A More Accurate Total Heat Transfer Rate Method of the Single-pipe Heat Exchanger Considering Heat Radiation

Shih-Shih Ku, King-Leung Wong, *IAENG member*, Ho-Chiao Chuang

Abstract—**The log mean temperature difference (LMTD) method is conventionally used to calculate the total heat transfer rate of heat exchangers. Because the heat radiation equation contains the 4th order exponential of temperature which is very complicate in calculations, thus LMTD method neglects the influence of heat radiation. From the recent investigation of a metal circular pipe in some practical situations, it is found that even in the situation of the temperature difference between outer pipe surface and surrounding is low to 1℃, the heat radiation effect cannot be ignored in the situations of ambient air with lower convection heat coefficient and oxidized metal pipe with higher surface emissivity. In this investigation, the log mean heat transfer rate (LMHTR) method considering the influence of heat radiation is developed to calculate the total heat transfer rate of single-pipe heat exchanger and much more accurate results can be obtained.**

Index Terms—**single-pipe heat exchanger, heat radiation, log mean heat transfer rate (LMHTR) method, LMTD method**

I. Introduction

Heat exchangers are widely applied to the industries and living surroundings. The Mean Temperature Difference method neglecting heat radiation was introduced to design heat exchanger by Bowman et al. [1]. Stevens et al. [2] applied Mean Temperature Difference method in one, two, three pass cross-flow heat exchangers. The log mean temperature difference (LMTD) method has been applied widely to calculate the total heat transfer rate of heat exchangers in most heat transfer text books [such as 3-4], air conditioning and refrigeration text books [such as 5-6], as well as heat exchanger hand books [such as 7-10]. Because the heat radiation equation contains the 4th order exponential of temperature which is very troublesome in calculations, thus LMTD method neglects the influence of heat radiation. Recently, Wong et al. [11] found out the inaccuracy of heat transfer characteristics of a metal pipe neglecting heat radiation effect in ambient air; they reported that in some practical situations, even when the temperature difference between the fluid inside a metal circular pipe and the ambient air is low to 1℃, the errors generated by neglecting heat radiation are still very large and hence, cannot be ignored. Unfortunately, it can be seen from Table 1 that most surface emissivities of oxidized metal are greater than 0.64; and Table 2 shows that the heat convection coefficients of ambient air with medium wind speed are with quite small; the value of 8.3 $Wm^{-2}K^{-1}$ is conventionally adopted in air conditioning design.

In this present investigation, the log mean heat transfer rate (LMHTR) method which considering the influence of heat radiation, is developed to calculate the total heat transfer rate of single-pipe heat exchangers.

Fig. 1 Condenser or evaporator of air conditioner is a single-pipe heat exchanger

Fig. 2 Radiator applied in vehicles and inside buildings is a single-pipe heat exchanger

Manuscript received October 8, 2011; revised November 8, 2011. This work was supported by the National Science Council of Taiwan under the project NSC **100**-2221-E-168-035.

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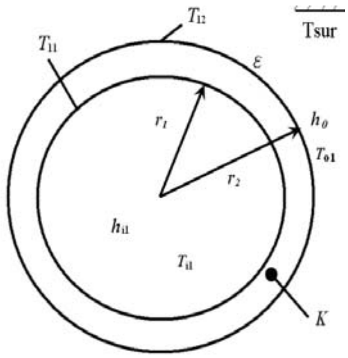
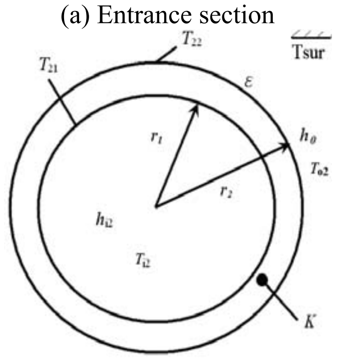
ISBN: 978-988-19251-9-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

II. PROBLEM FORMULATION

The condenser or evaporator of an air conditioner is a good application example of single-pipe heat exchanger as shown in Fig. 1. The radiators applied to vehicles and to heating system inside buildings is other application example of single-pipe heat exchanger as shown in Fig. 2. Fig. 3 shows that an circular single-pipe heat exchanger is with length L, inner radius r1, outer radius r2, wall conductivity K, outer surface emissivity ε, internal fluid with convection heat transfer coefficient hi1 and temperature Ti1 at entrance section, convection heat transfer coefficient hi2 and temperature Ti2 at exit section, respectively; and it is exposed to ambient air with convection heat transfer coefficient ho, ambient air temperatures To1 at entrance section and To2 at exit section, respectively . Practically, hi1 =hi2= hi can be assumed. The surroundings temperature is Tsur. The outer surface area of heat exchanger is much smaller than that of surroundings.



(a) Entrance section



(b) Exit section

Fig. 3 A single-pipe heat exchanger and its relative parameters at entrance and exit sections

A. LMTD method neglecting the influence of heat radiation

While the influence of outer surface heat radiation is not considered, the log mean temperature difference (LMTD) method [1-10] is conventionally used to calculate the total heat transfer rate of heat exchangers. From the relative temperatures of entrance and exit sections as shown in Fig. 3, LMTD can be expressed as:

$$\mathrm{LMTD} = \frac{(T_{i1}-T_{o1})-(T_{i2}-T_{o2})}{\ln\dfrac{(T_{i1}-T_{o1})}{(T_{i2}-T_{o2})}} \tag{1}$$

The total thermal resistance of the circular pipe shown in Fig. 3 can be written as:

$$R_{th} = \frac{1}{h_i 2\pi r_1 L} + \frac{\ln\dfrac{r_2}{r_1}}{2\pi K L} + \frac{1}{h_o 2\pi r_2 L} \tag{2}$$

The total heat transfer rate of the long circular pipe neglecting the heat radiation by LMTD method is:

$$\mathrm{Q} = \frac{LMTD}{R_{th}} \tag{3}$$

The unit length heat transfer rate, q_1, at the entrance section is:

$$q_1 = \frac{T_{i1}-T_{o1}}{R_{th}L} = \frac{T_{s1}-T_{o1}}{\dfrac{1}{h_o 2\pi r_2}} \tag{4}$$

The unit length heat transfer rate, q_2, at the exit section is:

$$q_2 = \frac{T_{i2}-T_{o2}}{R_{th}L} = \frac{T_{s2}-T_{o2}}{\dfrac{1}{h_o 2\pi r_2}} \tag{5}$$

The values of total heat transfer rate Q, the average surface temperature at the entrance section T_{s1}, the average surface temperature at the exit section T_{s2}, can obtained from Eqs. (1)~(5) under the given values of h_i, h_o, r_1, r_2, K, L, T_{i1}, T_{i2}, T_{o2} and L

B. Situations considering the influence of heat radiation

While the influence of outside surface heat radiation is considered, the complete unit length heat transfer rate at the entrance section is:

$$q_{a1} = \frac{T_{i1}-T_{o1}}{\dfrac{1}{h_i 2\pi r_1} + \dfrac{\ln\dfrac{r_2}{r_1}}{2\pi K}} \tag{6}$$

The unit length surface convective heat transfer rate at the entrance section is:

$$q_{c1} = h_o 2\pi r_2 (T_{21} - T_{o1}) \tag{7}$$

The unit length surface radiation heat transfer rate at the entrance section is:

$$q_{r1} = \sigma\varepsilon 2\pi r_2 (T_{21}{}^4 - T_{sur}{}^4) \tag{8}$$

The following equation is obtained from heat balance at the entrance section:

$$q_{a1} = q_{c1} + q_{r1} \tag{9}$$

The values of q_{a1}, q_{r1}, q_{c1} and T_{21} can obtained from Eqs. (6)~(9) under the given values of h_{i1}, h_o, r_1, r_2, K, T_{i1}, T_{o1}, ε and T_{sur}.

Similarly, the complete unit length heat transfer rate at the exit section is:

$$q_{a2} = \frac{T_{i2}-T_{o2}}{\dfrac{1}{h_i 2\pi r_1} + \dfrac{\ln\dfrac{r_2}{r_1}}{2\pi K}} \tag{10}$$

ISBN: 978-988-19251-9-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

The unit length surface convective heat transfer rate at the exit section is:

$$q_{c2} = h_o 2\pi r_2 (T_{22} - T_{o2}) \quad (11).$$

The unit length surface radiation heat transfer rate at the exit section is:

$$q_{r2} = \sigma\varepsilon 2\pi r_2 (T_{22}{}^4 - T_{sur}{}^4) \quad (12)$$

The following equation is obtained from heat balance at the exit section:

$$q_{a2} = q_{c2} + q_{r2} \quad (13)$$

The values of q_{a2}, q_{r2}, q_{c2} and T_{22} can obtained from Eqs. (10)~(14) under the given values of h_{i2}, h_o, r_1, r_2, K, T_{i2}, T_{o2}, ε and T_{sur}.

The total heat transfer rate of the single-pipe heat exchanger considering the heat radiation by log mean heat transfer rate (LMHTR) method is defined as:

$$Q_a = \frac{q_{a1} - q_{a2}}{\ln \dfrac{q_{a1}}{q_{a2}}} L \quad (14)$$

The above LMHTR method (considering heat radiation) under the same concept as LMTD method (neglecting heat radiation) is developed in this study. While the heat radiation is not considered, assume the temperatures T_{i1} and T_{o1} keep constant at the entrance section, and T_{i2} and T_{o2} keep constant at the exit section, then the following relations can be obtained in the situation of neglecting heat radiation (or $\varepsilon = 0$):

$$q_{a1} = q_1 = \frac{T_{i1} - T_{o1}}{R_{th} L} \quad (15)$$

$$q_{a2} = q_2 = \frac{T_{i2} - T_{o2}}{R_{th} L} \quad (16)$$

Then the following relation between LMTD method and LMHTR method in the situation of neglecting heat radiation (or $\varepsilon = 0$) can be proven:

$$Q_a = \frac{q_{a1} - q_{a2}}{\ln \dfrac{q_{a1}}{q_{a2}}} L = \frac{\dfrac{(T_{i1} - T_{o1})}{R_{th} L} - \dfrac{(T_{i2} - T_{o2})}{R_{th} L}}{\ln \dfrac{\dfrac{(T_{i1} - T_{o1})}{R_{th} L}}{\dfrac{(T_{i2} - T_{o2})}{R_{th} L}}} L$$

$$= \frac{\dfrac{(T_{i1} - T_{o1}) - (T_{i2} - T_{o2})}{\ln \dfrac{(T_{i1} - T_{o1})}{(T_{i2} - T_{o2})}}}{R_{th}} = \frac{LMTD}{R_{th}} \quad (17)$$

It can be proved from Eq. (17) that the results obtained from LMTD method and LMHTR method in the situation of neglecting heat radiation (or **ε=0)** are the same value.

The error of total heat transfer rate generated by neglecting heat radiation effect (while $\varepsilon \neq 0$) is defined as:

$$\mathrm{QR} = \left(\frac{Q - Q_a}{Q_a}\right) \times 100\% \quad (18)$$

The error of average surface temperature at the entrance section generated by neglecting heat radiation is:

$$\mathrm{TR}_1 = \left(\frac{T_{s1} - T_{21}}{T_{21}}\right) \times 100\% \quad (19)$$

The error of average surface temperature at the exit section generated by neglecting heat radiation is:

$$\mathrm{TR}_2 = \left(\frac{T_{s2} - T_{22}}{T_{22}}\right) \times 100\% \quad (20)$$

III. The reliability of the numerical results

The nature of results of heat equation for a circular single-pipe heat exchanger is one-dimension exact solution. The exact numerical heat transfer results of a circular pipe can be obtained by any one-dimensional computer code (such as LabVIEW programming in this study). The computer aid results are obtained within half second computing time by a common PC.

In order to check if the numerical results are reliable, the following checking methods are used:

(1). Let outer surface emissivity ε=0, make sure if the results of QR=0.

(2).Let surface emissivity ε=1, outer convection coefficient h_o=100,000Wm^{-2}K^{-1}, make sure if the results of QR close to zero.

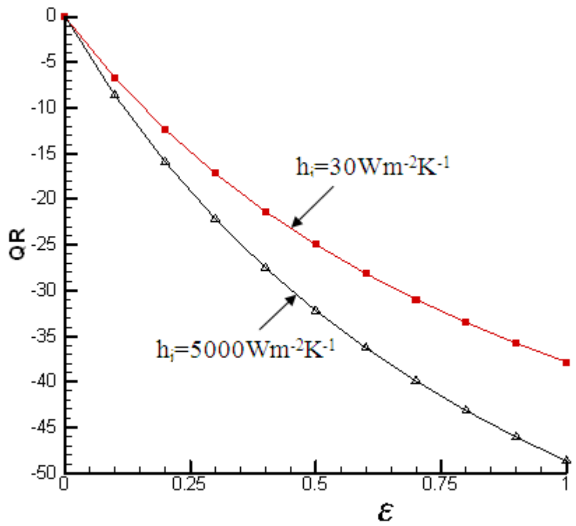


Fig. 4 The relations between heat transfer rate error, QR, and emissivity, ε, for a heater

ISBN: 978-988-19251-9-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

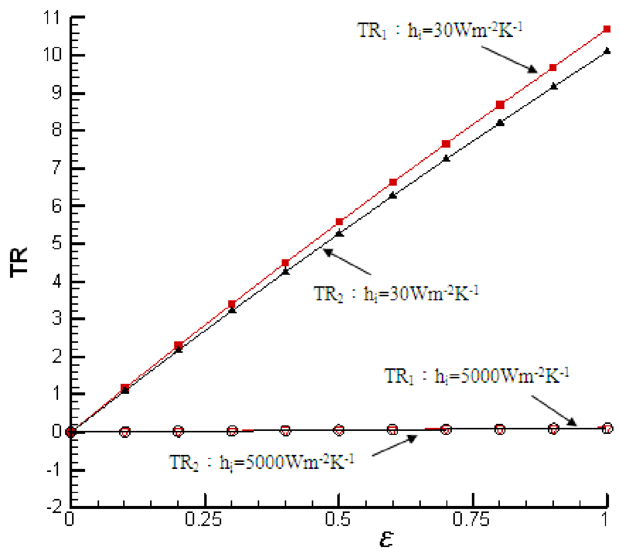


Fig. 5 The relations among surface temperature errors, TR_1 and TR_2, and emissivity, ε, for a heater

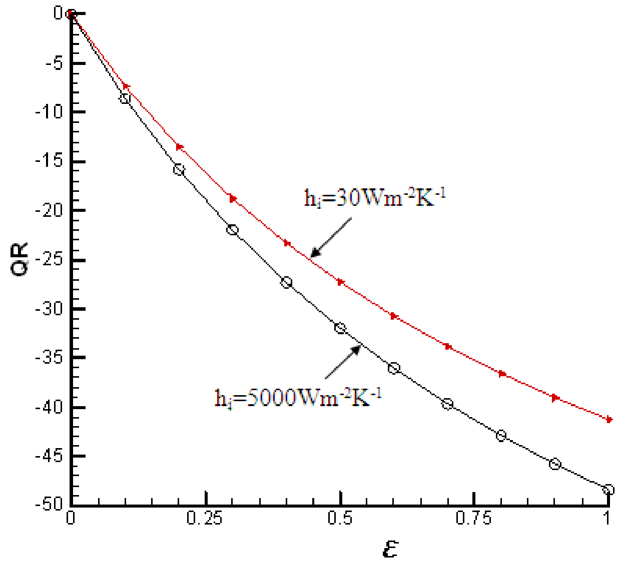


Fig. 6 The relations between heat transfer rate error, QR, and emissivity, ε, for a chiller

IV. Results and discussions

The following practical examples is use to demonstrate what the differences and relationships of the results between the LMTD method and LMHTR method are.

Example 1:

A single-pipe heater constructed by a circular pipe with conductivity $\boldsymbol{K}$=77 $Wm^{-1}K^{-1}$ and various surface emissivities ε=0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1 (refer Table 1), length $\boldsymbol{L}$=10m, inner radius r_1=198mm, outer radius r_2=200mm, the hot water is flowing inside the heater with entrance temperature T_{i1}=65℃ and convective heat coefficient h_i=30 and 5000 $Wm^{-2}K^{-1}$ (refer Table 2), with exit temperature T_{i2}=60℃, the ambient air at the entrance section with temperature T_{o1}=22℃ and convective heat coefficient h_o=8 $Wm^{-2}K^{-1}$ (refer Table 2 and practical application in air conditioning), the ambient air at the exit section with temperature T_{o2}=24℃, and the heater is located in a big room with wall or surrounding temperature T_{sur} =20℃. The results are shown in Table 3 and Figs. 4-5.

Example 2:

A chiller constructed by a circular pipe with conductivity K_A=77 $Wm^{-1}K^{-1}$ and various surface emissivities ε=0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1 (refer Table 1), length $\boldsymbol{L}$=10m, inner radius r_1=198mm, outer radius r_2=200mm, the cold water is flowing inside the heater with entrance temperature T_{i1}=7℃ and convective heat coefficient h_i=30 and 5000 $Wm^{-2}K^{-1}$ (refer Table 2), with exit temperature T_{i2}=12℃, the ambient air at the entrance section with temperature T_{o1}=24℃ and convective heat coefficient h_o=8 $Wm^{-2}K^{-1}$ (refer Table 2 and practical application in air conditioning), the ambient air at the exit section with temperature T_{o2}=26℃, and the heater is located in a big room with wall or surroundings temperature T_{sur} =30℃. The results are shown in Table 4 and Figs. 6-7.

From Table 3-4 and Figs. 4-7 show that in situations of ε =0 , $\boldsymbol{QR}$=0, TR_1=0 and TR_2=0; the greater the ε is, the greater the absolute values of $\boldsymbol{QR}$, TR_1 and TR_2 will be; the greater the h_o are, the smaller the TR_1 and TR_2 will be; while ε >**0**, all the $\boldsymbol{QR}$ are negative, it means in the situations of neglecting heat radiation, the smaller absolute values of heat transfer rate, $|Q|$, will be obtained (i.e., $|Q|<|Q_a|$), and most absolute values of QR are so big and out of the engineering acceptable range; and in situations of ε >**0**, the greater the values of h_i, the greater the absolute values of QR will be, but the smaller the TR_1 and TR_2 will be; for the heater, TR_1 and TR_2 are positive (i.e., $T_{s1}> T_{21}$ and $T_{s2}> T_{22}$), it means in the situations of neglecting heat radiation, the greater surface temperatures (over-prediction of hot surface) will be obtained; for the chiller, TR_1 and TR_2 are negative (i.e., $T_{s1}< T_{21}$ and $T_{s2}< T_{22}$), it means in the situations of neglecting heat radiation, the smaller surface temperatures (over-prediction of cold surface) will be obtained, these predictions may lead to misjudge the condensation occurred on the surface.

Meanwhile, it can be seen in Table 1 that most surface emissivities of oxidized metal heat exchanger are greater than 0.64 (where ε=0.83 for aluminum, ε= 0.78 for copper, ε=0.64 for iron, ε=0.85 for stainless steel and ε=0.79 for steel). Some metal single-pipe heat exchangers, condenser and evaporator of air conditioner (Fig.1), such as radiator (Fig. 2), are applied in the natural air or low speed air with very low values of convection coefficient (refer Table 2 and practical application in air conditioning). Thus, in above situations, neglecting the influence of heat radiation will obtain the inaccurate heat transfer results for a single-pipe heat exchanger.

ISBN: 978-988-19251-9-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

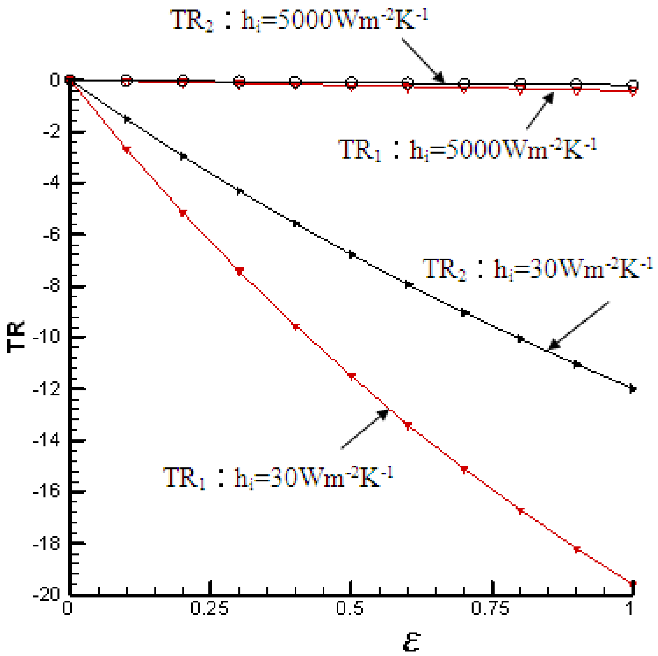


Fig. 7 The relations among surface temperature errors, TR_1 as well as TR_2, and emissivity, ε, for a chiller

Table 1 The emissivities ε of various metal surfaces [1]

Gold	0.02
Silver	0.02
Aluminum	Weathered=0.83;Foil (bright)=0.04 Disk, rough=0.96
Copper	polished=0.05; oxidized=0.78
Iron	cast(ox)=0.64; sheet, rusted=0.69
Stainless steel	polished=0.16; oxidized=0.85
Steel	polished=0.07; oxidized=0.79
Nickel	Electro pole=0.05

V. Conclusion

From the practical numerical results of this study, it demonstrates that one will take a very big risk to neglect the influence of heat radiation especially in the situations of oxidized metal single-pipe heat exchanger with greater surface emissivity applying in ambient air with lower outer convection coefficient. Thus, in order to obtain accurate results of a heat exchanger, the log mean heat transfer rate (LMHTR) method (considering heat radiation) should be applied instead of the conventional log mean temperature difference (LMTD) method (neglecting heat radiation). The exact numerical heat transfer results of a circular single-pipe heat exchanger can be obtained by any one-dimensional computer code (such as LabVIEW programming in this study) within half second computing time by a common PC

Table 2 Referred approximate values of convection heat transfer [1]

Free convection, h($Wm^{-2}K^{-1}$)	
Temperature difference =30 ℃ Vertical plate 0.3 in high in air	4.5
Temperature difference =30 ℃ Vertical plate 0.3 in high in air	6.5
Horizontal cylinder, 2 cm diameter, in water	890
Heat transfer across 1.5 cm vertical air gap with TD=60 ℃	2.64
Forced convection	
Air flow at 2 m/s over 0.2-m square plate	12
Air flow at 35 m/s over 0.75-m square plate	75
Air at 2 atm flowing in 2.5 cm diameter tube at 10 m/s	65
Water at 0.5 kg/s flowing in 2.5 cm diameter tube	3500
Air flow across 5 cm diameter cylinder with V= 50 m/s	180
Boiling water	
In a pool or container	2500–35,000
Flowing in a tube	5000-100,000
Condensation of water vapor in 1 atm	
Vertical surfaces	4000–11,300
Outside horizontal tubes	9500–25,000

Table 3. A heater constructed by a circular pipe with K=77Wm^{-1}K^{-1}, L=10m, r_1=198mm, r_2=200mm, T_{i1}=65℃, T_{i2}=60℃, T_{o1}=22℃ and h_o =8 Wm^{-2}-K^{-1} ; T_{o2}=24℃ and; T_{sur} =20℃

(a) h_i=30 Wm^{-2}K^{-1}

ε	Q Watt	Q_a Watt	QR %	T_{S2} ℃	T_{22} ℃	TR_2 %
0.0	3119	3119	0.0	52.4	52.4	0.0
0.6	3119	4342	-28	52.4	49.3	6.3
0.7	3119	4519	-31	52.4	48.8	7.3
0.8	3119	4692	-34	52.4	48.4	8.2

(b) h_i =5000 Wm^{-2}K^{-1}

ε	Q Watt	Q_a Watt	QR %	T_{S2} ℃	T_{22} ℃	TR_2 %
0.0	3953	3953	0.0	59.9	59.9	0.0
0.6	3953	6205	-36	59.9	59.8	0.06
0.7	3953	6579	-40	59.9	59.8	0.07
0.8	3953	6956	-43	59.9	59.8	0.08

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Table 4. A chiller constructed by a circular pipewith K_A=77Wm^{-1}K^{-1}, L=10m, r_1=198mm, r_2=200mm, T_{i1}=7℃, T_{i2}=12℃, T_{o1}=24℃ and h_o=8W Wm^{-2}-K^{-1} ; T_{o2}=26℃ and; T_{sur} =30℃

(a) h_i=30 Wm^{-2}-K^{-1}

ε	Q Watt	Q_a Watt	QR %	T_{S2} ℃	T_{22} ℃	TR_2 %
0.0	-1223	-1223	0.0	14.9	14.9	0.0
0.6	-1223	-1766	-31	14.9	16.3	-7.9
0.7	-1223	-1848	-34	14.9	16.5	-9.0
0.8	-1223	-1928	-37	14.9	16.6	-10.1

(b) h_i =5000 Wm^{-2}-K^{-1}

ε	Q Watt	Q_a Watt	QR %	T_{S2} ℃	T_{22} ℃	TR_2 %
0.0	-1550	-1550	-0.0	12.0	12.0	0.0
0.6	-1550	-2424	-36	12.0	12.0	-0.18
0.7	-1550	-2569	-40	12.0	12.0	-0.14
0.8	-1550	-2715	-43	12.0	12.0	-0.16

Acknowledgment

The authors would like to acknowledge the National Science Council of Taiwan, R.O.C.. This investigation is completed under the support of the project NSC **100**-2221-E-168-035.

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ISBN: 978-988-19251-9-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)