How Probability Weighting Affects Inventory Management with Supply Disruptions

Junlin Chen♭, Han Zhao, Xiaobo Zhao

Abstract—**There are many evidences that people tend to overweight low-probability events and underweight high-probability events. This risk attitude is generally expressed as an inverse-S shaped weighting function. This paper considers a risk-averse inventory manager who operates a continuous-review inventory system subject to supply disruption risk. Both Zero-Inventory-Order (ZIO) and Non-ZIO policies are considered. Based on the studies of [1], [2], [3], we provide an Economy Order Quantity under Disruption (EOQD) model with consideration of the inverse-S shaped weighting function. We conduct numerical studies to investigate the impacts of weighting function on optimal decisions and system costs. Our results show that with ZIO policy, ordering more may not be efficient for a risk-averse inventory manager to mitigate supply disruption risk; whereas with Non-ZIO policy, ordering less but more frequently is suggested to be applied.**

Index Terms—**supply disruption, EOQD, inventory, weighting function.**

I. Introduction

SUPPLY disruptions may be caused by diverse reasons including nature disasters, equipment failures or damaged facilities, during which a supplier cannot fulfill customer orders and then influence flows of the whole supply chain.

Numerous literature study about inventory systems with disruptions. Parlar and Berkin[1], Berk and Arreola-Risa [2] first introduce disruptions into Economy Order Quantity (EOQ) model, which is labeled by Economy Order Quantity under Disruption (EOQD) model. They examine an inventory system with lost sales, where the supply status may be up and down. The durations for both up and down statuses are assumed to be exponentially distributed. The optimal order quantities are determined by minimizing the expected cost per unit time. An extension work of [3] considers an EOQD model with back orders and developed models for single, two, and multiple suppliers. Gürler and Parlar [4] allow the duration of up/down periods as Erlang-k interfailure times/general repair times. Qi et al.[5] consider both the supplier and the retailer are subject to disruptions. A disruption to the supplier results in an unavailable supply, and a disruption to the retailer destroys the inventory at the retailer. Because it is not possible to develop closed form solutions to EOQD models, Snyder [6] develops an approximation to EOQD model in an inventory system with lost sales, that is the inventory is managed by Zero-Inventory-Order (ZIO) policy. Heimann and Waage [7] relax the assumption of the positive re-order point r and constructed an approximation to EOQD model in an inventory system with back orders, that is the inventory is managed by the non-ZIO policy.

Apart from EOQD models, supply disruption is widely analyzed in other inventory models. For example, Arreola-Risa and Decroix [8], Kalpakam and Sapna [9] analyze an environment dependent (s, S) inventory system to minimize the long-run expected cost. The environment goes through available and unavailable periods according to a two-state Markov chain. Parlar and Perry [10], and Mohebbi [11] formulate (R, Q)-type models for supply disruptions within continuous-review inventory systems with stochastic lead times and stochastic demands. For a comprehensive review of inventory models with supply disruptions, we refer interested readers to [12].

This paper studies a continuous-review inventory system with supply disruption risk. Both ZIO and Non-ZIO inventory management policies are examined. Unlike the the classical EOQD model, we consider the inventory manager is not perfectly rational but with probabilistic risk attitudes to vary within the probability interval. As illustrated by prospect theory in [13], people tend to overreact to small probability events, but underreact to medium and large probabilities, on the basis of which, the inventory manager in our system is likely to overestimate the hazard rate: the probability that the supplier is unavailable when inventory level reaches reorder point. Following [1], [2], [3], we theoretically presented EOQD models with probabilistic risk attitudes of inventory manager. Our numerical simulations demonstrate that for the ZIO policy, a risk averse (overestimating the hazard rate) inventory manager does not necessarily order a larger quantities to mitigate supply disruption risk, the

Manuscript received Dec. 8, 2011. This work was supported by NSF of China under Grant 71073005.

J. Chen, and X. Zhao are with the Department of Industrial Engineering, Tsinghua University, Beijing, 100084, China. ♭ e-mail: chenjl06@mails.tsinghua.edu.cn. H., Zhao is with the Department of Logistics Engineering, Beijing University of Posts and Telecommunications, Beijing, 100876, China.

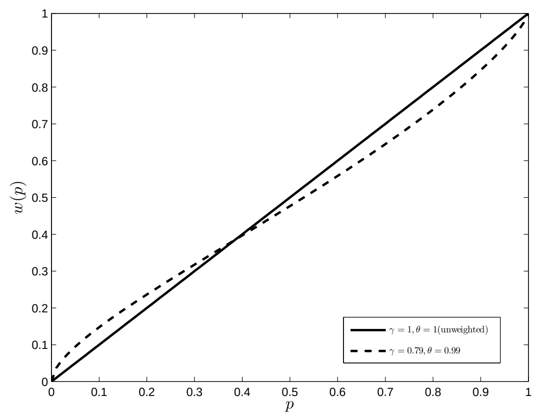

Fig. 1. A sample of two parameter weighting function (Prelec 1998)

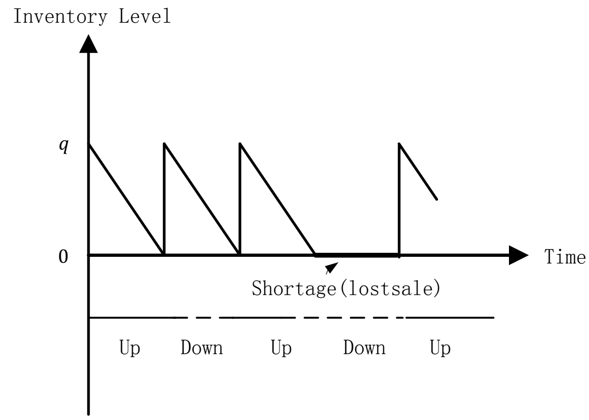

Fig. 2. Inventory level and the status process with ZIO policy

optimal order quantity generated from a cost utility function could be less than the counterpart generated from a cost function. Whereas for the Non-ZIO policy, a risk averse (overestimating the hazard rate) inventory manager is suggested to order more frequently but with less quantities. However, the risk attitude occurs cost, especially for the ZIO policy. A risk averse inventory manager always expends more than a risk natural inventory manager.

Many empirical experiments have proved the existence of people overweighting small probability and underweighting medium and large probabilities so that the weighting function is characterized by an inverse-S shape, e.g., [13], [14], [15], [16], as shown in Figure 1. Etchart-Vincent [17] studies the shape of weighting function under both small and large loss condition. Mattosa et al. [18] certificate that only when the risk aversion and loss aversion combined with weighting function will they have obvious influence to the decision maker. Gonzalez and Wu [19] conduct a study that permits nonparametric estimation of an individual's value function and weighting function. They interpret the shape and properties of the weighting function from the psychological perspective. For parametric weighting functions, see, [20], [21], [22], [23], who provide diverse parametric weighting functions with two or three parameters to describe the functions' shape and preference.

For applications of weighting functions, Ranjan and Shogren [24] incorporate the weighting function into a bargaining model to study the water market strategy. Berling and Peters [25] analyze the weighting function in a bargaining model with asymmetric information. Schweitzer and Cachon [26] propose the fluctuant of order quantity in the supply chain due to the prospect theory. To our knowledge, the model constructed in this paper has not been examined in the literature.

The remaining sections of this paper are organized as follows. In Section 2, we builds our model, and describe the structural property of the hazard rate function. In Section 3, we provide numerical studies, and in Section 4, we conclude the paper.

II. The Models

We model the continuous-review inventory system with an unreliable supplier using the economic order quantity (EOQ) model with disruptions. In the system, fixed ordering cost K per order, and holding cost h per unit per time are incurred. The demand is constant with rate D units per time. The unreliability of the supplier represents that the supplier may shut down for a certain period any time when it functions normally. The durations of both "UP"(functioning normally) and "DOWN"(shut down) periods are exponentially distributed with rate λ and μ respectively. We consider the EOQD problem (EOQ with disruptions) with ZIO Policy and Non-ZIO policy respectively. For ZIO policy, when the supplier is down, no order can be placed and all demands observed until the beginning of the next normal state are lost, with a stock out cost of π per unit. For Non-ZIO policy, unit stock out costs π. When the supplier is down, no order can be placed and all demands observed until the beginning of the next normal state are backordered. Figure 1 depicts a sample of the inventory level process over time for ZIO policy, and for Non-ZIO policy as in figure 3.

As constructed by [1] and [2], the average cost objective function with ZIO policy is given as follows

$$g_z(q) = \frac{K + hq^2/2D + \pi D\beta(q)/\mu}{q/D + \beta(q)/\mu}, \tag{1}$$

where

$$\beta(q) = \frac{\lambda}{\lambda+\mu}(1 - e^{-(\lambda+\mu)q/D}). \tag{2}$$

ISBN: 978-988-19251-9-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

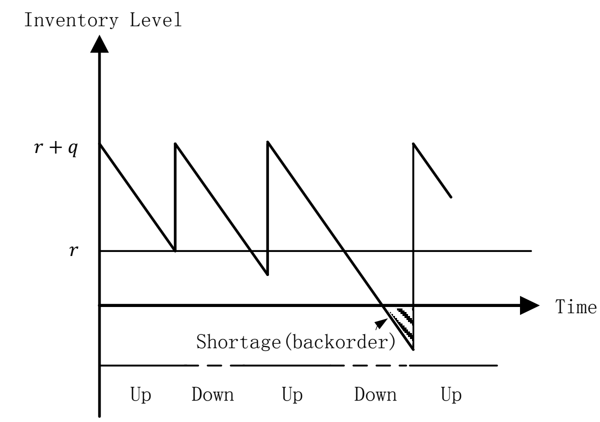

Fig. 3. Inventory level and the status process with Non-ZIO policy

Here, $\beta(q)$ is the probability that the supplier is unavailable when the inventory level reaches reorder point (zero in the ZIO case).

For EOQD model with Non-ZIO policy, [3] presented the average cost objective function as follows

$$g_n(q,r) = \frac{(K + hq^2/2D + hqr/D) + \beta(q)C(r)}{q + \beta(q)D/\mu}, \quad (3)$$

where

$$\beta(q) = \frac{\lambda}{\lambda+\mu}(1 - e^{-(\lambda+\mu)q/D}), \quad (4)$$

and

$$C(r) = \frac{D^2}{\mu^2}(\frac{h}{D}(\mu r/D - 1) + e^{-\mu r/D}(\pi\mu/D + h/D)).$$

Here, $\beta(q)$ is the probability that the supplier is unavailable when the inventory level reaches the reorder point r, and $C(r)$ is the expected cost incurred from the time when the inventory level reaches reorder point r and the state is "DOWN" to the beginning of the next "UP" state.

Then, we incorporate risk attitudes of the decision maker into the EOQD models. As in Figure 1, probability weighting functions are concave on an initial interval and convex beyond that, which implies that small probabilities are overweighted while large ones are underweighted. Moreover, the probability weighting function are asymmetric with a inflection point at about 1/3 where people(firm) switches from overestimating low probability to underestimating high probability. Following [23], we use a two-parameter weighting function as

$$w(p) = e^{-\theta(-lnp)^{\gamma}} \quad (5)$$

where parameter θ mainly affects the inflection (reference) point, and parameter γ mainly affects the curvature.

In our model, recall that $\beta(q)$ is the critical probability that the supplier is unavailable when the inventory level reaches zero. It is likely that decision makers may assign higher weights to the $\beta(q)$, and make decisions irrationally. Then, we incorporate the weighting function in equation (5) into the average cost objective functions for ZIO policy and Non-ZIO policy in equations (6) and (7), and get

$$g_Z(q) = \frac{K + hq^2/2D + \pi D w(q)/\mu}{q/D + w(q)/\mu}, \quad (6)$$

$$g_N(q,r) = \frac{(K + hq^2/2D + hqr/D) + w(q)C(r)}{q + w(q)D/\mu}, \quad (7)$$

where

$$w(q) = e^{-\theta(-ln\frac{\lambda}{\lambda+\mu}(1-e^{-(\lambda+\mu)q/D}))^{\gamma}}. \quad (8)$$

Here, $g_Z(q)$ is average cost objective function for ZIO policy, and $g_N(q,r)$ is average cost objective function for Non-ZIO policy. Then, we denote the order quantity by q and the re-order point by r when the decision maker is rational. And when the decision maker is irrational, those are denoted by q_w and r_w. The optimal order quantities and reorder points (for Non-ZIO policy) are obtained at the minimizers of the cost objective functions. That is $g_Z(q_w^*) = \min_{q\geq 0} g_Z(q)$, $g_z(q^*) = \min_{q\geq 0} g_z(q)$, $g_N({q_w}^*, {r_w}^*) = \min_{q\geq 0, r\in R} g_N(q,r)$, $g_n({q_w}^*, {r_w}^*) = \min_{q\geq 0, r\in R} g_n(q,r)$.

The inverse-S shape of weighting function describes that decision maker always tends to overweight the possibility of low probability events while underweight that of the high probability events. Since supply disruptions rarely happen, our model is built with the weighting function falling in the concave interval (below the inflection point), in which the decision maker overweights the disruption probability in the system.

Proposition 1: $w(q)$ is increasingly concave in q with $q \geq 0$.

Proof:

Because demand rate D is deterministic, and without loss of generality we take $D = 1$. Let $X = -\theta(-ln\frac{\lambda}{\lambda+\mu}(1 - e^{-(\lambda+\mu)q}))^{\gamma}$, then $w(q)$ in (8) equals to e^X.

It is clear that the first derivation of $w(q)$ on q is $e^X \cdot X'$, where $X' = \theta r[(-ln\frac{\lambda}{\lambda+\mu}(1 - e^{-(\lambda+\mu)q}))^{r-1}]\frac{(\lambda+\mu)e^{-(\lambda+\mu)q}}{\frac{\lambda}{\lambda+\mu}(1-e^{(\lambda+\mu)q})}$.

Hence, $e^X \cdot X' \geq 0$ for $q \geq 0$ and, by extension $w(q)$ increases for $q \geq 0$.

Then, we prove the concavity part. Taking the second derivation of $w(q)$ on q, $w''(q) = e^X \cdot X'(X' + X'')$, where $X'' = \theta r[(r -$

ISBN: 978-988-19251-9-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$1)(-ln\frac{\lambda}{\lambda+\mu}(1 - e^{-(\lambda+\mu)q}))^{r-2}][\frac{(\lambda+\mu)e^{-(\lambda+\mu)q}}{\frac{\lambda}{\lambda+\mu}(1-e^{(\lambda+\mu)q})}]^2 + \theta r[(-ln\frac{\lambda}{\lambda+\mu}(1 - e^{-(\lambda+\mu)q}))^{r-1}]\frac{-\lambda(\lambda+\mu)e^{-(\lambda+\mu)q}}{(\frac{\lambda}{\lambda+\mu}(1-e^{(\lambda+\mu)q}))^2}$.

The sum of $X' + X'' = \theta r[(-ln\frac{\lambda}{\lambda+\mu}(1 - e^{-(\lambda+\mu)q}))^{r-1}]\frac{(\lambda+\mu)e^{-(\lambda+\mu)q}}{\frac{\lambda}{\lambda+\mu}(1-e^{(\lambda+\mu)q})} + \theta r[(r-1)(-ln\frac{\lambda}{\lambda+\mu}(1 - e^{-(\lambda+\mu)q}))^{r-2}][\frac{(\lambda+\mu)e^{-(\lambda+\mu)q}}{\frac{\lambda}{\lambda+\mu}(1-e^{(\lambda+\mu)q})}]^2$.

For notational convenience, we let $Y = -ln\frac{\lambda}{\lambda+\mu}(1 - e^{-(\lambda+\mu)q})$, then $X' + X'' = \theta r(-Y'Y^{r-1} + (r-1)Y^{r-2}(-Y')^2 + Y^{r-1}(-Y')^2\frac{-\lambda}{(\lambda+\mu)e^{-(\lambda+\mu)q}})$. Then, we reduce the $X' + X''$ to $\theta r(-Y')Y^{r-1}(\frac{r-1}{Y}(-Y') + \frac{-\lambda}{\frac{\lambda}{\lambda+\mu}(1-e^{-(\lambda+\mu)q})} + 1)$.

Therefore, if the term $\frac{r-1}{Y}(-Y') + \frac{-\lambda}{\frac{\lambda}{\lambda+\mu}(1-e^{-(\lambda+\mu)q})} + 1$ is negative, we can obtain the second derivation of $w(q)$ is negative. It is clear that $\frac{\lambda}{\lambda+\mu}(1 - e^{-(\lambda+\mu)q}) \leq \lambda$, hence, we have $\frac{-\lambda}{\frac{\lambda}{\lambda+\mu}(1-e^{-(\lambda+\mu)q})} + 1 \leq 0$. Because $r - 1 \leq 0$, we obtain that the the second derivation of $w(q)$ is negative and, we conclude that $w(q)$ is increasingly concave in q. ■

Berk and Arreola-Risa [2] demonstrate that the objective function $g_0(q)$ for ZIO policy is unimodal. For the Non-ZIO policy, Palar and Berkin [3] do not provide an analytical proof for the unimodality of $g_0(q,r)$, but they use numerical tests to indicate that $g_0(q,r)$ may be unimodal. When incorporating the weighting function $w(q)$, it is intractable for us to prove the unimodality of the objective function $g_Z(q)$ in (1) and $g_N(q,r)$ in (7). Therefore, in the following section, we apply numerical studies to obtain optimal solutions and illustrate the impacts of risk attitudes on the optimal order decisions and system costs.

III. Numerical Simulation

In this section, we describe numerical examples to study the impacts of overestimation on the optimal order quantities and system costs. Table I illustrates the parameter values, where we set parameters of weighting function as $\gamma = 0.79$, and $\theta = 0.99$. These parameters lead to the inflection point of the weighting function as 0.385, which is consistent with [13]'s estimation of 0.38 for losses. Note that, for the rational case, $\gamma = 1$, and $\theta = 1$.

For ZIO policy, Figure 4 illustrates the optimal order quantities q^* and q_w^* for rational and overestimating cases as a function of π varying from 10 to 20. It is noted that both q^* and q_w^* increase in π, and q_w^* has a higher increasing speed than q. The intersection point is at about $\pi = 14$. Intuitively, one may expect to order more when overestimating the hazard rate (the critical probability that the supplier is unavailable when the inventory level reaches reorder point), meaning that $q^* < q_w^*$ when overestimating β. However, Figure 4 confutes this argument clearly: $q_w^* < q^*$ for π below the intersection point.

For Non-ZIO policy, we describe the patterns of q^* and q_w^* in Figure 5, and the patterns of r^* and r_w^* in Figure 6. It is noted that the q_w^*'s curve is below the q_w^*'s curve, and the r_w^*'s curve is above the r^*'s curve,

TABLE I
List of parameter values

Parameter	Symbol	Value
Demand rate	D	1
Holding cost rate	h	10
Ordering cost	K	10
Shortage cost/unit	π	10-20
Up period mean	$1/\lambda$	4
Down period mean	$1/\mu$	0.4
Probability Weighting Parameters	γ	0.79
Probability Weighting Parameters	θ	0.99

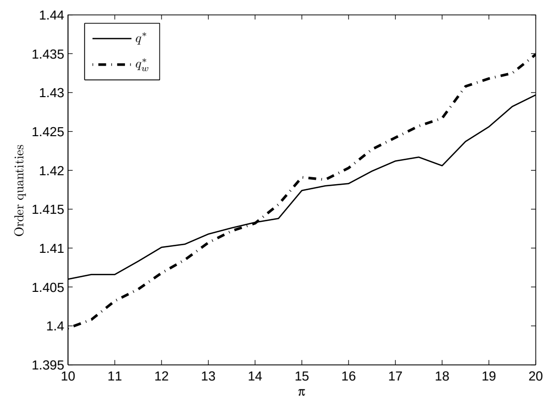

Fig. 4. Optimal order quantities q^* and q_w^* as functions of π with ZIO policy

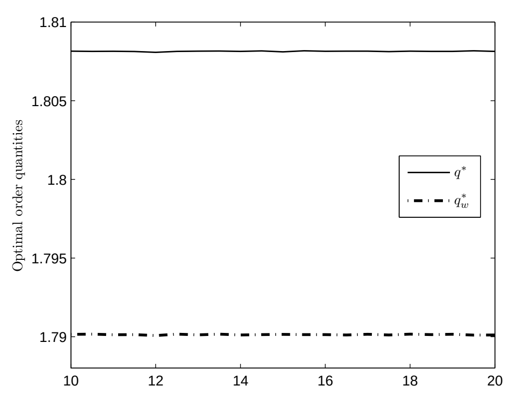

Fig. 5. Optimal order quantities q^* and q_w^* as functions of π with Non-ZIO policy

ISBN: 978-988-19251-9-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

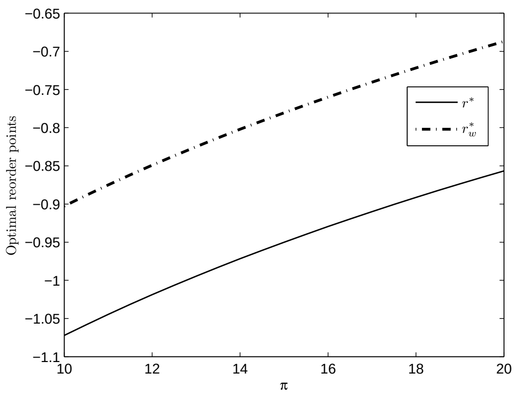

Fig. 6. Optimal reorder point r^* and r_w^* as functions of π with Non-ZIO policy

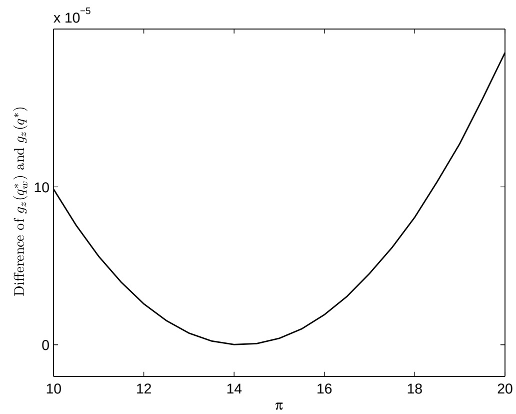

Fig. 7. Difference of costs: $g_z(q_w^*) - g_z(q^*)$ as a function of π with ZIO policy

which implies that when overestimating the hazard rate, the decisions become ordering more frequently but with smaller amount. Moreover, when back orders are allowed, r^* and r_w^* increases in π and q^* and q_w^* is not sensitive to different values of π.

Consequently, when overestimating the hazard rate, the order quantities do not necessarily become higher compared to the rational case for ZIO policy and, for Non-ZIO policy, irrational decision makers is likely to order smaller quantities but with higher frequency compared to rational decision makers.

Then, we introduce numerical examples to illustrate the impacts of weighting function on system costs. Recall that q_w^*,r_w^* are obtained at minimizers of utility functions $g_Z(q_w)$, $g_N(q_w, r_w)$, (Here, denoted by q_w^* is the optimal order quantity for $g_Z(q_w)$ when optimizing $g_Z(q_w)$, and is the optimal order quantity for $g_N(q_w)$ when optimizing $g_N(q_w)$.) We calculate the cost function $g_z(q_w^*)$ and compared with the rational minimizer of $g_z(q^*)$, the resulting difference of $g_z(q_w^*) - g_z(q^*)$ is used to measure the extra cost incurred by the weighting function in ZIO case. Similarly, we use a ratio of $\frac{g_n(q_w^*, r_w^*)}{g_n(q^*, r^*)}$ to measure the extra cost incurred by the weighting function in Non-ZIO case.

Figures 7 and 8 depict the results. As π increases from 10, the difference $g_z(q_w^*) - g_z(q^*)$ decreases to zero at point of $\pi = 14$, then becomes increasing. For Non-ZIO policy, the ratio $\frac{g_n(q_w^*, r_w^*)}{g_n(q^*, r^*)}$ decreases as π increases from 10. Despite the sensitivity of costs for ZIO and Non-ZIO policies on π, figures 7 and 8 clearly indicate that overestimation of the hazard rate would cause a higher cost for both ZIO and Non-ZIO policies.

We typically illustrate the impacts of weighting function on optimal decisions and system costs using a sensitivity analysis of single parameter π. Thus, clear diagrams are helpful to show the results. To interpret

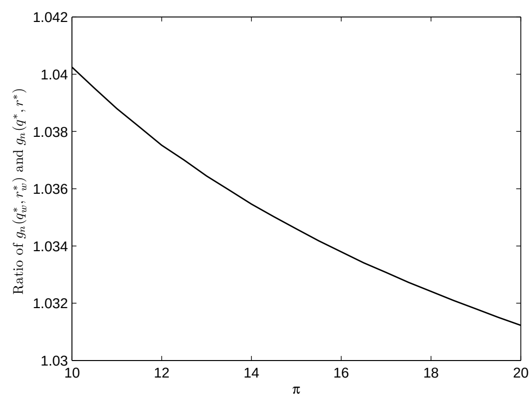

Fig. 8. Ratio of costs: $\frac{g_n(q_w^*, r_w^*)}{g_n(q^*, r^*)}$ as a function of π with Non-ZIO policy

TABLE II
LIST OF PARAMETER VALUES

Parameter	Symbol	Value
Demand rate	D	1,100,1000
Holding cost rate	h	10
Ordering cost	K	1,10,100,1000
Shortage cost/unit	π	1,10,100,1000
Up period mean	$1/\lambda$	0.1, 0.25, 0.5, 1, 2
Down period mean	$1/\mu$	10, 1, 0.5, 0.25, 0.1
Probability Weighting Parameters	γ	0.79
Probability Weighting Parameters	θ	0.99

the impacts more generally, we performed a numerical analysis on a test bed of 1200 cases. The parameters were selected to cover a broad range (see Table II). We observed the following properties:

1) For ZIO policy, $q_w^* - q^*$ to be positive or negative is dependent;

2) For Non-ZIO policy, q_w^* is smaller than q^*, r_w^* is larger than r^*;

ISBN: 978-988-19251-9-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

3) $g_z(q_w^*) \geq g_z(q^*)$ and $g_n(q_w^*, r_w^*) \geq g_n(q^*, r^*)$, which implies that a risk averse inventory manager expends more cost with either ZIO and Non-ZIO policy.

IV. Conclusion

We consider a risk-averse decision maker who operates a continuous-review inventory system subject to supply disruption risk, based on EOQD models with ZIO and Non-ZIO inventory policies. We introduce an inverse-S shaped weighting function into the model construction. Through numerical simulation, we provide managerial suggestions to the risk averse decision maker. As for future research, it would be a challenging work to characterize closed form optimal solutions to the EOQD models. Furthermore, the decision maker's risk attitude can also be examined in inventory systems with two or multi-suppliers.

References

[1] Palar, M., and Berkin, D., "Future Supply Uncertainty in EOQ Models," *Naval Research Logistics*, vol. 38, pp. 107-121, 1991.

[2] Berk, E., and Arreola-Risa, A., "Note on"Future Supply Uncertainty in EOQ Models", *International Journal of Computer Science*, vol. 37, no. 1, pp. 1-10, Jul. 1994.

[3] Palar, M., and Berkin, D., "Future Supply Uncertainty in EOQ Models," *Naval Research Logistics*, vol. 38, pp. 107-121, 1996.

[4] G ürler, Ü, and Mahmut Parlar, "An Inventory Problem with Two Randomly Available Suppliers," *Operations Research,* vol. 45, no. 6, pp. 904-918, 1997.

[5] Qi, L., Shen, Z. M., and Snyder, L. V., "A Continuous-Review Inventory Model with Disruptions at Both Supplier and Retailer," *Production and Operations Management,* vol. 18, no. 5, pp. 516C532, 2009.

[6] Snyder, L. V., "A tight approximation for a continuous-review inventory model with supplier disruptions," *Working Paper, P.C. Rossin College of Engineering and Applied Sciences, Lehigh University, Bethlehem, PA,* 2006.

[7] Heimann, D., and Waage, F., "A closed-form approximation solution for an inventory model with supply disruptions and non-ZIO reorder policy," *Journal of Systemics, Cybernetics, and Informatics,* vol. 5, no. 4, pp. 1-12, 2007.

[8] Arreola-Risa, A., and Decroix, G. A., "Inventory Management Under Radom Supply Disruptions and Partical Backorders," *Naval Research Logistics*, vol.45, pp. 688-703, 1998.

[9] Kalpakam, S., and K. P. Sapna, "A lost sales inventory system with supply uncertainty," *Computers &Mathematics with Applications,* vol. 33, no. 3, pp. 81-93, 1997.

[10] Parlar, M., and Perry, D. "Inventory Models of Future Supply Uncertainty with Single and Multiple Suppliers", *Naval Research Logistics,* vol. 43, pp. 191-210, 1996.

[11] Mohebbi, E, "Supply interruptions in a lost-sales inventory system with random lead time," *Computers and Operations Research,* vol. 30, pp. 411-426, 2003.

[12] Snyder L. V. et al, "OR/MS Models for Supply Chain Disruptions: A Review," *Working Paper,* 2010.

[13] Tversky, A., and Kahneman, D., "Advances in prospect theory: a cumulative representation of uncertainty," *Journal of Risk and Uncertainty,* vol. 5, pp. 291C323, 1992.

[14] Wu, G., and Gonzalez, R., "Common consequence decisions in decision-making under risk," *Journal of Risk and Uncertainty,* vol. 16, pp. 115C139, 1998.

[15] Gonzalez, R., and Wu, G., "On the shape of the probability weighting function," *Cognitive Psychology*, vol. 38, pp. 129C166, 1999.

[16] Humphrey, S. J., and Verschoor, A., "The probability weighting function: experimental evidence from Uganda, India and Ethiopia," *Economics Letters,* vol. 84, pp. 419C425, 2004.

[17] Etchart-Vincent, N., "Is Probability Weighting Sensitive to the Magnitude of Consequences? An Experimental Investigation on Losses," *The Journal of Risk and Uncertainty,* vol. 28, pp. 217-235, 2004.

[18] Mattosa, F., Garciaa, P., and Pennings, J. M. E., "How probability weighting affects participation in water markets," *Journal of Financial Markets,* vol. 11, pp. 433-452, 2008.

[19] Gonzalez, R., and Wu, G., "On the Shape of the Probability Weighting Function," Cognitive Psychology, vol. 38, pp. 129-166, 1999.

[20] Diecidue, E., Schmidt, U., and Zank, H., "Parametric weighting functions," Journal of Economic Theory, vol. 144, pp. 1102-1118, 2008.

[21] Wu, G., and Gonzalez, R., "Curvature of the probability weighting function," *Management Science,* vol. 42, pp. 1676-1690, 1996.

[22] Tversky, A., and Wakker, P., "Risk attitudes and decision weights," *Econometrica,* vol. 63, pp. 1255-1280, 1995.

[23] Prelec, D., "The probability weighting function," *Econometrica,* vol. 66, pp. 497-527, 1998.

[24] Ranjan, R., and Shogren, J. F., "How probability weighting affects participation in water markets," *Water Resources Research,* vol. 42, pp. 419-425, 2006.

[25] Berling, V. K., and Peters, H., "The effect of decision weights in bargaining problems," *Journal of Economic Theory,* vol. 110, pp. 154-175, 2002.

[26] Schweitzer, M. E., and Cachon, G. P., "Decision Bias in the Newsvendor Problem with a Known Demand Distribution," *Management Science,* vol. 46, pp. 404-420, 2000.

ISBN: 978-988-19251-9-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)