A More Accurate Total Heat Transfer Rate Method of the Single-pipe Heat Exchanger Considering Heat Radiation

Shih-Shih Ku, King-Leung Wong, IAENG member, Ho-Chiao Chuang

Abstract—The log mean temperature difference (LMTD) method is conventionally used to calculate the total heat transfer rate of heat exchangers. Because the heat radiation equation contains the 4th order exponential of temperature which is very complicated in calculations, thus LMTD method neglects the influence of heat radiation. Recently, Wong et al. [11] found out the inaccuracy of heat transfer characteristics of a metal pipe neglecting heat radiation effect in ambient air; they reported that in some practical situations, even when the temperature difference between the fluid inside a metal circular pipe and the ambient air is low to 1°C, the errors generated by neglecting heat radiation are still very large and hence, cannot be ignored. Unfortunately, it can be seen from Table 1 that most surface emissivities of oxidized metal are greater than 0.64; and Table 2 shows that the heat convection coefficients of ambient air with medium wind speed are quite small; the value of 8.3 Wm⁻²K⁻¹ is conventionally adopted in air conditioning design.

In this present investigation, the log mean heat transfer rate (LMHTR) method which considering the influence of heat radiation is developed to calculate the total heat transfer rate of single-pipe heat exchanger and much more accurate results can be obtained.

Index Terms—single-pipe heat exchanger, heat radiation, log mean heat transfer rate (LMHTR) method, LMTD method

I. INTRODUCTION

Heat exchangers are widely applied to the industries and living surroundings. The Mean Temperature Difference method neglecting heat radiation was introduced to design heat exchanger by Bowman et al. [1]. Stevens et al. [2] applied Mean Temperature Difference method in one, two, three pass cross-flow heat exchangers. The log mean temperature difference (LMTD) method has been applied widely to calculate the total heat transfer rate of heat exchangers in most heat transfer text books [such as 3-4], air conditioning and refrigeration text books [such as 5-6], as well as heat exchanger hand books [such as 7-10]. Because the heat radiation equation contains the 4th order exponential of temperature which is very troublesome in calculations,

Fig. 1 Condenser or evaporator of air conditioner is a single-pipe heat exchanger

Fig. 2 Radiator applied in vehicles and inside buildings is a single-pipe heat exchanger
II. PROBLEM FORMULATION

The condenser or evaporator of an air conditioner is a good application example of single-pipe heat exchanger as shown in Fig. 1. The radiators applied to vehicles and to heating system inside buildings is other application example of single-pipe heat exchanger as shown in Fig. 2. Fig. 3 shows that an circular single-pipe heat exchanger is with length \( L \), inner radius \( r_1 \), outer radius \( r_2 \), wall conductivity \( K \), outer surface emissivity \( \varepsilon \), internal fluid with convection heat transfer coefficient \( h_i \) and temperature \( T_{i1} \) at entrance section, convection heat transfer coefficient \( h_{i2} \) and temperature \( T_{i2} \) at exit section, respectively; and it is exposed to ambient air with convection heat transfer coefficient \( h_o \), ambient air temperatures \( T_{o1} \) at entrance section and \( T_{o2} \) at exit section, respectively. Practically, \( h_i = h_{i2} = h_{i} \) can be assumed. The surroundings temperature is \( T_{sur} \). The outer surface area of heat exchanger is much smaller than that of surroundings.

\[ R_{th} = \frac{1}{h_i 2\pi r_1 L} + \frac{1}{2\pi K L} + \frac{1}{h_i 2\pi r_2 L} \]  

The total thermal resistance of the long circular pipe neglecting the heat radiation by LMTD method is:

\[ Q = \frac{LMTD}{R_{th}} \]

The unit length heat transfer rate, \( q_1 \), at the entrance section is:

\[ q_1 = \frac{T_{i1} - T_{o1}}{R_{th} L} = \frac{T_{i1} - T_{o1}}{h_i 2\pi r_2} \]

The unit length heat transfer rate, \( q_2 \), at the exit section is:

\[ q_2 = \frac{T_{i2} - T_{o2}}{R_{th} L} = \frac{T_{i2} - T_{o2}}{h_i 2\pi r_2} \]

The values of total heat transfer rate \( Q \), the average surface temperature at the entrance section \( T_{s1} \), the average surface temperature at the exit section \( T_{s2} \), can obtained from Eqs. (1)~(5) under the given values of \( h_i, h_o, r_1, r_2, K, L, T_{i1}, T_{i2}, T_{o1}, T_{o2} \) and \( L \).

B. Situations considering the influence of heat radiation

While the influence of outside surface heat radiation is considered, the complete unit length heat transfer rate at the entrance section is:

\[ q_{a1} = \frac{T_{i1} - T_{o1}}{h_i 2\pi r_1} + \frac{1}{2\pi K} \]

The unit length surface convective heat transfer rate at the entrance section is:

\[ q_{c1} = h_i 2\pi r_2 (T_{21} - T_{o1}) \]

The unit length surface radiation heat transfer rate at the entrance section is:

\[ q_{r1} = \sigma \varepsilon 2\pi r_2 (T_{21}^4 - T_{sur}^4) \]

The following equation is obtained from heat balance at the entrance section:

\[ q_{a1} = q_{c1} + q_{r1} \]

The values of \( q_{a1}, q_{c1}, q_{r1} \) and \( T_{21} \) can obtained from Eqs. (6)~(9) under the given values of \( h_i, h_o, r_1, r_2, K, T_{i1}, T_{o1}, T_{sur} \) and \( \varepsilon \).

Similarly, the complete unit length heat transfer rate at the exit section is:

\[ q_{a2} = \frac{T_{i2} - T_{o2}}{h_i 2\pi r_2} + \frac{1}{2\pi K} \]

The values of total heat transfer rate \( Q \), the average surface temperature at the entrance section \( T_{s1} \), the average surface temperature at the exit section \( T_{s2} \), can obtained from Eqs. (1)~(5) under the given values of \( h_i, h_o, r_1, r_2, K, L, T_{i1}, T_{i2}, T_{o1}, T_{o2} \) and \( L \).
The unit length surface convective heat transfer rate at the exit section is:

\[ q_{c2} = h_c 2 \pi r_2 (T_{22} - T_{a2}) \]  

(11).

The unit length surface radiation heat transfer rate at the exit section is:

\[ q_{r2} = \sigma \varepsilon 2 \pi r_2 (T_{22}^4 - T_{sur}^4) \]  

(12)

The following equation is obtained from heat balance at the exit section:

\[ q_a2 = q_{c2} + q_{r2} \]  

(13)

The values of \( q_{a2}, q_{c2}, q_{r2} \) and \( T_{22} \) can obtained from Eqs. (10)–(14) under the given values of \( h_1, h_o, r_1, r_2, K, T_{12}, T_{a2}, \varepsilon \) and \( T_{sur} \).

The total heat transfer rate of the single-pipe heat exchanger considering the heat radiation by log mean heat transfer rate (LMHTR) method is defined as:

\[ Q_a = \frac{q_{a1} - q_{a2}}{\ln \frac{q_{a1}}{q_{a2}}} \]  

(14)

The above LMHTR method (considering heat radiation) under the same concept as LMTD method (neglecting heat radiation) is developed in this study. While the heat radiation is not considered, assume the temperatures \( T_{12} \) and \( T_{a2} \) keep constant at the entrance section, and \( T_{12} \) and \( T_{a2} \) keep constant at the exit section, then the following relations can be obtained in the situation of neglecting heat radiation (or \( \varepsilon = 0 \)):

\[ q_{a1} = \frac{T_{11} - T_{a1}}{R_{th} L} \]  

(15)

\[ q_{a2} = \frac{T_{12} - T_{a2}}{R_{th} L} \]  

(16)

Then the following relation between LMTD method and LMHTR method in the situation of neglecting heat radiation (or \( \varepsilon = 0 \)) can be proven:

\[ Q_a = \frac{q_{a1} - q_{a2}}{\ln \frac{q_{a1}}{q_{a2}}} = \frac{R_{th} L}{(T_{11} - T_{a1})} \frac{(T_{12} - T_{a2})}{R_{th} L} \]  

(17)

It can be proved from Eq. (17) that the results obtained from LMTD method and LMHTR method in the situation of neglecting heat radiation (or \( \varepsilon = 0 \)) are the same value.

The error of total heat transfer rate generated by neglecting heat radiation effect (while \( \varepsilon \neq 0 \)) is defined as:

\[ QR = \frac{Q_a - Q_{a0}}{Q_a} \times 100\% \]  

(18)

The error of average surface temperature at the entrance section generated by neglecting heat radiation is:

\[ TR_1 = \frac{(T_{11} - T_{a1})}{T_{21}} \times 100\% \]  

(19)

The error of average surface temperature at the exit section generated by neglecting heat radiation is:

\[ TR_2 = \frac{(T_{12} - T_{a2})}{T_{22}} \times 100\% \]  

(20)

**III. The reliability of the numerical results**

The nature of results of heat equation for a circular single-pipe heat exchanger is one-dimension exact solution. The exact numerical heat transfer results of a circular pipe can be obtained by any one-dimensional computer code (such as LabVIEW programming in this study). The computer aid results are obtained within half second computing time by a common PC.

In order to check if the numerical results are reliable, the following checking methods are used:

1. Let outer surface emissivity \( \varepsilon = 0 \), make sure if the results of \( QR = 0 \).
2. Let surface emissivity \( \varepsilon = 1 \), outer convection coefficient \( h_o = 100,000 \text{Wm}^{-2}\text{K}^{-1} \), make sure if the results of \( QR \) close to zero.

![Fig. 4 The relations between heat transfer rate error, QR, and emissivity, ε, for a heater](image-url)
IV. Results and discussions

The following practical examples is use to demonstrate what the differences and relationships of the results between the LMTD method and LMHTR method are.

Example 1:
A single-pipe heater constructed by a circular pipe with conductivity $K=77$ Wm$^{-1}$K$^{-1}$ and various surface emissivities $\varepsilon=0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$ and 1 (refer Table 1), length $L=10m$, inner radius $r_i=198mm$, outer radius $r_o=200mm$, the hot water is flowing inside the heater with entrance temperature $T_{i1}=65{^\circ}C$ and convective heat coefficient $h_i=30$ and $5000$ Wm$^{-2}$K$^{-1}$ (refer Table 2), with exit temperature $T_{i2}=60{^\circ}C$, the ambient air at the entrance section with temperature $T_{o1}=22{^\circ}C$ and convective heat coefficient $h_o=8$ Wm$^{-2}$K$^{-1}$ (refer Table 2 and practical application in air conditioning), the ambient air at the exit section with temperature $T_{o2}=24{^\circ}C$, and the heater is located in a big room with wall or surrounding temperature $T_{sul}=20{^\circ}C$. The results are shown in Table 3 and Figs. 4-5.

Example 2:
A chiller constructed by a circular pipe with conductivity $K_a=77$ Wm$^{-1}$K$^{-1}$ and various surface emissivities $\varepsilon=0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$ and 1 (refer Table 1), length $L=10m$, inner radius $r_i=198mm$, outer radius $r_o=200mm$, the cold water is flowing inside the heater with entrance temperature $T_{i1}=7{^\circ}C$ and convective heat coefficient $h_i=30$ and $5000$ Wm$^{-2}$K$^{-1}$ (refer Table 2), with exit temperature $T_{i2}=12{^\circ}C$, the ambient air at the entrance section with temperature $T_{o1}=22{^\circ}C$ and convective heat coefficient $h_o=8$ Wm$^{-2}$K$^{-1}$ (refer Table 2 and practical application in air conditioning), the ambient air at the exit section with temperature $T_{o2}=26{^\circ}C$, and the heater is located in a big room with wall or surrounding temperature $T_{sul}=30{^\circ}C$. The results are shown in Table 4 and Figs. 6-7.

From Table 3-4 and Figs. 4-7 show that in situations of $\varepsilon=0$, $QR=0$, $TR_1=0$ and $TR_2=0$; the greater the $\varepsilon$ is, the greater the absolute values of $QR$, $TR_1$ and $TR_2$ will be; the greater the $h_o$ are, the smaller the $TR_1$ and $TR_2$ will be; while $\varepsilon>0$, all the $QR$ are negative, it means in the situations of neglecting heat radiation, the smaller absolute values of heat transfer rate, $|Q|$, will be obtained (i.e., $|Q|<|Q_0|$), and most absolute values of $QR$ are so big and out of the engineering acceptable range; and in situations of $\varepsilon>0$, the greater the values of $h_i$, the greater the absolute values of $QR$ will be, but the smaller the $TR_1$ and $TR_2$ will be; for the heater, $TR_1$ and $TR_2$ are positive (i.e., $T_{s1}>T_{21}$ and $T_{s2}>T_{22}$), it means in the situations of neglecting heat radiation, the greater surface temperatures (over-prediction of hot surface) will be obtained; for the chiller, $TR_1$ and $TR_2$ are negative (i.e., $T_{s1}<T_{21}$ and $T_{s2}<T_{22}$), it means in the situations of neglecting heat radiation, the smaller surface temperatures (over-prediction of cold surface) will be obtained, these predictions may lead to the condensation occurred on the surface.

Meanwhile, it can be seen in Table 1 that most surface emissivities of oxidized metal heat exchanger are greater than 0.64 (where $\varepsilon=0.83$ for aluminum, $\varepsilon=0.78$ for copper, $\varepsilon=0.64$ for iron, $\varepsilon=0.85$ for stainless steel and $\varepsilon=0.79$ for steel). Some metal single-pipe heat exchangers, condenser and evaporator of air conditioner (Fig.1), such as radiator (Fig. 2), are applied in the natural air or low speed air with very low values of convection coefficient (refer Table 2 and practical application in air conditioning). Thus, in above situations, neglecting the influence of heat radiation will obtain the inaccurate heat transfer results for a single-pipe heat exchanger.
Table 1 The emissivities $\varepsilon$ of various metal surfaces [1]

<table>
<thead>
<tr>
<th>Material</th>
<th>$\varepsilon$</th>
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<tbody>
<tr>
<td>Gold</td>
<td>0.02</td>
</tr>
<tr>
<td>Silver</td>
<td>0.02</td>
</tr>
<tr>
<td>Aluminum</td>
<td>Weathered=0.83; Foil (bright)=0.04; Disk, rough=0.96</td>
</tr>
<tr>
<td>Copper</td>
<td>polished=0.05; oxidized=0.78</td>
</tr>
<tr>
<td>Iron</td>
<td>cast(ox)=0.64; sheet, rusted=0.69</td>
</tr>
<tr>
<td>Stainless steel</td>
<td>polished=0.16; oxidized=0.85</td>
</tr>
<tr>
<td>Steel</td>
<td>polished=0.07; oxidized=0.79</td>
</tr>
<tr>
<td>Nickel</td>
<td>Electro pole=0.05</td>
</tr>
</tbody>
</table>

V. Conclusion

From the practical numerical results of this study, it demonstrates that one will take a very big risk to neglect the influence of heat radiation especially in the situations of oxidized metal single-pipe heat exchanger with greater surface emissivity applying in ambient air with lower outer convection coefficient. Thus, in order to obtain accurate results of a heat exchanger, the log mean heat transfer rate (LMHTTR) method (considering heat radiation) should be applied instead of the conventional log mean temperature difference (LMTD) method (neglecting heat radiation). The exact numerical heat transfer results of a circular single-pipe heat exchanger can be obtained by any one-dimensional computer code (such as LabVIEW programming in this study) within half second computing time by a common PC.
Table 4. A chiller constructed by a circular pipe with \( K_a = 77 \text{Wm}^{-1}\text{K}^{-1} \), \( L = 10 \text{m} \), \( r_1 = 198 \text{mm} \), \( r_2 = 200 \text{mm} \), \( T_{in} = 7 \text{°C} \), \( T_{in} = 12 \text{°C} \), \( T_{ex} = 24 \text{°C} \) and \( h_p=8 \text{Wm}^{-2}\text{K}^{-1} \); \( T_{ex}=26 \text{°C} \) and; \( T_{sur}=30 \text{°C} \)

(a) \( h_f=30 \text{Wm}^{-2}\text{K}^{-1} \)

<table>
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<tr>
<th>( \epsilon )</th>
<th>( Q )</th>
<th>( Q_a )</th>
<th>( QR )</th>
<th>( T_{52} )</th>
<th>( T_{22} )</th>
<th>( TR_2 )</th>
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<td>-37</td>
<td>14.9</td>
<td>16.6</td>
<td>-10.1</td>
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(b) \( h_f=5000 \text{Wm}^{-2}\text{K}^{-1} \)

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<th>( \epsilon )</th>
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<th>( Q_a )</th>
<th>( QR )</th>
<th>( T_{52} )</th>
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References


