The Influence of the Quay Crane Traveling Time for the Quay Crane Scheduling Problem

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Abstract—The quay crane operation is one of the important operations for the container terminal logistics, which carries out loading a container from a truck to a vessel or unloading a container from a vessel to a truck. Normally the operation dominates the container terminal performance. A real data collected in Keelung harbor, Taiwan in 2010 showed that the quay crane operation takes approximately 70% of the container terminal operations. In the research field, the operation is called quay crane scheduling problem i.e. QCSP. In the past studies, most studies ignored the crane traveling time because it is treated a very small amount of time comparing to the whole makespan, while some researchers take the time factor into the approaches to make the solution models more complete. However, to my best knowledge to this QCSP, no researcher tried out quantifying the gap to examine if the gap is important or not. This study tried to use a model and a heuristic which can include or exclude a crane traveling time to compare small and large size instances to suggest whether the crane traveling time is worth to be included in a solution approach of QCSP.

Keywords: Quay Crane, QCSP, Crane Traveling Time

I. INTRODUCTION

According to the past statistics, over 90% of the world cargos were shipped by vessels and the majority was the container transportation. According to the container traffic forecast of the year 2007 United Nations report showed that annual growth rate for global container trade volumes from 2005 to 2015 was estimated to be 7.6%. Also the report showed that annual growth between 1987 and 2006 was 9.5%. Although the recent economic recession made the forecast a little bit inaccurate, however, major sea ports still had around 5% net growth. Therefore, the container transportation is a very important for the world cargo logistics, which makes a container terminal play an essential role in the transportation network.

Several major container terminal operations influence the port performance, which include the vessel berthing operation, the crane unloading/loading operation, the container delivery operation by trucks, the inspection operation, and the container storage operation. Of those operations, the crane operation is the key factor that determines the efficiency and effectiveness of a container terminal. In this study, real data were collected in an international harbor in Taiwan, i.e. Keelung harbor. The data analysis showed that the crane operation takes approximately 70% of a vessel berthing time, which indicates that an efficient and effective quay crane scheduling policy will facilitate container logistics, shorten container flow time, and improve the terminal operation performance.

There had been a lot of wonderful researches on the quay crane scheduling problem. [1] initialized QCSP. [2] followed the previous study and solved QCSP by applying the branch and bound technique. [5] considered the precedence relationships of jobs into in the mathematical model, applied the branch and bound algorithm to solve small size instances and adopted the greedy randomized adaptive search procedure to improve the branch and bound algorithm. [6] developed a branch and cut algorithm to further solve some instances that could not be solved by [5]. Because of the physical configurations of the quay crane, the non-crossing and non-interference characteristics limit the quay crane operation. Significant researches elaborated for the topics, such as [3], [4], [7], and [8].

All of the mentioned researches didn’t consider the crane traveling time in their solution approaches, which some other such as [9], [10], and [11] took the crane traveling time into the solution approaches. However, there is still unclear that whether the crane traveling time is an important issue for the analysis of QCSP. In this paper, the model and the heuristics will be adopted for the analysis to examine the influence of traveling time for the QCSP.

II. PROBLEM DESCRIPTION

QCSP stands for “Quay Crane Scheduling Problem”, which is a popular research topic for the container terminal operation. It is defined as there are multiple cranes serving a container vessel, as shown in Figure 1. $S$ is the number of working sections and $K$ is the total number of assigned cranes for the container vessel. When a container vessel is moored a berth, several cranes are arranged to load or unload containers for that vessel. Unloaded containers are transported by trucks and then go through other terminal operations. After finishing all unloading jobs, cranes will start load containers from land side on to the container vessel. Once a crane finishes its assigned jobs, it will continue other assigned jobs. If there is no further assignment, the crane will wait for the next assignments and stay at the present position. The vessel leaves the assigned berth after all jobs are finished.
According to the above descriptions, the assumptions are made for QCSP.

1. A container vessel has \( N \) containers needed to be loaded or unloaded in total.
2. There are \( K \) cranes are available to serve the moored vessel.
3. In order to specifically assign jobs for a crane, a vessel needs to be further divided into \( S \) sections (some references used “hold” instead of “section”). In this study, the same section width is considered, and for each section, there has certain amount of containers, which denotes \( W_{j,t} \), \( j = 1, \ldots, S \). Also, in order avoid cranes interfering with each other while working, the section width includes safety distance.
4. No two cranes are allowed working for the same section at the same time to avoid interfering with each other.
5. Without considering special events, once a crane starts to work on a section, it will not stop until it finishes loading or unloading all the containers in this section, that is, no-preemption is allowed. This assumption is not strong in reality. Different terminals have their rules for the crane operations. In this study, this assumption is considered in the approach. The model developed by [10] can be revised easily if this assumption is taken out.

6. Constant crane traveling time is considered.
7. Because cranes are mounted in the rail, therefore, no two cranes can cross with each other.

III. METHODOLOGIES

[10] developed a model that can include or exclude the crane traveling time factor in the solution approach. However, it focused on the analyses of the instances that included the crane traveling time factor. Also, a heuristic was designed that can analyze the cases that included the crane traveling time factor. The worst case analyses showed that this heuristic was effective because the worst case bound is less than two. [11] extended the study results to prove the worst case bound of a similar heuristic was also less than 2. The main purpose of this paper is to adopt the mathematical model developed by [10] and the heuristics developed by [10] and [11], to analyze lots of small and large size instances in order to find out how the crane traveling time factor influences the results of QCSP.

The following model and two heuristics showed in the following section, which are abstracted from [10] and [11].

The definitions of the symbols for the mathematical models

- \( j \): Section index
- \( t \): Time index
- \( S \): Number of sections of a vessel
- \( T \): The maximum time period
- \( Y \): Numbers of containers per time period that a crane can deal with
- \( \tau \): Makespan

\[ b_j = \begin{cases} 
1 & \text{if there is a crane at section } j \\
0 & \text{otherwise}
\end{cases} \]

\[ x_{j,t} = \begin{cases} 
1 & \text{if there is a job at section } j \text{ at time } t \\
0 & \text{otherwise}
\end{cases} \]

\[ w_{j,t}: \text{Number of remaining jobs at section } j \text{ at time } t \]

\[ z_{j,t+1} = \begin{cases} 
1 & \text{if a crane moves from section } j \text{ to section } j' \\
0 & \text{otherwise}
\end{cases} \]

\[ M: \text{A big number} \]

The mathematical model with considering the crane traveling time

\[
\begin{align*}
\min \tau & \quad \text{(1)} \\
x_{j,t} + 1 & \leq \tau \quad \forall j, t & \quad \text{(2)} \\
w_{j,t} - Mx_{j,t} & \leq 0 \quad \forall j, t & \quad \text{(3)} \\
w_{j,t-1} - \sum_{j \neq j'} w_{j,t} & \leq w_{j,t} \quad \forall j, t & \quad \text{(4)} \\
z_{j,t+1} + z_{j,t+1} + z_{j,t+1} & = b_j \quad \forall j, t = 1 \quad \text{(5)} \\
z_{j-1,t} + z_{j,1} + z_{j+1,1} - z_{j+1,1} - z_{j-1,1} - z_{j+1,1} & = 0 \quad \text{(6)} \\
\sum_{j=1}^{S} y_{j,0} & = \sum_{j=1}^{S} b_j \quad \forall t = T & \quad \text{(7)} \\
z_{j,t-1} + z_{j,t-1} & \leq 1 \quad \forall j, t \quad \text{(8)} \\
z_{j-1,t} + z_{j-1,t} & \leq 1 \quad \text{(9)} \\
z_{j,t} + z_{j,t} - T & \leq 1 \quad \text{(10)} \\
w_{j,t} & \geq 0 \quad \text{and } w_{j,t} \in \mathbb{Z}^+ \quad \text{(11)}
\end{align*}
\]

(1) is the objective function which minimizes the makespan, i.e. the time that the last job is complete. (2) is to determine the makespan. (3) establishes a relationship between \( x_{j,t} \) and \( w_{j,t} \). (4) describes \( Y \) jobs are proceeded every time period. (5) and (6) are flow balance constraints. (7) determines the initial cranes flow. (8) is the non-interference constraint. (9) is the non-crossing constraint, and (10) is the non-preemption constraint. (11) depicts \( w_{j,t} \) is an integer variable and \( z_{j,t}, z_{j,t+1} \) are binary variables.

The definitions of the symbols for the heuristics

\[ s^i_1: \text{The initial location of crane } j_i \]
\[ t^i_1(s): \text{The time to move crane } j_i \text{ from its initial location to section } s, \text{ i.e., } t^i_1(s) = \nu_i |s^i_1 - s_i| \]
\[ \nu_i: \text{The traveling time between two adjacent sections} \]
\[ \nu_o: \text{The time to load or unload a container.} \]
\[ s(i): \text{The section index for the } i^{th} \text{ container where} \]
i = 1, 2, \ldots, N,

m_j$: The first working section of crane $j$,

$n_j$: The last working section of crane $j$,

$\tau_j$: The total time that the $j^{th}$ crane spends on loading or unloading containers between sections $m_j$ and $n_j$.

$L_2$: The threshold for the heuristic without considering traveling time,

$T_1$: The time threshold for the heuristic with considering traveling time,

$T_2$: The time threshold for the heuristic without considering traveling time,

$\mathcal{Z}^{H_2}$ makespan for the heuristic with considering crane traveling time, and

$\mathcal{Z}^{H_1}$ makespan for the heuristic without considering crane traveling time.

$\mathcal{H}(i, j)$: The optimal value function (i.e., makespan) for the first $i$ containers served by the first $j$ cranes.

$H_1$ heuristic (considering with the crane traveling time)

Step 1: Find $\mathcal{H}(N, K)$ by TDP, which is shown in the following section. The threshold $T_1$ is $H(N, K) = \nu_1$

Step 2: Initialize $\tau_j = 0$ for each $j = 1, 2, \ldots, K$.

Step 3: Set the start section for the first crane $m_1 = 1$.

Step 4: For $j^{th}$ crane, finding $m_j$ and $n_j$ by using the following formulas

$$\nu_2 \sum_{i=m_j}^{n_j-1} W_i + [n_j - 1 - m_j] \nu_1 + \min\{t_{m_j}(i), t_{n_j}(n_j - 1)\} \leq T_1$$

and

$$\nu_2 \sum_{i=m_j}^{n_j} W_i + [n_j - m_j] \nu_1 + \min\{t_{m_j}(i), t_{n_j}(n_j)\} > T_1$$

$$\tau_j = \nu_2 \sum_{i=m_j}^{n_j} W_i + [n_j - m_j] \nu_1 + \min\{t_{m_j}(i), t_{n_j}(n_j)\}$$

Step 5: Update the crane index $j = j + 1$ and the start section of $j^{th}$ to be $m_{j+1} = m_{j} + 1$. Repeat Step 4 until $j > K$.

Step 6: The makespan $Z^{H_1} = \max\{\tau_j | 1 \leq j \leq K\}$

TDP algorithm

TDP stands for the abbreviation of “Threshold by Dynamic Programming” technique, which determines the lower bound of the $H_1$ heuristic and estimate the threshold to be a criterion to assign jobs for cranes. The following show the detail steps for the algorithm.

Step 1: Initialize the dynamic programming value function $H(i, 1) = \min\{\nu_1, s(1)\}$ for all $i = 1, 2, \ldots, N$

$H(1, j) = t_{m_j}(s(1)) + \nu_2$ for all $j = 1, 2, \ldots, K$

Step 2: Obtain the dynamic programming recursive function

For $i = 2, 3, \ldots, N$ and $j = 2, 3, \ldots, K$

$$H(i, j) = \min_{k=1}^{N} \{H(k, j - 1), \min\{t_{m_j}(s(k + 1)), t_{n_j}(s(i))\} + (s(i) - s(k + 1)) \nu_1 + (i - k) \nu_2$$

$H_2$ heuristic (considering without the crane traveling time)

Step 0: Find the lower bound of the numbers of the least job

$$L_2 = \frac{1}{K} \sum_{i=1}^{K} W_i$$

Step 1: Calculate the time threshold $T_2 = \nu_2 \times L_2$

Step 2: Initialize $\tau_j = 0$ for each $j = 1, \ldots, K$.

Step 3: Let $m_1 = 1$.

Step 4: For $j^{th}$ crane, finding $m_j$ and $n_j$ by using the following formulas,

$$\nu_2 \sum_{i=m_j}^{n_j-1} W_i \leq T_2$$

$$\nu_2 \sum_{i=m_j}^{n_j} W_i > T_2$$

$$\tau_j = \nu_2 \sum_{i=m_j}^{n_j} W_i$$

Step 5: Let $m_{j+1} = n_j + 1$ and $j = j + 1$, repeat Step 3 to Step 5 until $j = K$, then go to Step 6.

Step 6: Makespan is $Z^{H_2} = \max\{\tau_j | 1 \leq j \leq K\}$

IV. ANALYSES AND RESULTS

In this section, small and large size instances are generated randomly by specific settings. Since the nature of the QCSP is NP hard, $M_1$ (the model with considering the crane traveling time) and $M_2$ (the model without considering the crane traveling time) are used for analyzing the small size instances and $H_1$ and $H_2$ are used for analyzing the large size instances. Model simulation Settings are listed in the Table 1 and the analyzed results of the small size instances are shown in Table 2. All instances are solved by GAMS with CPLEX solver. While for the heuristics, Microsoft C++ is the tool for developing the heuristics.

### Table 1 Small size instances simulation settings

<table>
<thead>
<tr>
<th>Numbers of cranes</th>
<th>1 to 4 cranes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size of a vessel</td>
<td>5 to 9 sections</td>
</tr>
<tr>
<td>Number of containers per section in a vessel</td>
<td>Case I: 10 to 50</td>
</tr>
<tr>
<td>Model execution time limit</td>
<td>1800 seconds</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case</th>
<th>Results</th>
<th>$M_1$ Total number</th>
<th>$M_3$ Total number</th>
<th>Makespan Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Optimal</td>
<td>21</td>
<td>17</td>
<td>0.011%</td>
</tr>
<tr>
<td></td>
<td>Feasible</td>
<td>115</td>
<td>31</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Infeasible</td>
<td>164</td>
<td>252</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>Optimal</td>
<td>23</td>
<td>18</td>
<td>0.011%</td>
</tr>
<tr>
<td></td>
<td>Feasible</td>
<td>92</td>
<td>32</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Infeasible</td>
<td>185</td>
<td>250</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>Optimal</td>
<td>5</td>
<td>8</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>Feasible</td>
<td>17</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Infeasible</td>
<td>278</td>
<td>269</td>
<td></td>
</tr>
</tbody>
</table>
Because QCSP is a NP-Complete problem, most of instances are infeasible. Those optimal solution cases with the same settings by $M_1$ and $M_2$ are chosen to calculate relative difference between the two approaches. The results show that these instances almost reach the same makespans. The simulation settings of large size instances are all the same definitions in Table 1 except for the number of cranes and number of sections. In large-sized instance simulations, there are 5 and 10 cranes, 20, 30, 40, and 50 sections instead. The analyzed results are shown in Table 3 and Table 4. For each instance, 10 runs are executed. The results are the average of the 10 runs.

Table 3 Analyzed results of the large-sized instances of 5 cranes cases

<table>
<thead>
<tr>
<th>Case</th>
<th>Number of sections</th>
<th>$H_1$ Execution time (sec.)</th>
<th>$H_2$ Execution time (sec.)</th>
<th>Makespan difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>20</td>
<td>0.42</td>
<td>0.0</td>
<td>5.08%</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>1.45</td>
<td>0.0</td>
<td>6.04%</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>3.58</td>
<td>0.0</td>
<td>3.93%</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>5.10</td>
<td>0.0</td>
<td>3.80%</td>
</tr>
<tr>
<td>II</td>
<td>20</td>
<td>4.71</td>
<td>0.0</td>
<td>5.77%</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>11.1</td>
<td>0.0</td>
<td>5.53%</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>30.2</td>
<td>0.0</td>
<td>5.12%</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>52.1</td>
<td>0.0</td>
<td>4.22%</td>
</tr>
<tr>
<td>III</td>
<td>20</td>
<td>66.4</td>
<td>0.0</td>
<td>11.0%</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>147.3</td>
<td>0.0</td>
<td>4.24%</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>416.1</td>
<td>0.0</td>
<td>5.78%</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>871.8</td>
<td>0.0</td>
<td>3.56%</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
<td>5.34%</td>
</tr>
</tbody>
</table>

Table 4 Analyzed results of the large-sized instances of 10 cranes cases

<table>
<thead>
<tr>
<th>Case</th>
<th>Number of sections</th>
<th>$H_1$ Execution time (sec.)</th>
<th>$H_2$ Execution time (sec.)</th>
<th>Makespan difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>20</td>
<td>0.81</td>
<td>0.0</td>
<td>11.8%</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>2.40</td>
<td>0.0</td>
<td>12.8%</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>5.87</td>
<td>0.0</td>
<td>7.30%</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>9.80</td>
<td>0.0</td>
<td>6.72%</td>
</tr>
<tr>
<td>II</td>
<td>20</td>
<td>7.42</td>
<td>0.0</td>
<td>7.38%</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>25.5</td>
<td>0.0</td>
<td>9.75%</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>54.1</td>
<td>0.0</td>
<td>6.49%</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>88.6</td>
<td>0.0</td>
<td>5.17%</td>
</tr>
<tr>
<td>III</td>
<td>20</td>
<td>118.2</td>
<td>0.0</td>
<td>12.1%</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>351.7</td>
<td>0.0</td>
<td>9.76%</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>826.6</td>
<td>0.0</td>
<td>8.43%</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>1618.4</td>
<td>0.0</td>
<td>9.13%</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
<td>8.9%</td>
</tr>
</tbody>
</table>

Although the algorithm steps is similar between $H_1$ and $H_2$, the results of the execution time have great difference with each other. $H_1$ almost takes no time for an execution, whiles $H_2$ takes longer time for executions than $H_1$ does, especially for 10 cranes-50 sections cases. Averages, it takes almost half an hour to run one instance. For the results of relative makespan difference between these heuristics, the figure is larger than 5%, which indicates that while analyzing the large size instances by heuristics, the crane traveling time can not be ignored, which can not be treated as a small amount of values and be ignored from the analyzed process and solution approaches.

V. CONCLUSIONS

In this study, whether the crane traveling time influencing the analyzed result of QCSP is discussed. For small-sized instances by the mathematical model approach, the crane traveling time is not an issue for the study results. Oppositely, the crane traveling time will influences the results of QCSP for the large-sized instances. From simulations in this study, it has more than 5% influencing on the makespan of the crane operations. In the real world, the crane operation is close to the medium-sized instances. The crane traveling time will be an issue if the heuristic approach is taken. However, the time for execution of the heuristic will not take very long as the cases of large-sized instances. Therefore, the crane traveling time needs to be considered for the analysis of the real world QCSP.

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