Controller Design Base on Servo State Feedback for Two-wheeled Balancing Robot

P. Huantham, V. Kongratana, S. Gulphanich and V. Tipsuwanporn, *Member, IAENG*

Abstract **This paper presents the controller design for the two-wheeled balancing robot system. The proportional-derivative (PD) controller was designed to compare the performance with servo state feedback controller designed by pole placement technique. Both controllers are used to stabilize the two-wheeled balancing robot system. Simulation results show that servo state feedback design can stabilize the balancing robot. Moreover, the presence of its large deviation angle is also better than PD controller.**

Keywords **: Servo state feedback, Proportional-Derivative (PD), Two-wheeled balancing robot**

I. INTRODUCTION

When refer to the system which is used for testing the effectiveness of the controller, the invert pendulum system is the one of the most widely used as invert pendulum on car [2], Two-wheeled balancing robot [1][5][6], and rotational inverted pendulum [4] because has several advantages and the structure is simple including non-linear and the uncertainty on the features of the system.

In this paper is presented the principal of design the stable controlling two-wheeled balancing robot called invert pendulum. There are two types of stable controllers which applied to. PD controller and servo state feedback which is designed by pole-placement method in order to keep the stability of two-wheeled balancing robot system [2] [4] and tune the performance of system. To compare with two controllers to find out the best performance of controlling two-wheeled balancing robot system.

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This paper is divided as follows: section II is presented the mathematical model, section III given the control system structure while section IV provides the simulation results. Finally, section V the paper conclusion.

II. MATHEMATICAL MODEL

The two-wheeled balancing robot to be controlled is show in Fig.1.Its structure composes of diagram of two-wheeled balancing robot.

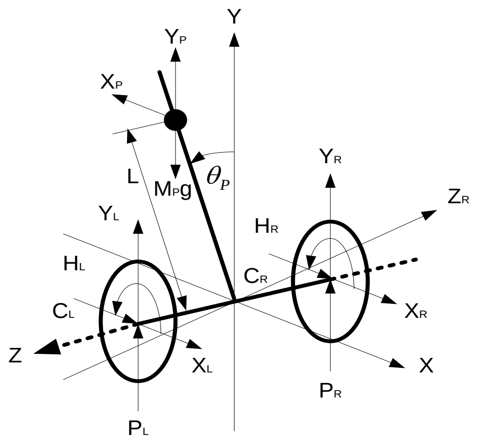

Fig. 1 The two-wheeled balancing robot.

In this the table.1 shows parameter values of two-wheeled balancing robot for experiment and simulation.

Parameter values of two-wheeled balancing robot

Symbol	Description	Unit
θ_P	Angular position of the pendulum	rad
x	Position of the robot	m
K_M	Motor torque constant	0.3674 (N*m/A)
K_E	Back-emf constant	0.7661 (V*s/rad)
M_P	Mass of the pendulum	1.5 (kg)
M_W	Mass of Wheels	0.2 (kg)
R_A	Motor armature resistance	14 Ω
r	Radius of the wheels	0.06 (m)
L	Length to pendulum	0.05 (m)
g	Acceleration of gravity	9.81 (m/sec^2)
I_W	Wheel moment of inertia	0.00036 (kg*m2)
I_P	Pendulum moment of inertia	0.003278 (kg*m^2)

Table.1 Parameter values of two-wheeled balancing robot

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Linear approximation of the state-space, estimated for $\theta_P = 0$, when θ_P is small angle on vertical axis.

$$\cos\theta_P = 1, \sin\theta_P = \theta_P \text{ and } \left(\frac{d}{dt}\theta_P\right)^2 \approx 0$$

The linear equation of motion for two-wheeled balancing robot is,

$$\ddot{\theta}_P = -\frac{M_P L\ddot{x}}{\alpha} + \frac{2K_M K_E \dot{x}}{R_A r\alpha} - \frac{2K_M V_A}{R_A r\alpha} - \frac{M_P g L\theta_P}{\alpha} \quad (1)$$

$$\ddot{x} = -\frac{2K_M V_A}{R_A r\beta} - \frac{2K_M K_E \dot{x}}{R_A r^2 \beta} - \frac{M_P L\ddot{\theta}_P}{\beta} \quad (2)$$

where

$$\alpha = I_P + M_P L^2 \quad , \quad \beta = 2M_W + \frac{2I_W}{r^2} + M_P$$

That a linear model can be obtained and linear state-space controller could be design and implemented.
State-space equation

$$\dot{\mathrm{x}} = \mathrm{Ax} + \mathrm{Bu} \quad (3)$$

where

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta}_P \\ \ddot{\theta}_P \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & A_{22} & A_{23} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & A_{42} & A_{43} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta_P \\ \dot{\theta}_P \end{bmatrix} + \begin{bmatrix} 0 \\ B_{21} \\ 0 \\ B_{41} \end{bmatrix} V_A \quad (4)$$

when

$$A_{22} = \left(-\frac{M_P L}{\beta}\right)\left(\frac{2K_M K_E}{R_A r\alpha}\right) - \frac{2K_M K_E}{R_A r^2 \beta}$$

$$A_{23} = \frac{(M_P L)^2 g}{\alpha\beta}$$

$$A_{42} = \left(-\frac{M_P L}{\alpha}\right)\left(-\frac{2K_M K_E}{R_A r\beta}\right) + \frac{2K_M K_E}{R_A r^2 \alpha}$$

$$A_{43} = -\frac{M_P L g}{\alpha}$$

$$B_{21} = \left(-\frac{M_P L}{\beta}\right)\left(-\frac{2K_M}{R_A r\alpha}\right) + \frac{2K_M}{R_A r\beta}$$

$$B_{41} = \left(-\frac{M_P L}{\alpha}\right)\left(\frac{2K_M}{R_A r\beta}\right) + \frac{2K_M}{R_A r\alpha}$$

$$u = V_a$$

transfer function of two-wheeled balancing robot

$$\frac{\theta_P(s)}{V_A(s)} = \frac{101.8s + 173.5}{s^3 + 2.63s^2 - 82.64s - 307.3}$$

III. CONTROL SYSTEM STRUCTURE

The overall structure of the proposed system is shown in Fig.2. Servo state feedback controller for stabilizing of two-wheeled balancing robot [3], described as follows.

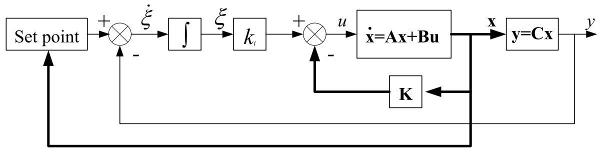

Fig.2 Servo state feedback control system.

The objective task of the servo state feedback model shown in Fig.2, it is seen that

$$\xi = r - y = r - \mathrm{Cx} \quad (6)$$

we assume that the transfer function of the plant can be give by

$$G_p(s) = \mathrm{C}(s\mathrm{I} - \mathrm{A})^{-1}\mathrm{B}$$

To avoid the possibility of the inserted integrator being canceled by the zero at the origin of the plant, we assume that $G_p(s)$ has no zero at the origin.

Assume that the reference input (step function) is applied at $t = 0$.Then, for $t > 0$, the system dynamics can be described by

$$\begin{bmatrix} \dot{\mathrm{x}}(t) \\ \dot{\xi}(t) \end{bmatrix} = \begin{bmatrix} \mathrm{A} & 0 \\ \text{-C} & 0 \end{bmatrix} \begin{bmatrix} \mathrm{x}(t) \\ \xi(t) \end{bmatrix} + \begin{bmatrix} \mathrm{B} \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t) \quad (7)$$

We shall design an asymptotically stable system such that $\mathrm{x}(\infty)$, $\xi(\infty)$ approach constant values, respectively. Then, at steady state, $\dot{\xi}(t) = 0$, and we get $y(\infty) = r$. Notice that at steady state we have

$$\begin{bmatrix} \dot{\mathrm{x}}(\infty) \\ \dot{\xi}(\infty) \end{bmatrix} = \begin{bmatrix} \mathrm{A} & 0 \\ \text{-C} & 0 \end{bmatrix} \begin{bmatrix} \mathrm{x}(\infty) \\ \xi(\infty) \end{bmatrix} + \begin{bmatrix} \mathrm{B} \\ 0 \end{bmatrix} u(\infty) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(\infty) \quad (8)$$

Noting that $r(t)$ is a step input, we have $r(\infty) = r(t) = r$ (constant) for $t > 0$. By subtracting Equation (8) from Equation (7), we obtain

$$\begin{bmatrix} \dot{\mathrm{x}}(t) - \dot{\mathrm{x}}(\infty) \\ \dot{\xi}(t) - \dot{\xi}(\infty) \end{bmatrix} = \begin{bmatrix} \mathrm{A} & 0 \\ \text{-C} & 0 \end{bmatrix} \begin{bmatrix} \mathrm{x}(t) - \mathrm{x}(\infty) \\ \xi(t) - \xi(\infty) \end{bmatrix} + \begin{bmatrix} \mathrm{B} \\ 0 \end{bmatrix} [u(t) - u(\infty)] \quad (9)$$

Define

$$\mathrm{x}(t) - \mathrm{x}(\infty) = \mathrm{x}_e(t)$$

$$\xi(t) - \xi(\infty) = \xi_e(t)$$

$$u(t) - u(\infty) = u_e(t)$$

The Equation (9) can be written as

$$\begin{bmatrix} \dot{x}_e(t) \\ \dot{\xi}_e(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x_e(t) \\ \xi_e(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_e(t) \quad (10)$$

where

$$u_e(t) = -Kx_e(t) + k_I \xi_e(t) \quad (11)$$

Define a new $(n+1)$ th-order error vector $e(t)$ by

$$e(t) = \begin{bmatrix} x_e(t) \\ \xi_e(t) \end{bmatrix} = (n+1) - \text{vector}$$

Then Equation (10) become

$$\dot{e} = \hat{A}e + \hat{B}u_e \quad (12)$$

where

$$\hat{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}, \qquad \hat{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

and Equation (11) become

$$u_e = -\hat{K}e \quad (13)$$

Where

$$\hat{K} = \left[\hat{K} \;\vdots -k_I\right]$$

The state error equation can be obtain by substituting equation (13) into equation (12)

$$\dot{e} = \left(\hat{A} + \hat{B}K\right)e \quad (14)$$

If the desired eigenvalues of matrix $\hat{A} + \hat{B}\hat{K}$ (that is, the desired closed-loop poles) are specified as $\mu_1, \mu_2, \ldots, \mu_{n+1}$ then the state-feedback gain matrix K and the integral gain constant k_I can be determined by the pole-placement technique, provided that the system defined by Equation (12) is completely state controllable.

Sine an integral is added to tract the output of system without steady-state error, therefore, the following augmented system for designing a servo state feedback controller can be obtained as [2]

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\xi}(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \xi(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t)$$

$$y(t) = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \xi(t) \end{bmatrix} \quad (15)$$

and its control law will be given by

$$u(t) = -\begin{bmatrix} K & k_I \end{bmatrix} \begin{bmatrix} x(t) \\ \xi(t) \end{bmatrix} \quad (16)$$

where **K** is the state feedback gain matrix, k_I is the integral gain, ξ is the output of the integral controller and r is the reference signal. The gain K and k_I can either be assigned by pole-placement method.

IV. EXPERIMENTAL RESULTS

The Servo state feedback and the PD controller are used with linear for the balancing robot model. Since only on the stability of the closed-loop systems are focused. The simulations of the balancing robot, we can be designed PD controller for experiment is to the comparison on performance between is angle pendulum, Car position, signal control and disturbances signal for the balancing robot.

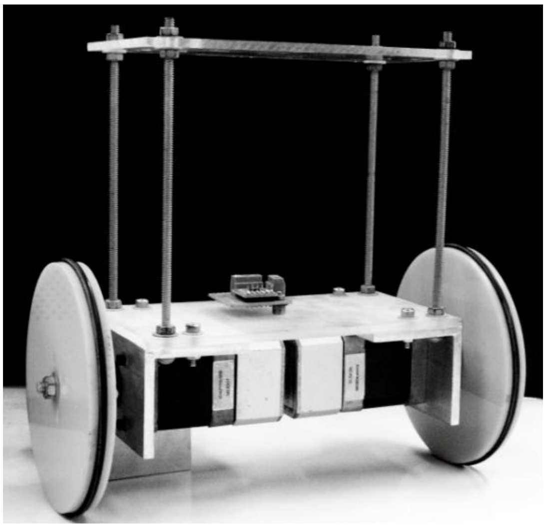

Fig. 3 Experimental apparatus.

PD Controller		Servo state feedback controller				
K_p	K_d	K_1	K_2	K_3	K_4	K_i
3.0000	0.2950	0.1000	0.3811	15.0000	1.0000	0.0200

Table.2 Parameter controller of two-wheeled balancing robot

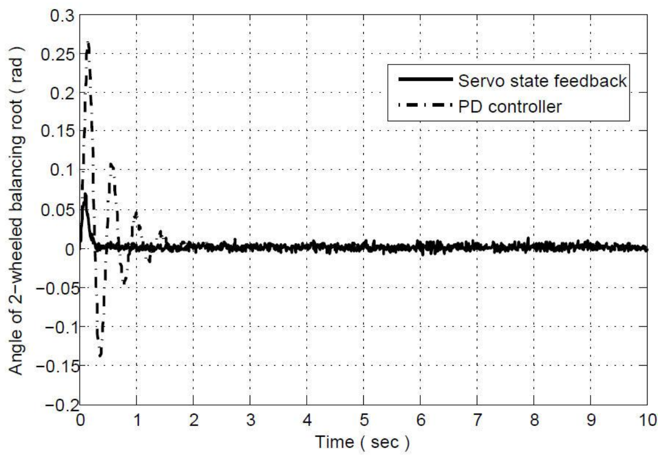

Fig.4 Angle pendulum for the PD with Servo state feedback.

When apply step disturbances t = 5 second in Fig.5, the step response of the controlled systems using State servo feedback controller and PD controller is shown in Fig. 6

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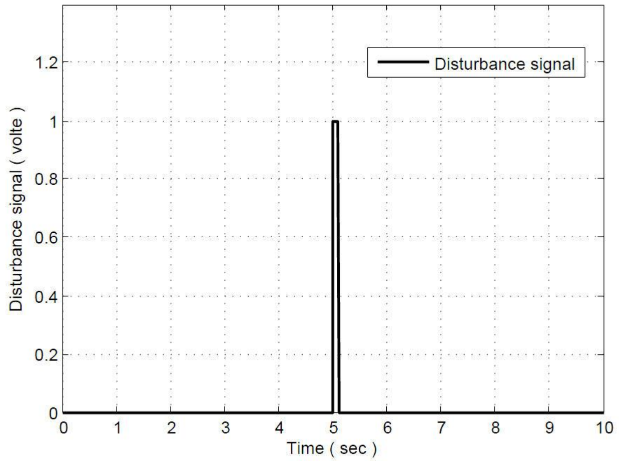

Fig.5 Disturbance signals.

and apply disturbance signals to systems.

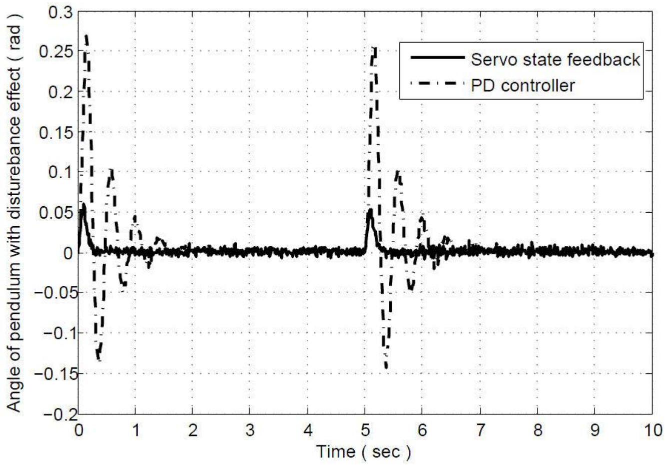

Fig.6 Angle pendulum with response disturbance effects.

and car position of 2-wheeled balancing robot.

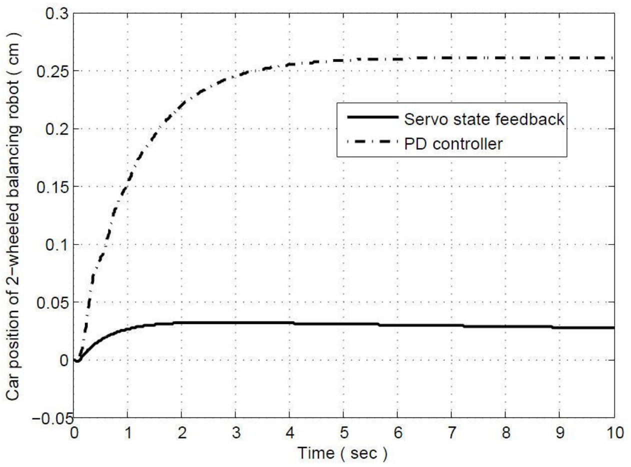

Fig.7 Car position for the PD with Servo state feedback.

Finally is the signal controller for 2-wheeled balancing robot systems in Fig.8.

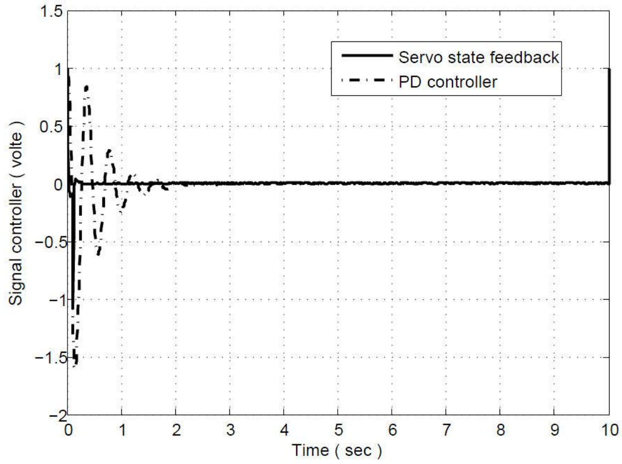

Fig.8 Signal controller for the PD with Servo state feedback.

V. CONCLUSION

This paper presented a method to design and control the two-wheeled balancing robot. Simulation and experimental results show the comparison between servo state feedback with PD controller. Servo state feedback controller has a swinging of car lesser than PD controller, Car position is steady not any movement for receiving more swing angle and control signal is also lesser than PD controller. We can conclude that the performance of servo state feedback controller is better than PD controller in order to control the two-wheeled balancing robot.

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