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Risk-adjusted Joint Optimization of Base-stock Levels and Component Allocation in an ATO System with Returns

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***Abstract*—We consider a multi-component, multi-product, periodic-review (re)assemble-to-order (RATO) system that uses an independent base-stock policy for the inventory replenishment of the components. At the beginning of each period, end-of-lease cores are returned. Because the quality of cores is random, they are tested, graded, and sorted into four pre-specified quality levels. Then, the random, jointly and continuously distributed demands for the products are realized. In our problem, partial fulfillment is not allowed. Furthermore, the system quotes a predetermined response time window for each product, and it penalizes the system if the demand is not satisfied within its time window. We model this problem through a risk-adjusted two-stage stochastic programming problem, where the first-stage decisions are the base-stock levels for all components, and the second-stage decisions are the allocations of components to different products. The risk adjustment is formulated through a chance constraint, which is then replaced by a conditional value-at-risk constraint. We solve the resulting problem through the sample average approximation method combined with the L-shaped method. We also present some encouraging numerical results.**

***Index Terms*—Monte Carlo simulation, reverse logistics, risk measures, stochastic programming.**

I. INTRODUCTION

PRODUCT recovery can be performed in many ways such as remanufacturing, reconditioning, recycling, cannibalizing and refurbishing of products. The reuse of product returns (also called cores) can be very profitable, especially for the high-tech products that have quite long product life cycles. For instance, the characteristic life cycle of a computer chip is 80,000 hours for which only 20,000 hours are used; therefore, that chip can still be economically used for 60,000 hours in some other products through, say, remanufacturing; see Geyer et al. (2007). Remanufacturing is one of the highly important field of product recovery. In the context of our paper, it refers to the process through which a core is brought to an as good as new condition by inspecting its components, and performing repairing, replacing, restoring operations and/or updating it with new specifications when necessary. This operation is widely found for high-valued industrial products like photocopiers, computers, cellular phones, aviation equipments, vehicle engines, and telecommunication or medical equipments.

Many firms have been looking for ways to decrease their response times to the market because the pressure for serving customers speedily and the impacts of product obsolescence increase. One way to deal with the aforementioned issues is to adopt an assemble-to-order (ATO) manufacturing strategy and/or its variations (i.e., reassemble-to-order (RATO), configure-to-order, etc.) instead of employing the traditional make-to-stock system. In an (R)ATO system, the inventories are held at component or part levels, which substantially reduces the inventory holding costs. This (R)ATO system further increases customer satisfaction through decreasing response times to the demands and increasing fill rates (i.e., the fraction of demands satisfied from on-hand inventory to the total demands).

Our paper contributes to the existing literature in the following ways:

- This paper considers the joint optimization of the base-stock levels and component allocation in case there are cores of uncertain quality.
- The problem is considered in a risk-adjusted manner by considering the so-called conditional-value-at-risk constraint.

The first item was also considered in Akçay and Xu (2004), but in a risk-neutral environment and without remanufacturing.

The rest of the paper is organized as follows. In Section 2, we present a description of the system, and in Section 3 we present the risk-adjusted two-stage optimization problem. In Section 4, we present our numerical results, and in Section 5 our conclusions and future research.

II. SYSTEM DESCRIPTION

We consider a hybrid assemble-to-order (ATO) and reassemble-to-order (RATO) system with m components, indexed by $i = 1, ..., m$, and n products, indexed by $j = 1, ..., n$. The following sequence of events is typical for the system for every period t, $t = 0, 1, 2, ...$ At the beginning of a period, the inventory position (i.e., on-hand inventory plus on-order inventory minus backorders) of each component is reviewed, and the component replenishment orders are placed according to the inventory policy. After the receipt of the replenishment for earlier orders and update of the inventory positions of the components, end-of-lease products (cores) are returned. These cores are subject to be tested, graded, and sorted into a number of quality levels, so that the random amounts of cores which fall into each quality level are revealed. Then, random orders for different products arrive through lease agreements.

The properties of the system are as follows. Each component i operates under a periodic-review, independent base-stock policy with the base-stock level (order-up-to level) for

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component i denoted by S_i. The replenishment lead time of component i, denoted by L_i, is deterministic and integer, which is an integer multiple of the review interval. These lead times can be different for different components.

The return time and quantity of the cores are considered as deterministic. However, the quality of the cores is difficult to predict. Hence, they are tested, graded, and sorted into four quality levels; see the Xerox Europe case study in [6]. The cores of quality level 1 (i.e., best cores) require only refurbishing, and the cores of quality level 2 can be remanufactured. Moreover, the cores of quality level 3 are disassembled, and after being repaired, some of their components enter the inventories of the brand-new components. Finally, the cores of quality level 4 are immediately disposed off at negligible costs.

In this study, we assume no market segmentation between remanufactured and manufactured products. Each manufactured (brand-new) product is assembled from multiple units of a subset of m components, and each core of quality level 2 is remanufactured by replacing a prespecified number of components. Let b_{ij} and b'_{ij2} denote the usage rates of component i to manufacture and to remanufacture unit demand of product j, respectively, where $b_{ij} \geq b'_{ij2}$. The system quotes a response time window, w_j, for product j. This time window is prespecified and fixed for every product type by the system. We assume that the system is penalized by a unit penalty, q_j, if a demand for product j cannot be filled within w_j periods after its arrival. Furthermore, a demand for product j is considered to be filled if that demand is allocated b_{ij} or b'_{ij2} units of component i (no partial shipment is allowed). All unfilled demands are completely backlogged. The system uses the following order to fill the demands: first, the cores of quality level 1, then the cores of quality level 2, and finally the manufactured products. This order is reasonable because the production times and component requirements increase in the same order.

The problem of interest is analyzed in two stages. The first-stage decisions are the optimal base-stock levels S_i for $i = 1, ..., m$, and these decisions are taken without observing the realizations of the random amounts of cores of different quality levels and the realizations of the random demands for the n products. After the cores are tested and graded, and customers' demands are received, the second-stage decisions, namely the amounts of inventories to be allocated to the unfilled demands, are made in each period. We assume that the inventories are allocated to the unfilled demands subject to a first-come first-served (FCFS) inventory commitment rule. Under the FCFS rule, no inventory is allocated to the demands received in later periods unless earlier backlogs for a component are completely satisfied. The FCFS rule has been adopted by [1], [2], and [8].

Now, we introduce new random variables, which depend on the joint random demands $(P_{1t}, P_{2t}, ..., P_{nt})$ for the n products and the joint random amounts of the cores that fall into four quality levels $(R_{jt1}, R_{jt2}, R_{jt3}, R_{jt4})$ for $j = 1, ..., n$ and $t = 0, 1, ...$ These new random variables will be used to derive an equation for the inventory on-hand. Let D_{it} and Q_{it} be the total demand for component i in period t and the total amount of component i disassembled from cores of quality level 3, respectively, for $i = 1, ..., m$. Then,

$$D_{it} = \sum_{j=1}^{n} \Big[b'_{ij2} \min \Big\{ (P_{jt} - R_{jt1})^{+}, R_{jt2} \Big\} + b_{ij} (P_{jt} - R_{jt1} - R_{jt2})^{+} \Big]$$

$$Q_{it} = \sum_{j=1}^{n} b'_{ij3} R_{jt3}$$

where, for any two random variables X and Y, $(X - Y)^{+} = \max\{X - Y, 0\}$. We further denote the amount of replenishment for component i in period t and the net inventory level (i.e., on-hand inventory minus backlog) of component i at the end of period t by A_{it} and I_{it}, respectively. Let $D_i[s,t]$, $A_i[s,t]$, and $Q_i[s,t]$ be the total demand, the total replenishment, and the total disassembled amount for component i from period s through period t inclusive, respectively, where

$$D_i[s,t] = \sum_{u=s}^{t} D_{iu} \quad A_i[s,t] = \sum_{u=s}^{t} A_{iu} \quad Q_i[s,t] = \sum_{u=s}^{t} Q_{iu}.$$

Now, we derive an equation for the inventory on-hand. Assume that k is a nonnegative integer such that $k \leq L_i$ for any lead time L_i. Because each component is operated under an independent base-stock level S_i, based on [7], the following equation for the net inventory level at the end of period $t + k$ can be written for $i = 1, ..., m$

$$I_{i,t+k} = S_i - D_i[t+k-L_i, t+k] + Q_i[t+k-L_i, t+k]. \tag{1}$$

Furthermore, using balance equations and FCFS inventory commitment rule, the net inventory level at the end of period $t+k$ is related to the one at the end of period $t-1$ as follows

$$I_{i,t+k} = I_{i,t-1} + A_i[t,t+k] - D_i[t,t+k] + Q_i[t,t+k]. \tag{2}$$

Substituting (2) in (1), we reach the following result

$$I_{i,t-1} + A_i[t,t+k] + Q_i[t,t+k] = S_i - D_i[t+k-L_i, t-1] + Q_i[t+k-L_i, t+k]. \tag{3}$$

Note that $I_{i,t-1} + A_i[t,t+k] + Q_i[t,t+k]$ is the net inventory level at the end of period $t + k$ after having received all replenishment orders and having disassembled all repairable components i from cores of quality level 3, but before allocating any inventory to the demands realized after period $t - 1$. Furthermore, because of the FCFS rule, if the amount $S_i - D_i[t+k-L_i, t-1] + Q_i[t+k-L_i, t+k]$ is positive, that inventory will be committed to the demands $(P_{1t}, P_{2t}, ..., P_{nt})$ of period t before any demands of the subsequent periods. Now, suppose that the response time windows for the n products can be ordered as $w_1 \leq w_2 \leq ... \leq w_n$. Then, the on-hand inventory of component i to be committed to $(P_{1t}, P_{2t}, ..., P_{nt})$ for $k = 0, 1, ..., w_n$ is given by

$$(S_i - D_i[t+k-L_i, t-1] + Q_i[t+k-L_i, t+k])^{+}. \tag{4}$$

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Before presenting the formulation, we assume the following: the longest response window w_n does not exceed the shortest of the lead times L_i; i.e., $w_n \leq \min_{1\leq i\leq m} L_i$. This is a plausible assumption because if there exists any product j for which the lead time of component i satisfies $L_i < w_j$, that component i can be replenished to fill the orders of product j before its response time window w_j. Hence, the component i will not be considered in the allocation problem for product j.

In the following, for ease of exposition, we shall focus on stationary random data so that the on-hand inventory level of component i in (4) becomes

$$\left(S_i - D_i\left[L_i - k\right] + Q_i\left[L_i + 1\right]\right)^+ . \tag{5}$$

III. Risk-Adjusted Two-Stage Stochastic Programming Formulation

We consider the following problem with a chance constraint:

$$\begin{array}{ll} \min\limits_{\mathbf{S}=(S_1,\ldots,S_m)^T\in\mathbb{R}^m} & \mathbf{c}^T\mathbf{S} + \mathbb{E}\left[Q\left(\mathbf{S},\boldsymbol{\varepsilon}\right)\right] \\ \text{s.t.} & \text{Prob}\left\{Q\left(\mathbf{S},\boldsymbol{\varepsilon}\right) \leq \eta\right\} \geq 1-\alpha \\ & \mathbf{S} \geq \mathbf{S}_{\text{safe}} \end{array} \tag{6}$$

where for a realization $\widetilde{\boldsymbol{\varepsilon}} = \left(\widetilde{P}_1, ..., \widetilde{P}_n, \widetilde{R}_{11}, ..., \widetilde{R}_{n4}\right)$ of $\boldsymbol{\varepsilon}$, the second-stage cost $Q\left(\mathbf{S}, \widetilde{\boldsymbol{\varepsilon}}\right)$ is given by

$$\begin{array}{lll} \min & \mathbf{q}^T\mathbf{u} & \\ \text{s.t.} & \sum\limits_{j=1}^{n}\sum\limits_{l=0}^{k}\left(b'_{ij2}x^r_{jl} + b_{ij}x^m_{jl}\right) \leq & \\ & \left(S_i - \widetilde{D}_i\left[L_i-k\right] + \widetilde{Q}_i\left[L_i+1\right]\right)^+ & \text{(7a)} \\ & \text{for } k = 0,1,...,w_n \text{ and } i = 1,...,m & \\ & \sum\limits_{l=0}^{w_j}\left(x^r_{jl} + x^m_{jl}\right) + u_j = \left(\widetilde{P}_j - \widetilde{R}_{j1}\right)^+ & \text{(7b)} \\ & \text{for } j = 1,...,n & \\ & \sum\limits_{l=0}^{w_j} x^r_{jl} \leq \widetilde{R}_{j2} \text{ for } j = 1,...,n & \text{(7c)} \\ & x^r_{jl} \geq 0, x^m_{jl} \geq 0, u_j \geq 0 & \\ & \text{for } l = 0,...,w_j \text{ and } j = 1,...,n. & \end{array}$$

In the following, we define the notation in (6) and (7). $\mathbf{c} = (c_1, ..., c_m)^T$ is vector of procurement costs per unit of the m components, $\mathbf{S} = (S_1, ..., S_m)^T$ is vector of base-stock levels of the m components, $\mathbf{S}_{\text{safe}} = (S_{1s}, ..., S_{ms})^T$ is vector of mean safety stock levels for the m components during their lead times, η is upper bound on random second-stage cost $Q\left(\mathbf{S}, \boldsymbol{\varepsilon}\right)$, and α is significance level where $\alpha \in (0, 1)$. The first-stage here-and-now decisions are the base-stock levels $(S_1, ..., S_m)$, and these decisions are made before observing the random data. Furthermore, $\mathbf{q} = (q_1, ..., q_n)^T$ is vector of shortage costs per unit of the n products, and $\mathbf{u} = (u_1, ..., u_n)^T$ is vector of shortage amounts of the n products. The second-stage wait-and-see decisions are the remanufactured and manufactured amounts x^r_{jl} and x^m_{jl} of product j ($j = 1, ..., n$), respectively, within its response time window $l = 0, 1, ..., w_j$, and the shortage amounts u_j. The second-stage decisions are made after observing the random demands and the random amounts of cores that fall into each of the four quality levels. Moreover, (7a) implies that the amount of component i used for remanufacturing and manufacturing within response time windows cannot exceed its on-hand inventory level; (7b) implies that the total remanufactured and manufactured amounts of product j within its response time window w_j plus the shortage amount has to be equal to the net demand $\left(\widetilde{P}_j - \widetilde{R}_{j1}\right)^+$ for product j, because the cores of quality level 1 (i.e., $\widetilde{R}_{j1}$) are ready to fill the demand for product j after only minor servicing. Furthermore, (7c) implies that the total remanufactured amounts of product j within w_j cannot exceed the remanufacturable amount $\widetilde{R}_{j2}$ for product j. Additionally, in case all penalty costs are $q_j = 1$, the objective function in (7) divided by the sum of the expected demands for all n products equals the expected average no-fill rate (i.e., the complement of fill rate with respect to one).

The formulation (6) provides a risk-adjusted approach to the problem; i.e., it minimizes the random second-stage cost $Q\left(\mathbf{S}, \boldsymbol{\varepsilon}\right)$ on average, while controlling its upper limit η for different realizations of the random data. A well-known problem of such a formulation is that chance constraints usually define non-convex feasible sets. It was suggested in [9] to replace chance constraints by conditional value-at-risk constraints, where the Conditional Value-at-Risk of a random variable Z at significance level α is defined as

$$\mathsf{CV@R}_\alpha\left[Z\right] := \inf_{t\in\mathbb{R}}\left\{t + \alpha^{-1}\mathbb{E}\left[Z-t\right]^+\right\}. \tag{8}$$

It was further shown in [9] that (8) is a convex conservative approximation to its corresponding chance constraint; i.e., the feasible set defined by $\mathsf{CV@R}_\alpha\left[Z\right] \leq \eta$ is contained in the feasible set defined by $\text{Prob}\left\{Z \leq \eta\right\} \geq 1-\alpha$. Therefore, in our analysis, we will replace the chance constraint in (6) by its corresponding $\mathsf{CV@R}_\alpha$ constraint.

Ignoring the chance constraint in (6), the formulations in (6) and (7) satisfy the well-known *relatively complete recourse* assumption; i.e., given any feasible first-stage solution $(S_1, ..., S_m)$, there exists a feasible second-stage solution $\left(x^r_{jl}, x^m_{jl}, u_j\right)$ ($j = 1, .., n$ and $l = 0, ..., w_j$) for almost every (a.e.) realization of $\boldsymbol{\varepsilon}$. To see this, consider the worst-case situation in which a feasible solution with $S_i \geq 0$ for each component i for (6) is given, but the right-hand-sides in (7a) are all zero; i.e., there is no on-hand inventory for any component i. Then, for a.e. realization of $\boldsymbol{\varepsilon}$, $x^r_{jl} = 0$, $x^m_{jl} = 0$, and $u_j = \left(\widetilde{P}_j - \widetilde{R}_{j1}\right)^+$ for all $j = 1, ..., n$ and $l = 0, ..., w_j$ constitutes a feasible solution for (7). However, the chance constraint and consequently the $\mathsf{CV@R}_\alpha$ constraint can make (6) infeasible. Therefore, we relax the $\mathsf{CV@R}_\alpha$ constraint as follows. Let

$$\begin{aligned} \rho_\lambda\left[Q\left(\mathbf{S},\boldsymbol{\varepsilon}\right)\right] := &\left(1-\lambda\right)\mathbb{E}\left[Q\left(\mathbf{S},\boldsymbol{\varepsilon}\right)\right] \\ &+\lambda\mathsf{CV@R}_\alpha\left[Q\left(\mathbf{S},\boldsymbol{\varepsilon}\right)\right] \end{aligned} \tag{9}$$

be a real-valued function of the random variable $Q\left(\mathbf{S}, \boldsymbol{\varepsilon}\right)$, where $\mathbb{E}\left[Q\left(\mathbf{S}, \boldsymbol{\varepsilon}\right)\right]$ is assumed to be well-defined and finite. In (9), $\lambda \in [0, 1]$ is a parameter that can be tuned for a tradeoff between minimizing on average and risk control. Using (8) and (9), we reformulate the first-stage problem (6) as follows, which we will use throughout the paper:

$$\min_{\mathbf{S}\geq\mathbf{S}_{\text{safe}},t\in\mathbb{R}} \quad \mathbf{c}^T\mathbf{S} + \lambda t + \mathbb{E}\{V\left(\mathbf{S},\boldsymbol{\varepsilon}\right)\}. \tag{10}$$

where $V\left(\mathbf{S}, \boldsymbol{\varepsilon}\right) = (1-\lambda)\,Q\left(\mathbf{S}, \boldsymbol{\varepsilon}\right) + \lambda\alpha^{-1}\left[Q\left(\mathbf{S}, \boldsymbol{\varepsilon}\right) - t\right]^+$. Now, the second-stage objective function in (7) is given

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by $(1-\lambda)\,\mathbf{q}^T\mathbf{u}+\lambda\alpha^{-1}\left[\mathbf{q}^T\mathbf{u}-t\right]^+$. By introducing a new variable v such that $v \geq \mathbf{q}^T\mathbf{u}-t$ and $v \geq 0$, the formulation in (7) becomes

$$\min\,(1-\lambda)\,\mathbf{q}^T\mathbf{u}+\lambda\alpha^{-1}v \tag{11}$$

$$\begin{aligned}
\text{s.t.}&\sum_{j=1}^{n}\sum_{l=0}^{k}\left(b'_{ij2}x^r_{jl}+b_{ij}x^m_{jl}\right)\leq \\
&\left(S_i-\widetilde{D}_i\left[L_i-k\right]+\widetilde{Q}_i\left[L_i+1\right]\right)^+ \\
&\text{ for } k=0,1,\ldots,w_n \text{ and } i=1,\ldots,m \\
&\sum_{l=0}^{w_j}\left(x^r_{jl}+x^m_{jl}\right)+u_j=\left(\widetilde{P}_j-\widetilde{R}_{j1}\right)^+ \\
&\text{ for } j=1,\ldots,n \\
&\sum_{l=0}^{w_j}x^r_{jl}\leq \widetilde{R}_{j2} \text{ for } j=1,\ldots,n \\
&\mathbf{q}^T\mathbf{u}-v\leq t \\
&x^r_{jl}\geq 0, x^m_{jl}\geq 0, u_j\geq 0 \\
&\text{ for } l=0,\ldots,w_j \text{ and } j=1,\ldots,n.
\end{aligned}$$

We assume that we can sample from the joint distributions of $(P_1,P_2,\ldots,P_n)$ and $(R_{j1},R_{j2},R_{j3},R_{j4})$ for $j=1,\ldots,n$ through Monte Carlo simulation and solve the problems in (10) and (11) through the sample average approximation method combined with the L-shaped algorithm. We do not give further details about our solution procedure and refer to [10] for the sample average approximation, and [3] for the L-shaped algorithm.

IV. Numerical Examples

We implement all experiments on a PC with Windows XP, Intel Pentium 4 CPU of 1.60 GHz, and 1.00 GB RAM. Because for now the instances that are detailed below are small, the CPU times are negligible, and hence they are not presented.

We consider the configure-to-order system in [4]. The lead times and the unit acquisition costs are given in Table 1, and the bill-of-materials structure is given in Table 2. Furthermore, we assume that the demands for the three products are multivariate normally distributed with the mean vector (150, 100, 125), the variances (750, 625, 675), and the correlations between the demands are randomly generated from the uniform distribution on (-1, 1). The response window times are first considered as $w_1=1$, $w_2=2$, $w_3=3$ for products 1, 2, and 3. Later, we also consider $w_1=w_2=w_3=0$, which enables us to observe immediate fill rates for products 1, 2, and 3. Moreover, the amounts of cores (returned products) are considered as (10, 15, 20) for products 1, 2, and 3. We assume that for each product, the numbers of cores that fall into the quality levels 1, 2, 3, and 4 follow multivariate normal distributions with the following mean vectors and variances: (1200, 1500, 2500, 2200) as the mean vector and (300, 500, 1000, 560) as the variances for product 1, (3500, 1200, 2200, 1800) as the mean vector and (1000, 600, 1200, 900) as the variances for product 2, and (1500, 1500, 1500, 300) as the mean vector and (900, 900, 900, 125) as the variances for product 3. For each product, the correlations between the quality levels are again randomly generated from the uniform distribution on (-1, 1). We denote the realizations

TABLE I
Components, lead times, and unit acquisition costs for the example configure-to-order systems in [4]

i	Component	Lead time	Unit acquisition cost
1	Base unit	5	215
2	128 MB card	15	232
3	450 MHz board	12	246
4	500 MHz board	12	316
5	600 MHz board	12	639
6	7 GB disk drive	18	215
7	13 GB disk drive	18	250
8	Preload A	4	90
9	Preload B	4	90
10	CD ROM	10	126
11	Video graphics card	6	90
12	Ethernet card	10	90

TABLE II
The bill-of-materials structure for the example configure-to-order systems in [4]

Component	Product 1	Product 2	Product 3
Base unit	1.0	1.0	1.0
128 MB card	1.0	1.0	1.0
450 MHz board	1.0	-	-
500 MHz board	-	1.0	-
600 MHz board	-	-	1.0
7 GB disk drive	1.0	0.4	-
13 GB disk drive	-	0.6	1.0
Preload A	0.7	0.5	0.3
Preload B	0.3	0.5	0.7
CD ROM	1.0	1.0	1.0
Video graphics card	-	0.3	0.6
Ethernet card	-	0.2	0.5

of these multivariate normally distributed random variables by $\widetilde{\boldsymbol{\delta}}_1=\left(\widetilde{\delta}_{1,1},\widetilde{\delta}_{1,2},\widetilde{\delta}_{1,3},\widetilde{\delta}_{1,4}\right)$, $\widetilde{\boldsymbol{\delta}}_2=\left(\widetilde{\delta}_{2,1},\widetilde{\delta}_{2,2},\widetilde{\delta}_{2,3},\widetilde{\delta}_{2,4}\right)$, and $\widetilde{\boldsymbol{\delta}}_3=\left(\widetilde{\delta}_{3,1},\widetilde{\delta}_{3,2},\widetilde{\delta}_{3,3},\widetilde{\delta}_{3,4}\right)$. Note that because for a fixed product j, the sum of the fractions of cores that fall into the four quality levels has to be equal to one, we compute these four fractions from the $\widetilde{\boldsymbol{\delta}}_j$ through $\widetilde{\delta}_{j,1}/\widetilde{\rho}_j$, $\widetilde{\delta}_{j,2}/\widetilde{\rho}_j$, $\widetilde{\delta}_{j,3}/\widetilde{\rho}_j$, and $\widetilde{\delta}_{j,4}/\widetilde{\rho}_j$, where $\widetilde{\rho}_j=\widetilde{\delta}_{j,1}+\widetilde{\delta}_{j,2}+\widetilde{\delta}_{j,3}+\widetilde{\delta}_{j,4}$. Then, for example, we find the realizations of the cores of quality levels 1, 2, 3, and 4 for product 1 by $10\times\widetilde{\delta}_{1,1}/\widetilde{\rho}_1$, $10\times\widetilde{\delta}_{1,2}/\widetilde{\rho}_1$, $10\times\widetilde{\delta}_{1,3}/\widetilde{\rho}_1$, and $10\times\widetilde{\delta}_{1,4}/\widetilde{\rho}_1$, respectively. The penalty costs are $\mathbf{q}^T=(22480, 25200, 33020)$.

We consider λ and α in (10) and (11) as parameters, and solve these problems for several values of λ and α. We first solve the problems for $w_1=1$, $w_2=2$, $w_3=3$ (instance 1), and then repeat the experiments for $w_1=w_2=w_3=0$ (instance 2). We obtain very similar results for both instances, hence we present results in Figures 1 and 2 only for instance 1. Note that after sampling the demands and the random amounts of cores that fall into each quality levels, the problems in (10) and (11) are formulated as two linear programming problems, which are then solved through CPLEX 12.2.

Note that increasing λ or decreasing α would increase the relative importance of the risk adjustment term, and hence would lead to a more conservative system; for α, this can

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also be seen from the chance constraint in (6). Both Figures 1 and 2 reflect the increase in the conservatism of the system because the optimal objective value of the first-stage problem in (10) increases as λ increases in Figure 1, and it increases as α decreases in Figure 2.

V. Conclusions and Future Research

In this paper, we consider a multi-component, multi-product, periodic-review (re)assemble-to-order system, and find the joint optimal base-stock levels and component allocation policies in a risk-adjusted environment. We model this problem through a risk-adjusted two-stage stochastic programming problem, where the first-stage decisions are the base-stock levels for all components, and the second-stage decisions are the allocations of components to different products. Risk adjustment is achieved through the conditional-value-at-risk constraint. We solve the resulting problem through the sample average approximation combined with the L-shaped method. Our preliminary numerical results are intuitively sound: as we make the system more conservative (by increasing the parameter λ or by decreasing the parameter α), our expected total optimal objective value increases.

Further research should include more numerical results by using different multivariate distributions for demands. Consideration of market segmentation between remanufactured and manufactured products is a further important issue.

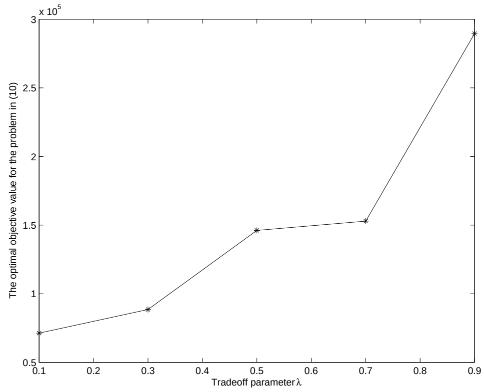

Fig. 1. Effects of changing λ on the optimal objective value of the first-stage problem in (10): $\alpha = 10\%$ and fixed

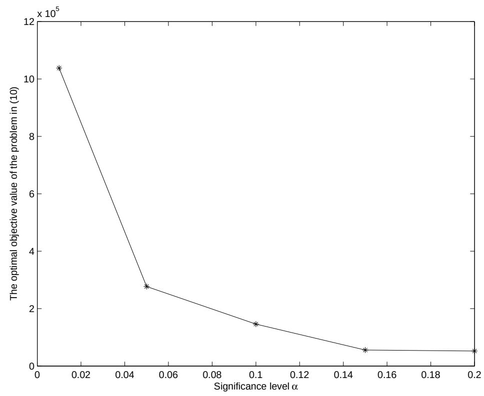

Fig. 2. Effects of changing α on the optimal objective value of the first-stage problem in (10): $\lambda = 0.5$ and fixed

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