IAENG*, Ushakumari S. and Lakaparampil Z. V.

***Abstract*--Pulse Width Modulation (PWM) inverters play a major role in the field of power electronics. Space Vector Modulation (SVM) is the popular PWM method and possibly the best among all the PWM techniques as it generates higher voltages with low total harmonic distortion and works very well with field oriented (vector control) schemes for motor control. High quality output spectra can be obtained by eliminating several lower order harmonics by adopting a suitable harmonic elimination technique. Selective Harmonic Elimination (SHE) technique offers an apt control for the harmonic spectrum of a given voltage waveform generated by an inverter for induction machine drives. In this paper, a mathematical model of space vector modulated three phase inverter is generated. Further, a method to eliminate the most significant harmonic components of line voltage waveforms of the inverter by SHE is also formulated. The harmonic spectrum analysis of the proposed model shows the effect of eliminating the lower order harmonics in SVM. Simulation results are implemented to confirm the effectiveness of the proposed model.**

***Index Terms*--Space Vector Modulation, Selective Harmonic Elimination, Modulation Index, Weighted Total Harmonic Distortion.**

I. INTRODUCTION

Inverters are commonly employed for Variable Speed AC Drives (VSD), Uninterruptible Power Supplies (UPS), Static Frequency Changers (SFC) etc. Among these, VSD continues to be the fastest growing applications of inverters. An inverter used for this purpose should have the capability of varying both voltage and frequency in accordance with speed and other control requirements.

Voltage Source Inverters are generally classified into two types viz, square-wave and pulse width modulated. These inverters were introduced in early 1960s when force-commutation technique was developed. The major disadvantage of this inverter is that, for low or medium power applications the output voltage contains lower order harmonics. This type of drives has been largely superseded by Pulse Width Modulation (PWM) drives and there have been a number of clear trends in the development of PWM concepts and strategies since 1970s [1]. In the mid of 1980s, a form of PWM called Space Vector Modulation (SVM) was proposed which claimed to offer significant advantages over natural and regular sampled PWM in terms of performance, ease of implementation and maximum transfer ratio [2].

The classic SVM strategy, first proposed by Holtz [3, 4] and Van der Broeck [5], is now very popular due to its simplicity and good operating characteristics, leads to generation of specific sequence of states of the inverter. In this method the converter is treated as a single unit, which can assume a finite number of states depending on the combination of states of its controlled switches. In a two-level converter, combinations result in six active (non-zero) states and two zero states. In digital implementation of the SVM, the aim is to find an appropriate combination of active and zero vectors so that the reference space vector (which represents the reference voltage) is best approximated [6]. This technique is also applied to three-level inverters and expanded to the multiple levels [7, 8]. The application of SVM to a variable speed electric drive is applied in [9], and a switching sequence is proposed for a multilevel multiphase converter such that it minimizes the number of switching.

One of the major issues faced in power electronic design is the reduction of harmonic content in inverter circuits. All PWM schemes generate inverter voltage waveforms which contain a rich harmonic spectrum [10]. This spectrum ordinarily contains the desired fundamental component and clusters of sidebands about integer multiples of the switching frequency. As the switching frequency is usually limited only by the maximum permissible level of inverter switching losses, the harmonic voltages are at reasonably high frequencies in modern inverters and do not affect the fundamental system behaviour. However, each of the harmonic voltages introduced by the modulation process does contribute undesirable high frequency copper, iron and stray-load losses in three phase machine. One of the solutions to enhance the harmonic free environment in high power converters is to use PWM control techniques. Several techniques of modulation have been proposed for this purpose. According to the manner in which the techniques operate, they can be divided into analog and digital techniques. In the former technique, the switching angles occur at the crossing of two waves, thus giving to the PWM wave. In the latter technique the switching angles are chosen so as to directly affect the harmonic content [11]. This is done by eliminating some harmonics [12, 13] and by minimizing a figure of current distortion in the load [14, 15]. The commonly available technique is Selective Harmonic Elimination (SHE) method at fundamental frequency, for which transcendental equations characterizing harmonics are solved to compute switching angles. Newton-Raphson method is the popular method used to solve the set of

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equations. Recently, there are renewed interests to solve traditional calculus based problems using non-conventional methods such as Evolutionary algorithm (EA) [16, 17].

The paper is organized as follows. Section II provides a brief review of mathematical model of SVM. Section III presents a brief description of the selective harmonic elimination technique. Section IV provides MATLAB/ Simulink model of space vector modulated three phase inverter and the simulation results.

II. MATHMATICAL MODEL OF THREE PHASE INVERTER BASED ON SVM

Any three time varying quantities, which always sum to zero and are spatially separated by 120° can be expressed as space vector. As time increases, the angle of the space vector increases, causing the vector to rotate with frequency equal to the frequency of the sinusoids. A three phase system defined by $V_a(t)$, $V_b(t)$, $V_c(t)$) can be represented uniquely by a rotating vector,

$$V = V_a(t) + V_b(t)e^{j2\pi/3} + V_c(t)e^{-j2\pi/3} \tag{1}$$

where,

$$V_a(t) = V_m \sin\omega t$$
$$V_b(t) = V_m \sin(\omega t - 2\pi/3)$$
$$V_c(t) = V_m \sin(\omega t + 2\pi/3)$$

In space vector pulse width modulation, the three phase stationary reference frame voltages for each inverter switching state are mapped to the complex two phase orthogonal α-β plane. The mathematical transform for converting the stationary three phase parameters to the orthogonal plane is known as the Clark's transformation. The reference voltage is represented as a vector in this plane. In a three-phase system, the vectorial representation is achieved by the transformation given in Fig. 1.

$$\begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} = \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} \tag{2}$$

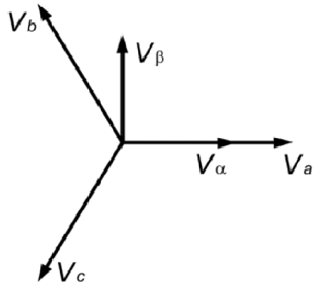

Fig. 1. Relationship between stationary reference frame and α-β reference frame

where, (V_α, V_β) are forming an orthogonal two phase system as $V = V_\alpha + jV_\beta$. Fig. 2 shows a typical two-level three phase Voltage Source Inverter (VSI) configuration. In the three phase system, each pole voltage node can apply a voltage between $+V_{DC}/2$ and $-V_{DC}/2$.

The principle of SVM is based on the fact that there are only eight possible switching combinations for a three phase inverter and the basic inverter switching states are shown in Fig. 3.

The vector identification uses a '0' to represent the negative phase voltage level and '1' to represent the positive phase voltage level. Six non-zero vectors (V_1 and V_6) shape the axis of hexagonal and the angle between any adjacent two non-zero vectors is 60° [18]. Two of these states (V_0 and V_7) correspond to a short circuit on the output, while the other six can be considered to form stationary vectors in the α-β complex plane as shown in Fig. 4. The eight vectors are called the basic space vectors.

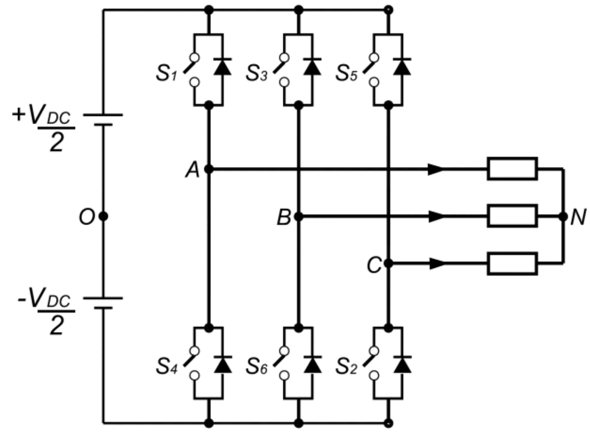

Fig. 2. Three phase voltage source inverter configuration

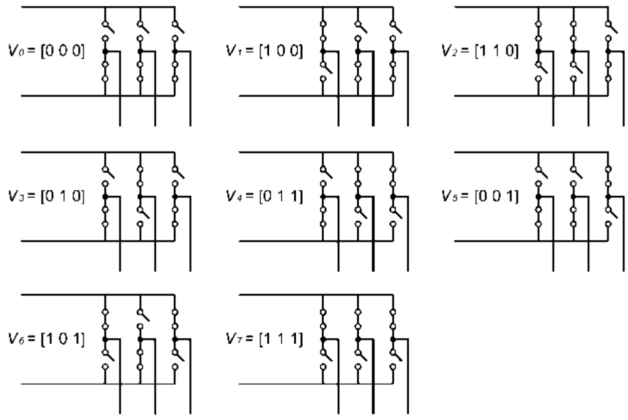

Fig. 3. Inverter switching topology

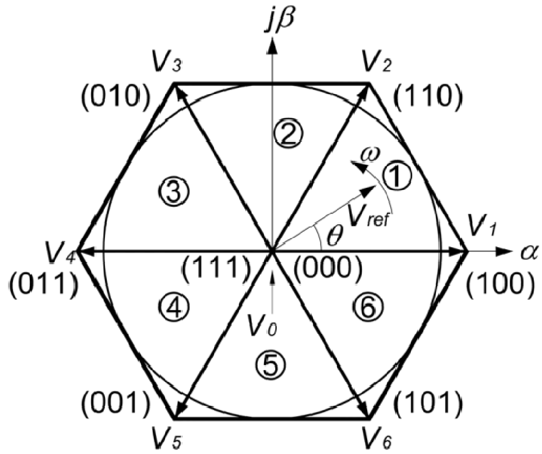

Fig. 4. Space vector hexagon

Each stationary vector corresponds to a particular fundamental angular position as shown in Fig. 5.

An arbitrary target output voltage vector, V_{ref} is formed by the summation of a number of these space vectors within

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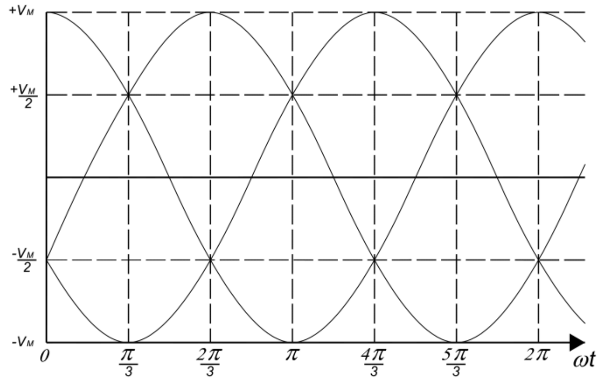

Fig.5 Inverter phasor angular positions in fundamental cycle

one switching period, which is shown in Fig. 6 for a target phasor in the first 60° segment of the plane.

Any space vector lies in the hexagon can be constructed by time averaging of the adjacent two active space vectors and zero vectors. For each switching period T_s,

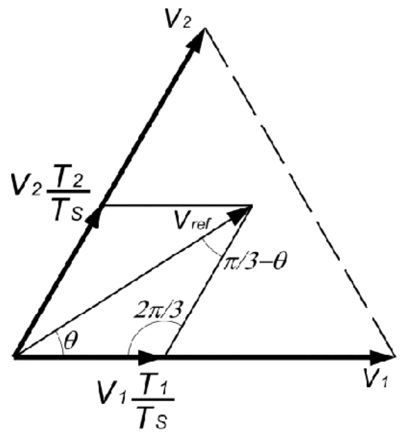

Fig. 6. Reference vector as a geometric summation of two nearest space vectors

the geometric summation can be expressed mathematically as,

$$V_{ref} = V_{ref}\angle\theta = \frac{T_1}{T_s}V_1 + \frac{T_2}{T_s}V_2 + \frac{T_0}{T_s}V_0 \tag{3}$$

where, T_1 is the time for which space vector V_1 is selected and T_2 is the time for which space vector V_2 is selected. The block diagram for generating SVM pulses is shown in Fig.7.

SVM can be implemented through the following steps:

1. Computation of reference voltage and angle (θ)

The space vector, V_{ref} is normally represented in complex plane and the magnitude as,

$$\begin{aligned} |V_{ref}| &= \sqrt{V_\alpha^{\ 2} + V_\beta^{\ 2}} \\ \theta &= \tan^{-1}\frac{V_\beta}{V_\alpha} \\ V_\alpha &= V_{ref}\cos\theta \\ V_\beta &= V_{ref}\sin\theta \end{aligned} \tag{4}$$

2. Identification of sector number

The six active vectors are of equal magnitude and are mutually phase displaced by $\pi/3$. The general expression can be represented by,

$$V_n = V_{DC}.e^{j(n-1)\pi/3}, n = 1, 2, ..., 6 \tag{5}$$

3. Computation of space vector duty cycle

The duty cycle computation is done for each triangular sector formed by two state vectors. The individual duty cycles for each sector boundary state vector and the zero state vector are given by,

$$\int_0^{T_s} V_{ref}\,dt = \int_0^{T_1} V_1 dt + \int_{T_1}^{T_2} V_2 dt + \int_{T_2}^{T_S} V_0 dt \tag{6}$$

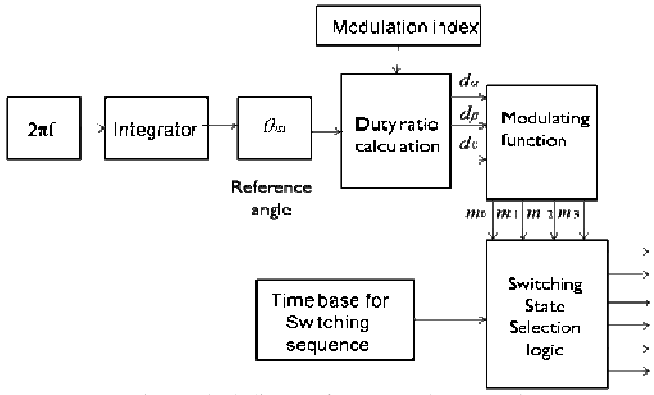

Fig. 7. Block diagram for SVM pulse generation

$$\begin{aligned} d_\alpha &= \frac{V_{ref}}{V_{DC}}.\frac{\sin(\pi/3-\theta)}{\sin(\pi/3)} = m\sin(\pi/3-\theta) \\ d_\beta &= \frac{V_{ref}}{V_{DC}}.\frac{\sin\theta}{\sin(\pi/3)} = m\sin\theta \\ d_0 &= 1 - d_\alpha - d_\beta \end{aligned} \tag{7}$$

where,

$$d_\alpha = \frac{T_1}{T_s}, d_\beta = \frac{T_2}{T_s}, d_0 = \frac{T_0}{T_s}$$

This gives switching times T_0, T_1 and T_2 for each inverter state for a total switching period, T_s. Applying both active and zero vectors for the time periods given in (7) ensures that average voltage has the same magnitude as desired.

4. Computation of modulating function

The four modulating functions, m_0, m_1, m_2 and m_3, in terms of the duty cycle for the space vector modulation scheme can be expressed as,

$$\begin{aligned} m_0 &= \frac{d_0}{2} \\ m_1 &= m_0 + d_\alpha \\ m_2 &= m_1 + d_\beta \\ m_3 &= m_0 + d_\beta \end{aligned} \tag{8}$$

5. Generation of SVPWM pulses

The required pulses can be generated by comparing the modulating functions with the triangular waveform. A symmetric seven segment technique is to alternate the null vector in each cycle and to reverse the sequence after each null vector. The switching pulse pattern for the three phases in the six sectors can be generated. A typical seven segment switching sequence for generating reference vector in sector one is shown in Fig. 8.

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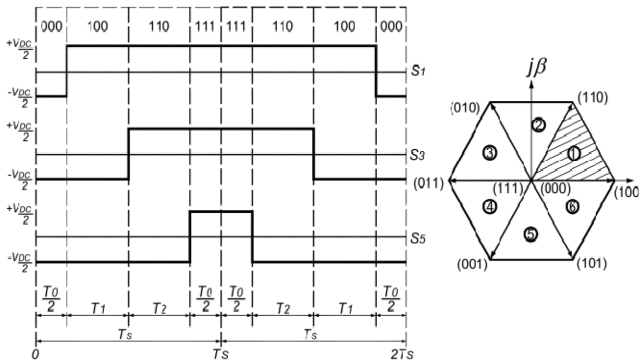

Fig. 8. Switching logic signals

III. SELECTIVE HARMONIC ELIMINATION

The undesirable lower order harmonics can be eliminated by the popular selective harmonic elimination method, which is based on the harmonic elimination theory developed by Patel [9, 10]. Fig. 9 shows the gating signals and output voltage of an inverter, in which S_1 is turned on at various switching angles α_1, α_3,….., α_{N-1} and S_4 is turned off at α_2, α_4,….., α_N per quarter cycle. At each instant, three required firing pulses are generated for upper switches. These three switches can be simply inverted to obtain the other three pulses for bottom switches. The firing commands are programmed in such a way that they provide three-phase symmetry, half-wave symmetry and quarter-wave symmetry. As shown in Fig. 10, the quarter-wave symmetry is preserved which will eliminate the even harmonics. The fundamental component can be controlled and a selected low order harmonics can be eliminated by the proper choice of SVM switching angles.

By applying Fourier series analysis, the output voltage can be expressed as,

$$V(t)=\sum_{n=1,3,5}^{\infty}\frac{4}{n\pi}[V_1\cos(n\alpha_1)\pm V_2\cos(n\alpha_2)\pm\ldots\pm V_N\cos(n\alpha_N)]\sin(n\omega t) \tag{9}$$

where, N is the number of switching angles per quarter cycle, and V_1, V_2 … V_N are the level of DC voltages. In this expression, the positive sign implies the rising edge, and the negative sign implies the falling edge. The solution must satisfy the condition.

$$0\le\alpha_1\le\alpha_2\le\ldots\ldots\le\alpha_N\le\frac{\pi}{2}$$

However, if the switching angles do not satisfy the condition, this method no longer exists. From (9), it can be seen that the output voltage has no even harmonics because the output voltage waveform is odd quarter-wave symmetry. It can also be seen that the peak values of these odd harmonics are expressed in terms of the switching angles α_1, α_2,..,α_N. Based on the harmonic elimination theory, for eliminating the n^{th} harmonics,

$$\cos(n\alpha_1)\pm\cos(n\alpha_2)\pm\ldots\ldots\pm\cos(n\alpha_N)=0 \tag{10}$$

Therefore, an equation with N switching angles will be used to control the N different harmonic values. Generally, an equation with N switching angles is used to determine the fundamental frequency value, and to eliminate $N-1$ low order harmonics [18]. The set of equations given by (9) is

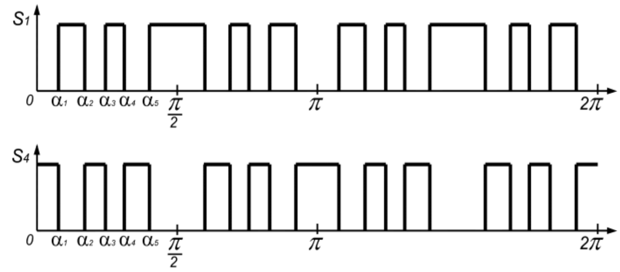

Fig. 9. Gating signals

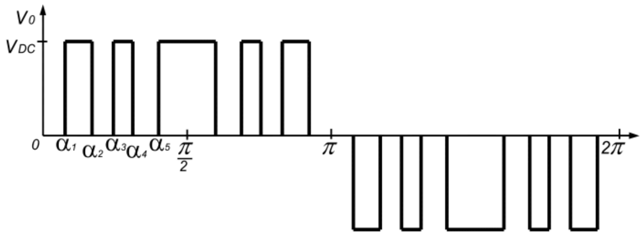

Fig. 10. Output line voltage

nonlinear since they are trigonometric functions of the variables α_1, α_2,………,α_N . Newton's iteration method is the convenient way to calculate the angles [19]. In this, N equations can be written in vector form as,

$$f_i(\alpha_1,\alpha_2\ldots.\alpha_N)=\begin{bmatrix} f_1(\alpha_1,\alpha_2\ldots..\alpha_N) \\ f_2(\alpha_1,\alpha_2\ldots..\alpha_N) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ f_M(\alpha_1,\alpha_2\ldots.\alpha_N) \end{bmatrix} \tag{11}$$

it can be linearized by,

$$\begin{aligned} &f_i(\alpha_1,\alpha_2\ldots..\alpha_N) \\ &=f(\alpha_{1,0},\alpha_{2,0}\ldots..\alpha_{N,0})+grad f(\alpha_1,\alpha_2\ldots..\alpha_N)\Delta\alpha \end{aligned} \tag{12}$$

The algorithm for the Newton's method is as follows [17]:

1) Assign a set of initial values for switching angles.
$$\alpha_0=\left[\alpha_{1,0},\alpha_{2,0}\ldots..\alpha_{N,0}\right]$$
2) Calculate, $f(\alpha_0)$
3) Determine *grad* function
4) Compute $\Delta\alpha$ by using Gauss elimination method
$$f(\alpha_{1,0},\alpha_{2,0}\ldots..\alpha_{N,0})+grad f(\alpha_1,\alpha_2\ldots..\alpha_N)\Delta\alpha=0$$
5) Update the value of α
$$\alpha_0=\alpha_0+\Delta\alpha$$
6) Repeat (1) to (5) until $\Delta\alpha$ converges to a very small value.

These can be achieved in off line and the values of switching angles for particular modulation index are stored as look up tables. These tables are stored in a programmable memory and for example, microcomputer or microcontroller board which has been programmed to accept the value of modulation index and generate the corresponding switching angles.

The Weighted Total Harmonic Distortion (WTHD) method is used to indicate the quantity of harmonics contents in the output waveforms. It is calculated in the same way as THD, but each harmonic component is divided by its order, so that higher order harmonics receive lower weight and contribute less in this figure of merit [20]. An upper limit of 50 is often recommended for the calculation

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of the WTHD [21]. The definition of WTHD in an inverter circuit is given as,

$$WTHD = \frac{\sqrt{\sum_{n=2}^{\infty}\left(\frac{V_n}{n}\right)^2}}{V_1} \tag{13}$$

which can be simplified by computing for n = *5, 7, 11, 13, ...49* due to the reasons that the even and the triplen harmonics do not appear in the line voltages.

IV. Simulation Model and Results

A three-phase inverter with a balanced star connected RL load is considered. A complete mathematical model of the SVPWM is developed and simulated using MATLAB/ Simulink to investigate the performance of a three phase inverter. The model was run according to the following table:

TABLE I
SIMULATION PARAMETERS

DCVoltage (V_{DC})	Fundamental Frequency(f)	Switching Frequency(fs)	m	R (Ω)	L (mH)
400V	50Hz	800Hz	0.8	10.0	15.0

Fig. 11 shows the developed model of three phase inverter. For motor drive applications, it is necessary to eliminate low order harmonics up to 19. Hence, the 5th, 7th, 11th, 13th, 17th and 19th harmonics need to be theoretically eliminated from line voltage.

Hence Eq. (10) can be rewritten as,

$$\begin{aligned}
m &= \cos(\alpha_1) + \cos(\alpha_2) + ... + \cos(\alpha_6) \\
0 &= \cos(5\alpha_1) + \cos(5\alpha_2) + ... + \cos(5\alpha_6) \\
0 &= \cos(7\alpha_1) + \cos(7\alpha_2) + ... + \cos(7\alpha_6) \\
&: \\
&: \\
0 &= \cos(17\alpha_1) + \cos(17\alpha_2) + ... + \cos(17\alpha_6) \\
0 &= \cos(19\alpha_1) + \cos(19\alpha_2) + ... + \cos(19\alpha_6)
\end{aligned} \tag{15}$$

where,

m is the modulation index defined as

$$m = \frac{\pi V_1}{4V_{DC}} \qquad (0 < m \le 1) \tag{16}$$

Solution of the above set of nonlinear equation determines the six switching angles, namely, α_1, α_2, α_3, α_4, α_5 and α_6.

Simulation Results: Sector corresponds to the location of voltage in the circular locus traced by it and is divided into six sectors of 60° each which is shown in Fig. 12. Figs. 13, 14 and 15 show the generated phase voltage, line voltage and pole voltage respectively. In a two level VSC the voltage levels are $+V_{DC}/2$ and $-V_{DC}/2$, where V_{DC} is the DC link voltage. The line voltage alternates between $+V_{DC}$ and $-V_{DC}$ while the phase voltage can assume the values of $\pm 2V_{DC}/3$ and $\pm V_{DC}/3$. Fig. 16 shows the line current, because of the inductive nature of the load, higher order harmonics have been filtered out and the current waveform is sinusoidal in nature.

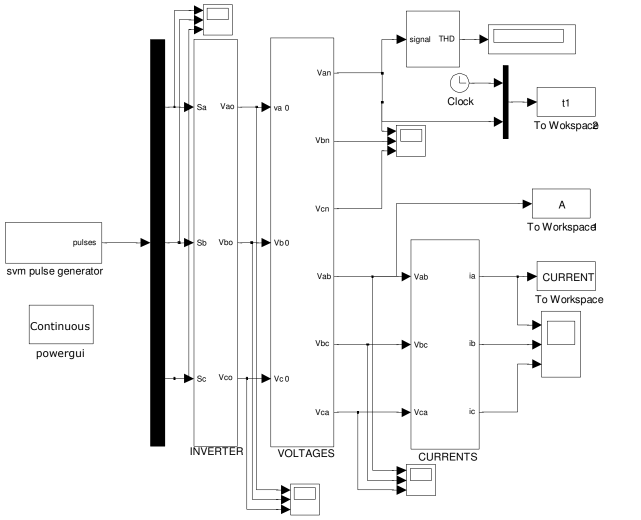

Fig.11. MATLAB/Simulink model of three phase inverter based on SVM

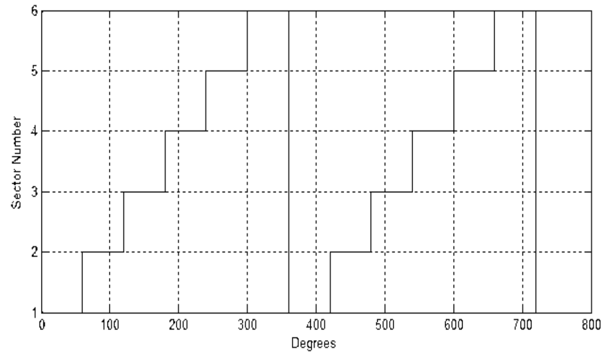

Fig.12. Sector selection

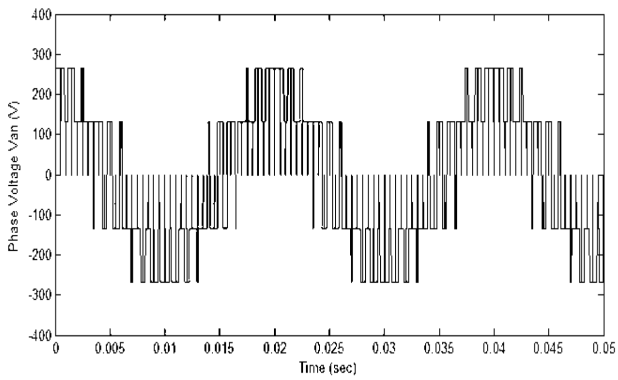

Fig.13. Line to neutral Voltage (m = 0.8)

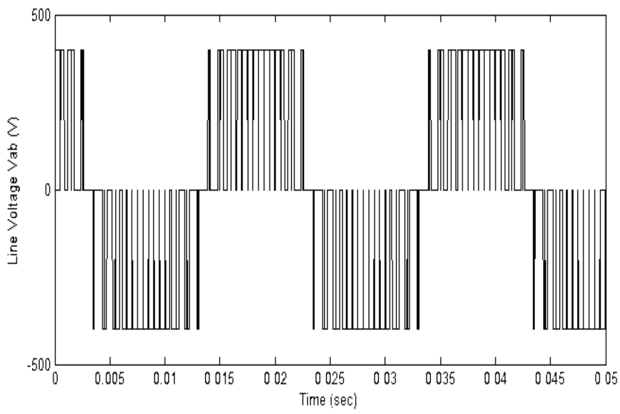

Fig.14. Line to line Voltage (m = 0.8)

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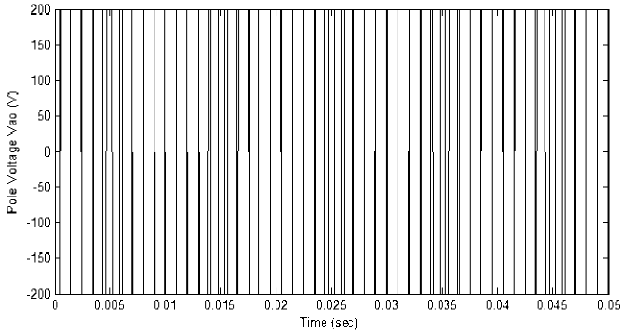

Fig.15. Pole voltage (m = 0.8)

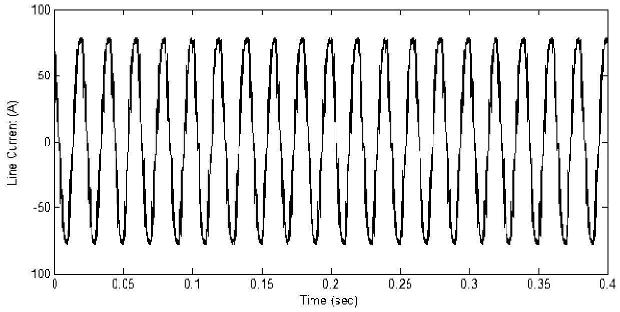

Fig.16. Line current for m = 0.8

Number of switching angles Vs switching frequency:
Table II shows the relationship between the number of switching angles per quarter cycles (*N*) of the output waveform and the switching frequency (*fs*) from the simulation result. For eliminating harmonics up to 19, *N* must be not less than 7. From table II, the suitable value of switching frequency corresponds to N = 7 as 800Hz.

TABLE II
SWITCHING ANGLES FOR SWITCHING FREQUENCIES

fs (Hz)	600	800	1000	1500	2000	3000
N	5	7	9	13	17	31

Selective harmonic elimination: Fig. 17 shows harmonic spectrum of line voltage before elimination. Fig. 18 shows the line voltage after selected harmonic elimination of 5^{th}, 7^{th}, 11^{th}, 13^{th}, 17^{th} and 19^{th} with number of switching angles per quarter cycle equal to 7, for different modulation indices of 0.8, 0.4 and 0.2. The normalized magnitudes before and after the elimination for the above values of modulation index are presented in Table III. The harmonic spectrum after the elimination of harmonics is shown in Fig. 19.

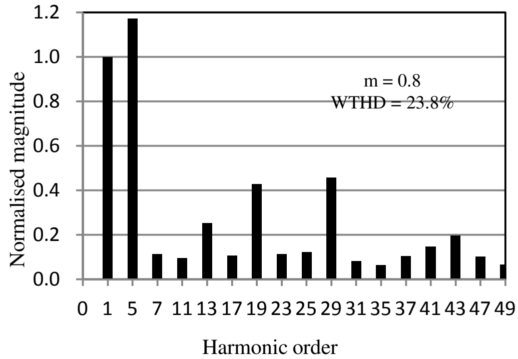

Fig.17. Harmonic spectrum for Line Voltage (before elimination)

Line voltage WTHD Vs modulation index: A comparison of line voltage WTHD before and after harmonic elimination for different values of modulation index of 0.8, 0.4 and 0.2

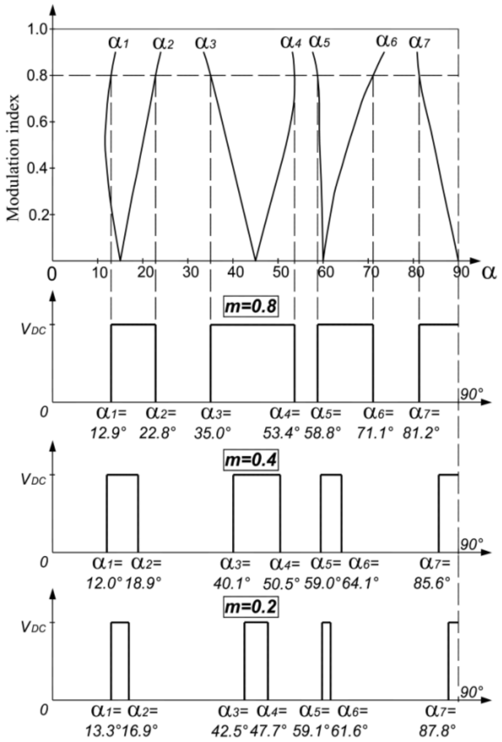

Fig.18. Selective Harmonic Elimination (m = 0.8, 0.4 & 0.2)

TABLE III
NORMALISED MAGNITUDE OF LINE VOLTAGE

Harmonic order	m=0.2		m=0.4		m=0.8	
	before	after	before	after	before	after
1	1.00	1.00	1.00	1.00	1.00	1.00
5	1.08	0.00	1.13	0.00	1.17	0.00
7	0.06	0.00	0.04	0.00	0.11	0.00
11	0.21	0.00	0.14	0.00	0.10	0.00
13	0.88	0.00	0.73	0.00	0.25	0.00
17	0.33	0.00	0.36	0.00	0.11	0.00
19	1.20	0.00	1.17	0.00	0.43	0.00
23	0.56	0.84	0.06	0.40	0.11	0.27
25	0.13	0.90	0.05	0.61	0.12	0.01
29	0.80	0.03	0.09	0.17	0.46	0.03
31	0.36	0.11	0.23	0.25	0.08	0.14
35	0.03	0.05	0.14	0.06	0.06	0.07
37	0.26	0.06	0.16	0.13	0.11	0.04
41	0.39	0.08	0.18	0.15	0.15	0.04
43	0.51	0.01	0.40	0.00	0.20	0.07
47	1.03	0.50	0.10	0.14	0.10	0.14
49	0.02	0.59	0.15	0.05	0.07	0.06

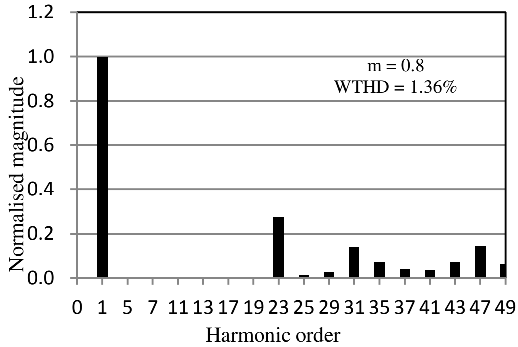

Fig.19. Harmonic spectrum for Line Voltage (after elimination)

are presented in Table IV. The variation of WTHD in line voltage as a function of modulation index for the model after eliminating 5^{th}, 7^{th}, 11^{th}, 13^{th}, 17^{th} and 19^{th} harmonics is shown in Fig. 20. These curves show that a marginal

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improvement of WTHD is possible by eliminating harmonics up to 19th order.

TABLE IV
COMPARISON OF WTHD

	m= 0.2		m= 0.4		m= 0.8	
	before	after	before	after	before	after
WTHD	24.2%	5.38%	24.1%	3.22%	23.8%	1.36%

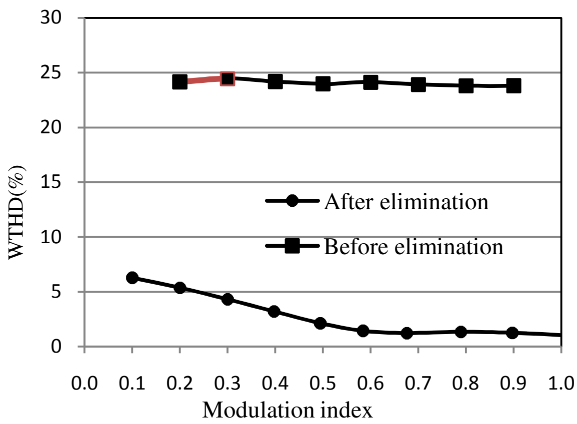

Fig. 20. Line voltage WTHD Vs Modulation index

V. CONCLUSIONS

In this paper, mathematical model of a space vector modulated three phase inverter is formulated and simulated using MATLAB/Simulink. Selective harmonic elimination method is systematically applied to the generated waveform and the harmonic performance of the waveform is studied.

Simulation results reported in this paper confirm that the developed model generates waveforms with complete symmetry. Amplitude of line to line voltage is as high as DC bus voltage in SVM technique. With the increased output voltage, the user can design the motor control system with reduced current rating, which helps to reduce inherent conduction loss of the voltage source inverter.

It is found that by adopting the selective harmonic elimination technique, least harmonic distortion (using WTHD) is achievable with modulation index ranging from 0.5 to 0.9. This has considerable practical use for variable-speed AC motor drive applications, where harmonics in the output of the inverter pose serious problems in the motor performance. The developed model allows the user to select parameters according to their design requirements. The model can be adapted for applications like UPS with fixed frequency or in application where the inverter has to track the grid.

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REFERENCES

[1] B. K. Bose, “Adjustable speed AC drives - A technology status review,” *Proceedings of the IEEE- IECON*, vol. 70, pp. 116-135, Feb. 1982.

[2] D. G. Holmes and T. A. Lipo, “Pulse Width Modulation for Power Converters: Principles and Practice,” *New Jersey: Wiley* IEEE Press, 2003.

[3] J. Holtz, P. Lammert and W. Lotzkat, “High speed drive system with ultrasonic MOSFET PWM inverter and single-chip microprocessor control,” *IEEE Transactions on Industry Applications*, vol. 23, pp. 1010-1015, 1987.

[4] J. Holtz, “Pulse width modulation - a survey,” *IEEE Transactions on Industrial Electronics,* vol. 39, no. 5, pp. 410-420, Oct. 1992.

[5] H. W. Van der Broeck, H. C. Skudelny and G.V. Stanke," Analysis and realisation of a pulse width modulator based on voltage space vectors", *IEEE Transactions on Industry Applications*, vol. 24, pp. 142-150, Jan./Feb. 1988.

[6] A. Mehrizi-Sani and S. Filizadeh, “Digital Implementation and Transient Simulation of Space Vector Modulated Converters,” *IEEE Power Engineering Society General Meeting*, Montreal, QC, Canada, Jun. 2006.

[7] Fang Zheng Peng, Jih-Sheng Lai, et. al, “A Multilevel Voltage-Source Inverter with Separate DC Sources for Static Var Generation,” *IEEE Transactions on Industry Applications*, vol. 32, no. 5, pp. 1130-1138, Sep./Oct. 1996.

[8] Jose Rodriguez, J. S. Lai, and F. Z. Peng, “Multilevel Inverters: A Survey of Topologies, Controls, and Applications,” *IEEE Transactions on Industrial Electronics*, vol. 49, no. 4, pp. 724-738, Aug. 2002.

[9] O. Lopez, J. Alvarez, J. Doval-Gandoy, and F. D. Freijedo, “Multilevel multiphase space vector PWM algorithm,” *IEEE Transactions on Industrial Electronics*, vol. 55, no. 5, pp. 244-251, May 2008.

[10] J. F. Moynihan, M. G. Egan and J. M. D. Murphy,” Theoretical spectra of space vector modulated waveforms,” *IEE Proc. Electrical Power Appl*ications, vol. 145, no. 1, Jan. 1998.

[11] G. S. Buja, ”Optimum Output Waveforms in PWM Inverters,” *IEEE Transactions on Industry Applications*, vol. 16, pp. 830-836, Nov/Dec. 1980.

[12] H. S. Patel and R. G. Hoft, ’’Genealized techniques of harmonic elimination and voltage control in thyristor inverters: Part I- Harmonic elimination,” *IEEE Transactions on Industry Applications*, vol. 9, pp. 310-317, May/June 1973.

[13] H. S. Patel and R. G. Hoft, “Genealized techniques of harmonic elimination and voltage control in thyristor inverters: Part II- Voltage control techniques,” *IEEE. Transactions on Industry Applications*, vol. 10, pp. 666-673, Sept/Oct. 1974.

[14] G. S. Buja and G. B. Indri, ”Optimal pulse width modulation for feeding ac motors,” *IEEE Transactions on Industry Applications*, vol. 13, pp. 38-44, Jan/Feb. 1977.

[15] J. B. Casteel and R. G. Hoft,” Optimum PWM waveforms of a microprocessor controlled inverter,” *IEEE Power Electronic Specialists Conference*, Jun. 1978.

[16] M. S. A. Dahidah and V.G. Agelidis, “Selective harmonic elimination PWM control for cascaded multilevel voltage source converters: A generalized formula,” *IEEE Transactions on Power Electronics*, vol. 23, no. 4, pp. 1620 – 1630, 2008.

[17] B. Ozpineci, L. M. Tolbert and J. N. Chiasson, “Harmonic optimization of multilevel converters using genetic algorithms, *IEEE Power Electronics Letters*, vol.3, no. 3, pp. 92-95, 2005.

[18] V. G. Agelidis, A. I. Balouktsis, and M. S. A. Dahidah, “A five level symmetrically defined selective harmonic elimination PWM strategy: Analysis and experimental validation,” *IEEE Transactions on Power Electronics,* vol. 23, no. 1, pp. 19-26, Jan. 2008.

[19] J. W. Chen and T. J. Liang, “A Novel Algorithm in Solving Nonlinear Equations for Programmed PWM Inverter to Eliminate Harmonics,” *23rd International Conference on Industrial Electronics, control and Instrumentation IEEE IECON* ‘97, vol. 2, pp. 698-703, 1997.

[19] J. R. Wells, B. M. Nee, P. L. Chapman and P. T. Krein,“ Selective harmonic control: a general problem formulation and selected solutions,” *IEEE Transactions on Power Electronics,* vol. 20, pp. 1337-1345, Nov. 2005.

[20] A. Mehrizi-Sani and S. Filizadeh, “An optimized Space Vector Modulation Sequence for Improved Harmonic Performance,” *IEEE Transactions on Industrial Electronics,* vol. 56, no. 8, pp. 2894-2903, Aug. 2009.

[21] *IEEE Recommended Practices and Requirements for Harmonic Control in Electrical Power Systems*, IEEE Standard 519, 1992.