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Path Embedding in a Faulty Folded Hypercube

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***Abstract*—Let F denote the faulty vertices in an n-dimensional folded hypercube FQ_n. In this paper, we show that FQ_n contains a fault-free path with length of at least $2^n - 2|F| - 1$ (respectively, $2^n - 2|F| - 2$) between two arbitrary vertices x and y of odd (respectively, even) Hamming distance in $FQ_n - F$ if $|F| \leq n - 1$, where $n \geq 3$. Since FQ_n is $(n + 1)$-regular and is bipartite when n is odd, both the number of faults tolerated and the length of a longest fault-free path obtained are worst-case optimal.**

***Index Terms*—Folded hypercubes; bipartite graph; fault tolerant embedding; hypercube; interconnection network**

I. Introduction

The hypercube is one of the most versatile and efficient interconnection networks (networks for short) discovered to date for parallel computation. The hypercube is ideally suited to both special-purpose and general-purpose tasks, and can efficiently simulate many other same sized networks [15]. We usually use Q_n to denote an n-dimensional hypercube. Many variants of the hypercube have been proposed. One variant is the folded hypercube [1]. An n-dimensional folded hypercube, denoted by FQ_n, is an extension of Q_n, constructed by adding a link to every pair of nodes with complementary addresses. The folded hypercube is superior to the hypercube in many measurements, such as diameter, fault diameter, connectivity, and so on (see [1], [22]). Previous works relating to the folded hypercube can be found in [1], [4], [8], [10], [11], [12], [13], [16], [17], [18], [19], [22], [24], [25], [26].

An embedding of one guest graph G into another host graph H is a one-to-one mapping f from the node set of G to the node set of H [15]. An edge of G corresponds to a path of H under f. Linear arrays and rings, which are two of the most fundamental networks for parallel and distributed computation, are suitable for designing simple algorithms with low communication costs. Numerous efficient algorithms designed on linear arrays and rings for solving various algebraic problems and graph problems can be found in [15]. Linear arrays and rings can also be used as control/data flow structures for distributed computation in arbitrary networks. All of these motivate the embedding of linear arrays and rings in networks.

The fault-tolerant problem has been one of the most important studies on interconnection networks since faults may happen when a network put into use. Some results of fault-tolerant embedding on Q_n or FQ_n can be found in [2], [3], [5], [6], [7], [8], [9], [10], [12], [13], [14], [20], [21], [22], [23]. Let F denote the faulty vertices in an n-dimensional folded hypercube FQ_n. In this paper, we show that FQ_n contains a fault-free path with length of at least $2^n - 2|F| - 1$ (respectively, $2^n - 2|F| - 2$) between two arbitrary vertices x and y of odd (respectively, even) Hamming distance in $FQ_n - F$ if $|F| \leq n - 1$, where $n \geq 3$. Since FQ_n is $(n + 1)$-regular and bipartite when n is odd [24], both the number of faults tolerated and the length of a longest fault-free path obtained are worst-case optimal.

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II. Preliminaries.

Let G be a graph and let $u, v \in V(G)$. We use (u, v) to denote an edge whose endpoints are u and v. A *path* $P[x_0, x_t] = \langle x_0, x_1, \cdots, x_t\rangle$ is a sequence of nodes such that two consecutive nodes are adjacent. Moreover, a path $\langle x_0, x_1, \cdots, x_t\rangle$ may contain other subpaths, denoted as $\langle x_0, x_1, \cdots, x_i, P[x_i, x_j], x_j, \cdots, x_t\rangle$, where $P[x_i, x_j] = \langle x_i, x_{i+1}, \cdots, x_{j-1}, x_j\rangle$. A *cycle* is a path with $x_0 = x_t$ and $t \geq 3$.

An n-cube is an undirected graph with 2^n nodes each labeled with a distinct binary string $b_1b_2\ldots b_n$. Nodes $b_1\ldots b_i\ldots b_n$ and $b_1\ldots \overline{b_i} \ldots b_n$ are joined by an edge along dimension i, where $1 \leq i \leq n$ and $\overline{b_i}$ represents the one complement of b_i. Moreover, suppose $x = x_1x_2\ldots x_n$ and $y = y_1y_2\ldots y_n$. In the rest of the paper, $x^{(i)}$ is used to denote the binary string $x_1\ldots \overline{x_i} \ldots x_n$ and $d_H(x, y)$ is used to denote the *Hamming distance* between x and y, namely, the number of different bits between x and y.

An n-dimensional folded hypercube FQ_n is Q_n augmented by adding more links among its nodes. More specifically, FQ_n is obtained by adding a link between two nodes whose addresses are complementary to each other in Q_n; i.e., for a node whose address is $b = b_1b_2\ldots b_n$, it has one more link to connect to node $\overline{b} = \overline{b_1}\,\overline{b_2}...\overline{b_n}$, in addition to its original n links. So FQ_n has 2^{n-1} more links than a regular links. Fig. 1 illustrates a 2-dimensional and a 3-dimensional folded hypercubes.

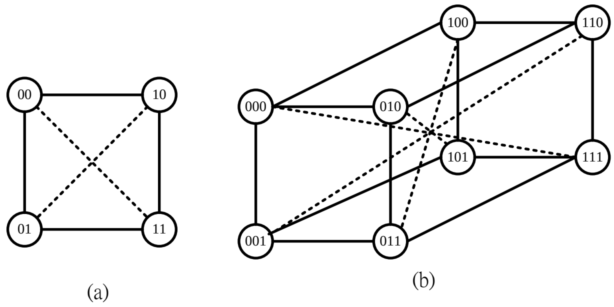

Fig. 1. The topologies of (a) FQ_2 and (b) FQ_3

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Conveniently, FQ_n can be represented with $\overbrace{**\ldots*}^{n} = *^n$, where $* \in \{0, 1\}$ means "don't care". Hence, $*^{n-1}1$ and $*^{n-1}0$, which contain the nodes with rightmost bits 1 and 0, respectively. An j-partition of $FQ_n = *^n$ partitions FQ_n along dimension j for some $j \in \{1, 2, \ldots, n\}$ into two *subcubes* $Q^0_{n-1} = *^{j-1}0*^{n-j}$ and $Q^1_{n-1} = *^{j-1}1*^{n-j}$, where Q^0_{n-1} (respectively, Q^1_{n-1}) is the subgraph of FQ_n induced by $\{b_1b_2\ldots b_i\ldots b_n \in V(FQ_n) | \ b_i = 0\}$ (respectively, $\{b_1b_2\ldots b_i\ldots b_n \in V(FQ_n) | \ b_i = 1\}$). Note that Q^0_{n-1} and Q^1_{n-1} are isomorphic to an $(n-1)$-cube Q_{n-1}. Note that, for each vertex b in Q^0_{n-1} (respectively, Q^1_{n-1}), there are two nodes $b^{(i)}$ and $\bar{b}$ in Q^1_{n-1} (respectively, Q^0_{n-1}) adjacent to it.

The following lemmas, which were shown in [6], [7], will be used in the following section.

Lemma 1. [6] Let $F \subset V(Q_n)$ denote the faulty vertices of Q_n, where $|F| \le n-2$. Suppose that x and y are two arbitrary nodes in $Q_n - F$, where $n \ge 3$. If $d_H(x, y)$ is odd (respectively, even), then there exists a path $P[x, y]$ with length of at least $2^n - 2|F| - 1$ (respectively, $2^n - 2|F| - 2$) in $Q_n - F$.

Lemma 2. [7] Let $F \subset V(Q_n)$ denote the faulty vertices of Q_n, where $|F| \le n-1$. Let $x \in V(Q_n) - F$, where $n \ge 3$. Suppose $n-1$ neighbors of x are in F. Then, there exist two paths $P[x, y_1]$ and $P[x, y_2]$ with lengths at least $2^n - 2(n-1)$ in $Q_n - F$ such that $d_H(x, y_1) = d_H(x, y_2) = 2$.

Lemma 3. [7] Let $F \subset V(Q_n)$ denote the faulty vertices of Q_n, where $|F| \le n-1$. Let $x \in V(Q_n) - F$, where $n \ge 3$. Suppose that at least two neighbors of x are in $Q_n - F$. Then there exists a fault-free cycle with length at least $2^n - 2|F|$ that contains x in Q_n.

III. Longest Fault-Free Paths with Node Faults

In this section, we have the main theorem as follows.

Theorem 1. Let $F \subset V(FQ_n)$ denote the faulty vertices of FQ_n, where $|F| \le n-1$ and $n \ge 3$.. Suppose that x and y are two arbitrary nodes in $FQ_n - F$. If $d_H(x, y)$ is odd (respectively, even), then there exists a path $P[x, y]$ with length of at least $2^n - 2|F| - 1$ (respectively, $2^n - 2|F| - 2$) in $FQ_n - F$.

Proof. By Lemma 1, the theorem holds when $|F| \le n-2$ (since $Q_n \subset FQ_n$). In the rest of the proof, we assume that $|F| = n-1$. We can partition FQ_n over some dimension j into two $(n-1)$-dimension hypercube Q^0_{n-1} and Q^1_{n-1} such that $|F_0| \ge 1$ and $|F_1| \ge 1$, where $F_0 = F \cap Q^0_{n-1}$ and $F_1 = F \cap Q^1_{n-1}$. Without loss generality, we assume that $|F_0| \ge |F_1|$. Thus, we have $|F_1| \le \left\lfloor \frac{n-1}{2} \right\rfloor$. We have the following case:

Case 1: $x, y \in Q^0_{n-1}$. Two cases are further considered:

Case 1.1: $|F_0| \le n-3$. It is not difficult to see that $n \ge 5$; for otherwise $|F_0| \le n-3 \le 1 < (n-1)-(n-3) = 2 = |F_1|$, which contradicts to the assumption that $|F_1| \le |F_0|$. By Lemma **1**, there exists a path $P[x, y]$ of length at least $2^{n-1} - 2|F_0| - 1$ (respectively, $2^{n-1} - 2|F_0| - 2$) if $d_H(x, y)$ is odd (respectively, even) in $Q^0_{n-1} - F_0$. We can choose an edge $(u, v) \in E(P[x, y])$ such that $u^{(j)}, v^{(j)} \notin F_1$[1]. Let $P[x, u]$ and $P[v, y]$ be two subpaths of $P[x, y]$ in Q^0_{n-1}. Also, By Lemma 1, there exists a path $P[u^{(j)}, v^{(j)}]$ of length at least $2^{n-1} - 2|F_1| - 1$ in $Q^1_{n-1} - F_1$. Thus, $\langle x, P[x, u], u, u^{(j)}, P[u^{(j)}, v^{(j)}], v^{(j)}, v, P[v, y], y\rangle$ is the desired path of length at least $(2^{n-1} - 2|F_0| - 1) - 1 + 2 + (2^{n-1} - 2|F_1| - 1) = 2^n - 2(|F_0| + |F_1|) - 1 = 2^n - 2|F| - 1$ (respectively, $2^n - 2(n-1) - 2$) if $d_H(x, y)$ is odd (respectively, even) (see Fig. 2(a)).

Case 1.2: $|F_0| = n-2$ (i.e., $|F_1| = 1$). We have two scenarios as follows:

Case 1.2.1: $n = 3$. We have $|F_0| = |F_1| = 1$. Without loss of generality, let $x = 000$. The desired path $P[x, y]$ are listed below:

Node y	The node in F_0	The node in F_1	$P[x, y]$ ($d_H(x, y)$ = even)	$P[x, y]$ ($d_H(x, y)$ = odd)
001	010	100		<000, 111, 011, 001>
		110		<000, 100, 101, 001>
		111		<000, 100, 101, 001>
		101		<000, 100, 110, 001>
	011	100		<000, 111, 101, 001>
		110		<000, 100, 101, 001>
		111		<000, 100, 101, 001>
		101		<000, 100, 110, 001>
011	001	100	<000, 010, 011>	
		110	<000, 010, 011>	
		111	<000, 010, 011>	
		101	<000, 010, 011>	
	010	100	<000, 001, 011>	
		110	<000, 001, 011>	
		111	<000, 001, 011>	
		101	<000, 001, 011>	

Case 1.2.2: $n \ge 4$. Let $w \in F_0$. Then, $|F_0 - \{w\}| = n-3$. By Lemma 1, there exists a path $P[x, y]$ of length at least $2^{n-1} - 2(n-3) - 1$ (respectively, $2^{n-1} - 2(n-3) - 2$), if $d_H(x, y)$ is odd (respectively, even) in $Q^0_{n-1} - (F_0 - \{w\})$.

If $w \notin V(P[x, y])$, let $(u, v) \in E(P[x, y])$ such that $u^{(j)}, v^{(j)} \notin F_1$. Let $P[x, u]$ and $P[v, y]$ are two subpaths of $P[x, y]$. Also, By Lemma 1, there exists a path $P[u^{(j)}, v^{(j)}]$ of length at least $2^{n-1} - 2 \times 1 - 1$ in $Q^1_{n-1} - F_1$. Thus, $\langle x, P[x, u], u, u^{(j)}, P[u^{(j)}, v^{(j)}], v^{(j)}, v, P[v, y], y\rangle$ is the desired path with length of at least $(2^{n-1} - 2(n-3) - 1) - 1 + 2 + (2^{n-1} - 2 \times 1 - 1) = 2^n - 2(n-2) - 1 > 2^n - 2|F| - 1$ (respectively, $2^n - 2(n-2) - 2 > 2^n - 2|F| - 2$) if $d_H(x, y)$ is odd (respectively, even) (see Fig. 2(a)).

If $w \in V(P[x, y])$, let $(u, w), (w, v) \in E(P[x, y])$, and let $P[x, u]$ and $P[v, y]$ be two subpaths of $P[x, y]$. Clearly, We have $u^{(j)}, v^{(j)} \notin F_1$ or $\bar{u}, \bar{v} \notin F_1$. Let $u' \in \{u^{(j)}, \bar{u}\} - F_1$ and $v' \in \{v^{(j)}, \bar{v}\} - F_1$. By Lemma 1, there exists a path $P[u', v']$ of length at least $2^n - 2 \times 1 - 2$ in $Q^1_{n-1} - F_1$. Thus, $\langle x, P[x, u], u, u', P[u', v'], v', v, P[v, y], y\rangle$ is the desired path with length of at least $2^{n-1} - 2(n-3) - 1 - 2 + 2 + 2^{n-1} - 2 \times 1 - 2 = 2^n - 2(n-1) - 1$ (respectively, $2^n - 2(n-1) - 2$) if $d_H(x, y)$ is odd (respectively, even). (see Fig. 2(b)).

[1] If edge (u, v) does not exist, then $|F_1| \ge \lceil (2^{n-1} - 2|F_0| - 2)/2 \rceil = 2^{n-2} - |F_0| - 1 \ge 2^{n-2} - (n-3) - 1 > n-3$ for $n \ge 5$, which contradicts to the assumption that $|F_1| \le |F_0| \le n-3$

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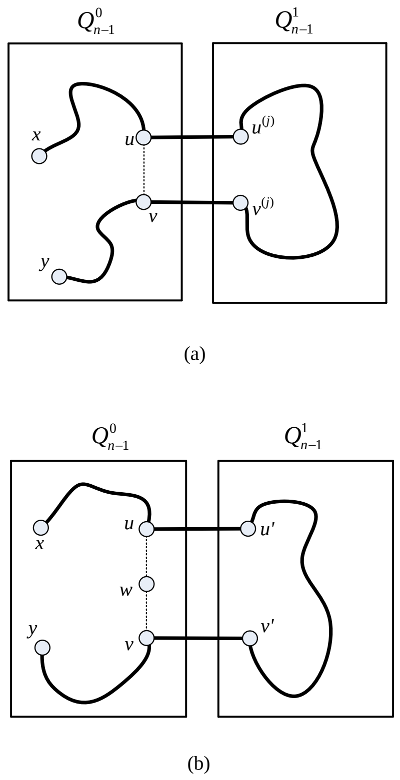

Fig. 2. The construction of a path $P[x, y]$ in $FQ_n - F$ when $x, y \in Q^0_{n-1}$

Case 2: $x \in Q^0_{n-1}$, $y \in Q^1_{n-1}$. Two cases are further considered:

Case 2.1: $|F_0| \le n - 3$. It is not difficult to see that $n \ge 5$; for otherwise $|F_0| \le n - 3 \le 1 < (n - 1) - (n - 3) = 2 = |F_1|$, which contradicts to the assumption that $|F_1| \le |F_0|$. Let $u \in Q^0_{n-1} - \{x\} - F_0$ such that $d_H(x, u)$ is odd and $u^{(j)} \in Q^1_{n-1} - F_1 - \{y\}$‡. By Lemma 1, there exists a path $P[x, u]$ of length at least $2^{n-1} - 2|F_0| - 1$ in $Q^0_{n-1} - F_0$. Moreover, By Lemma 1 there exists a path $P[y, u^{(j)}]$ of length at least $2^{n-1} - 2|F_1| - 1$ (respectively, $2^{n-1} - 2|F_1| - 2$), if $d_H(y, u^{(j)})$ is odd (respectively, even) in $Q^1_{n-1} - F_1$. Clearly, $d_H(x, y)$ is odd if and only if $d_H(y, u^{(j)})$ is odd. Thus, $\langle x, P[x, u], u, u^{(j)}, P[u^{(j)}, y], y\rangle$ is a path with length of at least $(2^{n-1} - 2|F_0| - 1) + 1 + (2^{n-1} - 2|F_1| - 1) = 2^n - 2(|F_0| + |F_1|) - 1 = 2^n - 2|F| - 1$ (respectively, $2^n - 2|F| - 2$) if $d_H(x, y)$ is odd (respectively, even) (see Fig. 3(a)).

Case 2.2: $|F_0| = n - 2$ (i.e., $|F_1| = 1$). We have two scenarios as follows:

Case 2.2.1: $n = 3$. We have $|F_0| = |F_1| = 1$. Without loss of generality, let $x = 000$. The desired path $P[x, y]$ are listed below:

The node in F_0	The node in F_1	Node y	$P[x, y]$ ($d_H(x, y)$ =even)	$P[x, y]$ ($d_H(x, y)$ =odd)
001	100	110	<000, 010, 110>	
		101	<000, 010, 101>	
		111		<000, 010, 110, 111>
	110	100		<000, 010, 101, 100>
		101	<000, 010, 101>	
		111		<000, 010, 101, 111>
	101	100		<000, 010, 110, 100>
		110	<000, 010, 110>	
		111		<000, 010, 110, 111>
	111	100		<000, 010, 110, 100>
		101	<000, 010, 101>	
		110	<000, 100, 110>	
011	100	110	<000, 010, 110>	
		101	<000, 001, 101>	
		111		<000, 010, 110, 111>
	101	100		<000, 010,110, 100>
		110	<000, 010, 110>	
		111		<000, 010, 110, 111>
	110	100		<000, 010, 101, 100>
		101	<000, 100, 101>	
		111		<000, 100, 101, 111>
	111	100		<000, 010, 110, 100>
		101	<000, 010, 101>	
		110	<000, 010, 110>	

‡ Let $V' = \{z \mid d_H(x, z)$ is odd, $z \in V(Q^0_{n-1})\}$. Note that $|V'| = 2^{n-2}$. If none of the vertices in Q^0_{n-1} meets the requirements of such a vertex u, then $V' = F_0 \cup \{w \mid w^{(j)} \in F_1\} \cup \{y^{(j)}\}$. As a result, $|V'| = 2^{n-2} = |F_0 \cup \{w \mid w^{(j)} \in F_1\} \cup \{y^{(j)}\}| \le |F_0| + |\{w \mid w^{(j)} \in F_1\}| + |\{y^{(j)}\}| = |F_0| + |F_1| + |\{y^{(j)}\}| = (n - 1) + 1 = n$, which is a contradiction. Therefore, we can always find such a vertex u.

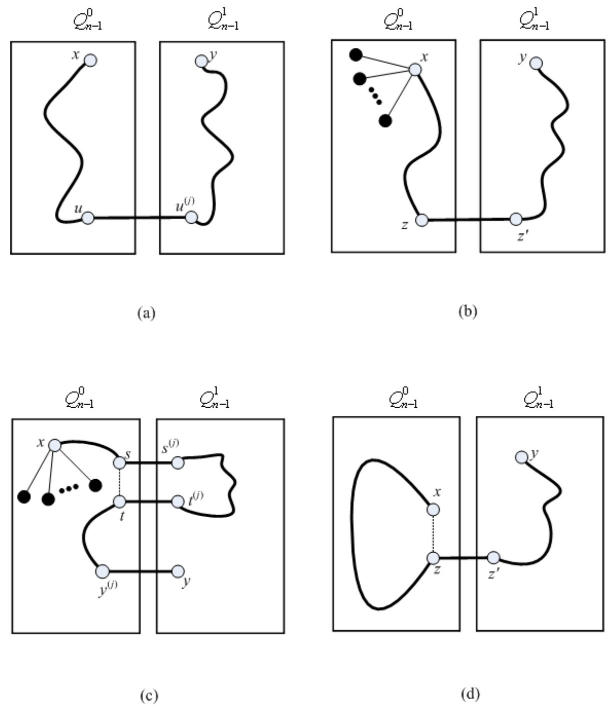

Fig 3. The construction of a path $P[x, y]$ in $FQ_n - F$ when $x \in Q^0_{n-1}$, $y \in Q^1_{n-1}$

Case 2.2.2: $n \ge 4$. Remember that $|F_1| = 1$. Assume that $F_1 = \{g\}$. First, consider that only one neighbor of x in Q^0_{n-1} is not in F_0; that is, $n - 2$ neighbors of x in Q^0_{n-1} are in F_0. By Lemma **2**, there exist two path $P[x, r]$ and $P[x, w]$ with length at least $2^{n-1} - 2(n - 2)$ in $Q^0_{n-1} - F_0$ such that $d_H(x, r) = d_H(x, w) = 2$.

If $r^{(j)} \ne \overline{w}$ and $w^{(j)} \ne \overline{r}$, then $|\{r^{(j)}, \overline{r}, w^{(j)}, \overline{w}\}| = 4$. We have $\{r^{(j)}, \overline{r}\} \ne \{g, y\}$ or $\{w^{(j)}, \overline{w}\} \ne \{g, y\}$. Let $z \in \{r, w\}$ such that $\{z^{(j)}, \overline{z}\} \ne \{g, y\}$ and let $z' \in \{z^{(j)}, \overline{z}\} - \{g, y\}$. Note that $d_H(x, z) = 2$ and $d_H(z, z') = 1$. Thus, we have $d_H(x, z')$ is odd. Consequently, if $d_H(x, y)$ is even (respectively, odd), then $d_H(y, z')$ is odd (respectively, even). By Lemma 1, there exists a path $P[y, z']$ of length at least $2^{n-1} - 2 \times 1 - 1$ (respectively, $2^n - 2 \times 1 - 2$) in $Q^1_{n-1} - F_1$. Thus, $\langle x, P[x, z], z, z', P[z', y], y\rangle$ is a path with length of at least $(2^{n-1} - 2(n -$

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$2)) + 1 + (2^{n-1} - 2 \times 1 - 1) = 2^n - 2(n-1)$ (respectively, $2^n - 2(n-1) - 1$) if $d_H(x, y)$ is odd (respectively, even) (see Fig. 3(b)).

If $r^{(j)} = \overline{w}$ or $w^{(j)} = \overline{r}$, then $n = 5$[2]. It is not difficult to see that if $r^{(j)} = \overline{w}$, then $w^{(j)} = \overline{r}$. When $\{r^{(j)}, w^{(j)}\} \neq \{g, y\}$. Let $z \in \{r, w\}$ and $z' \in \{r^{(j)}, w^{(j)}\} - \{g, y\}$. The construction is similar as that of in Fig. 3(b). When $\{r^{(j)}, w^{(j)}\} = \{g, y\}$, we have $y^{(j)} \in \{r, w\}$ (thus, $d_H(x, y^{(j)}) = 2$ and $d_H(x, y)$ is odd). Since $n = 5$ and $d_H(x, y^{(j)}) = 2$, by Lemma **2**, we have a path $P[x, y^{(j)}]$ of length at least $2^{n-1} - 2(n-2)) = 2^4 - 2(4-1) = 10$ in $Q_{n-1}^0 - F_0$. Since $|P[x, y^{(j)}]| \geq 10$, we can choose an edge $(s, t) \in E(P[x, z])$ such that $\{s, t\} \cap \{r, w\} = \varnothing$. Clearly, $\{s^{(j)}, t^{(j)}\} \cap \{r^{(j)}, w^{(j)}\}$ $(= \{y, g\}) = \varnothing$. Without loss of generality, let $P[x, s]$ and $P[t, y^{(j)}]$ be two subpaths of $P[x, y^{(j)}]$ in Q_{n-1}^0. By Lemma 1, there exists a path $P[s^{(j)}, t^{(j)}]$ of length at least $2^{n-1} - 2 \times 2 - 1$ in $Q_{n-1}^1 - \{g, y\}$. Thus, $\langle x, P[x, s], s, s^{(j)}, P[s^{(j)}, t^{(j)}], t^{(j)}, t, P[t, y^{(j)}], y^{(j)}, y\rangle$ is a path with length of at least $(2^{n-1} - 2(n-2)) - 1 + 3 + (2^{n-1} - 2 \times 2 - 1) = 2^n - 2(n-1) - 1$ (see Fig. 3(c)).

Now, consider that at least two neighbors of x in Q_{n-1}^0 are $Q_{n-1}^0 - F_0$. By Lemma 3, there exists a cycle C with length at least $2^{n-1} - 2(n-2)$ that contains x in $Q_{n-1}^0 - F_0$. Let r, w denote two neighbor of x in C. Note that since $d_H(r, w) = 2$, we have $r^{(j)} \neq \overline{w}$ and $w^{(j)} \neq \overline{r}$, i.e., $|\{r^{(j)}, \overline{r}, w^{(j)}, \overline{w}\}| = 4$. Thus, $\{r^{(j)}, \overline{r}\} \neq \{g, y\}$ or $\{w^{(j)}, \overline{w}\} \neq \{g, y\}$. Let $z \in \{r, w\}$ such that $\{z^{(j)}, \overline{z}\} \neq \{g, y\}$ and let $z' \in \{z^{(j)}, \overline{z}\} - \{g, y\}$. Moreover, let $P[x, z] = C - \{(x, z)\}$. Since $d_H(x, z') = 2$, we have $d_H(y, z')$ is odd (respectively, even) if $d_H(x, y)$ is odd (respectively, even). By Lemma 1, there exists a path $P[y, z']$ of length at least $2^{n-1} - 2 \times 1 - 1$ (respectively, $2^n - 2 \times 1 - 2$) in $Q_{n-1}^1 - F_1$. Thus, $\langle x, P[x, u], u, u^{(j)}, P[u^{(j)}, y], y\rangle$ is a path with length of at least $(2^{n-1} - 2(n-2) - 1) + 1 + (2^{n-1} - 2 \times 1 - 1) = 2^n - 2(n-1) - 1$ (respectively, $2^n - 2(n-1) - 2$) if $d_H(x, y)$ is odd (respectively, even) (see Fig. 3(d)). □

IV. Discussion and Conclusion

Fault tolerance is an important research subject in the area of the multi-process computer system, and many studies have focused on the vertex-fault tolerant or edge-fault tolerant properties of various networks. In this paper, we show that $FQ_n - F$ contains a path $P[x, y]$ with length at least $2^n - 2|F| - 1$ (respectively, $2^n - 2|F| - 2$) between two arbitrary vertices x and y of odd (respectively, even) Hamming distance, where $|F| \leq n - 1$ and $n \geq 3$.

[2] Suppose $r = r_1r_2\ldots r_{j-1}0r_{j+1}\ldots r_n$, $w = w_1w_2\ldots w_{j-1}0w_{j+1}\ldots w_n$, where $r_i, w_i \in \{0, 1\}$, for $i \in \{1, 2, \ldots, n\} - \{j\}$. We have $r^{(j)} = r_1r_2\ldots r_{j-1}1r_{j+1}\ldots r_n$, $w^{(j)} = w_1w_2\ldots w_{j-1}1w_{j+1}\ldots w_n$, $\overline{r} = \overline{r_1}\,\overline{r_2}\cdots\overline{r_{j-1}}\,1\,\overline{r_{j+1}}\cdots\overline{r_n}$, and $\overline{w} = \overline{w_1}\,\overline{w_2}\cdots\overline{w_{j-1}}\,1\,\overline{w_{j+1}}\cdots\overline{w_n}$. Note that $d_H(r, w) = 2$ or 4. If $r^{(j)} = \overline{w}$ or $w^{(j)} = \overline{r}$, then $r_k \neq w_k$ for all $k \in \{1, 2, \ldots, n\} - \{j\}$, we have $d_H(r, w) = n - 1$. When $d_H(r, w) = 2 = n - 1$, we have $n = 3$, which contradicts to the fact that $n \geq 4$. When $d_H(r, w) = 4 = n - 1$, we have $n = 5$.

References

[1] A. El-Amawy and S. Latifi, "Properties and performance of folded hypercubes," *IEEE Transactions on Parallel and Distributed Systems*, vol. 2, no. 1, pp. 31-42, 1991.

[2] M. -Y. Chan and S. -J. Lee, "On the existence of Hamiltonian circuits in faulty hypercubes," *SIAM Journal on Discrete Mathematics*, vol. 4, no. 4, pp. 511-527, 1991.

[3] T. Dvořák and V. Koubek, "Long paths in hypercubes with a quadratic number of faults," *Information Sciences*, vol. 179, no. 21-18, pp. 3763-3771, 2009.

[4] J. -F. Fang, "The bipanconnectivity and *m*-panconnectivity of the folded hypercube," *Theoretical Computer Science*, vol. 385, no. 1-3, pp. 286-300, 2007.

[5] J. Fink and P. Gregor, "Long paths and cycles in hypercubes with faulty vertices," *Information Sciences*, vol. 179, no. 20-29, pp. 3634-3644, 2009.

[6] J. -S. Fu, "Fault-tolerant cycle embedding in the hypercube", *Parallel computing*, vol. 29, no. 6, pp. 821-832, 2003.

[7] J. -S. Fu, "Longest fault-free paths in hypercubes with vertex faults," *Information Sciences*, vol. 176, pp.759-771, 2006.

[8] J. -S. Fu, "Fault-free cycles in folded hypercubes with more faulty elements," *Information Processing Letters*, vol. 108, no. 5, pp.261-263, 2008.

[9] S. -Y. Hsieh, "Fault-tolerant cycle embedding in the hypercube with more both faulty vertices and faulty edges", *Parallel Computing*, vol. 32, no. 1, pp. 84-91, 2006.

[10] S. -Y. Hsieh, "Some edge-fault-tolerant properties of the folded hypercube," *Networks*, vol. 51, no. 2, pp. 92-101, 2008.

[11] S. -Y. Hsieh and C. -N. Kuo "Hamilton-connectivity and strongly Hamiltonian-laceability of folded hypercubes," *Computers and Mathematics with Applications*, vol. 53, no. 7, pp. 1040-1044, 2007.

[12] S. -Y. Hsieh, "A note on cycle embedding in folded hypercubes with faulty elements," *Information Processing Letters*, vol. 108, no. 2, pp. 81, 2008.

[13] C. -N. Kuo and S. -Y. Hsieh, "Panclclicity and bipancyclicity of conditional faulty folded hypercubes," *Information Sciences*, vol. 180, no. 15-1, pp. 2904-2914, 2010.

[14] S. Latifi, S. Q. Zheng, N. Bagherzadeh, "Optimal ring embedding in hypercubes with faulty links," *Proceedings of the IEEE Symposium on Fault-Tolerant Computing*, pp. 178-184, 1992.

[15] F. T. Leighton, *Introduction to Parallel Algorithms and Architecture: Arrays· Trees· Hypercubes*, Morgan Kaufman, CA, 1992.

[16] C. -N. Lai, G. -H. Chen, and D. -R. Duh "Constructing one-to-many disjoint paths in folded hypercubes," *IEEE Transactions on Computers*, vol. 51, no. 1, pp. 33-45, 2002.

[17] C. -N. Lai, G. -H. Chen, "Strong Rabin numbers of folded hypercubes," *Theoretical Computer Science*, vol. 341, Issue 1-3, pp. 196-215, 2005.

[18] C. -N. Lai, G. -H. Chen, "*w*-Rabin numbers and strong w-Rabin numbers of folded hypercubes," *Networks*, vol. 51, no. 3, pp. 171 – 177, 2008.

[19] M. Ma, "The spanning connectivity of folded hypercubes," *Information Science*, in press, 2010.

[20] C. -H. Tsai, J. -M. Tan, T. Liang, L.-H. Hsu, "Fault-tolerant Hamiltonian laceability of hypercubes," *Information Processing Letters*, vol. 83, 301-306, 2002.

[21] Y. -C. Tseng, "Embedding a ring in a hypercube with both faulty links and faulty nodes," *Information Processing Letters*, vol. 59, no. 4, pp. 217-222, 1996.

[22] D. Wang, "Embedding Hamiltonian cycles into folded hypercubes with faulty links," *Journal of Parallel and Distributed Computing*. vol. 61, pp. 545-564, 2001.

[23] H. -L. Wang, J. -W. Wang, J. -N. M. Xu, "Edige-fault-tolerant bipanconnectivity of hypercubes," *Information Sciences*, vol. 179, vol. 4-1, pp. 404-409, 2009.

[24] J. -M. Xu and M. Ma, "Cycles in folded hypercubes," *Applied Mathematics Letters*, vol. 19, pp. 140-145, 2006.

[25] Q. Zhu, J. -M. Xu, X. Hou, M. Xu, "On reliability of the folded hypercubes," *Information Sciences*, vol. 177, vol. 8-15, pp. 1782-1788, 2007.

[26] Q. Zhu, S. -Y. Liu, M. Xu, "On conditional diagnosability of the folded hypercubes," *Information Sciences*, vol. 178, vol. 4-15, pp. 1069-1077, 2007.

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