

IMC-Based PID Controllers Design for Two-mass System

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Abstract—The paper presents the design of IMC-Based PID controllers for controlling the speed of two-mass system. The IMC-Based PID controller is easy to be adjusted to meet the desired control system performances both in transient state and steady state. The simulation results of the proposed IMC-Based PID controllers by using the same parameters proportional gain, integral gain, derivative gain and acceleration gain as the IMC-Based PID controllers is shown. The responses can be improved significantly.

Keywords: IMC-Based PID controller, two-mass system

I. INTRODUCTION

THIS automatic technology is important for general manufacture industries because there are plenty of competition in the business ways. Thereby, the producers need method which can quickly produce their products, increase efficiency and reduce the expenses in manufacturing process. They prefer to find method or something to help their systems. The better choice is the IMC-based PID because it is general used in the industries. In terms of selecting or adjusting controller parameter, they have to consider on each process in order to get the accuracy and efficiency results. This paper presents about the design method of IMC-Based PID control due to it's simple structure and can be adjusted parameter for a long time.

A simple IMC-based PID controller design method is studied. The design is composed of 2 steps which are IMC controller design and PID approximation [1]. In the IMC controller design process, a low pass filter selection is very important because it affects the performance of the system. For several common plant models, the low pass filter

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selections are proposed in [4] but these filters cannot be used directly with the two-mass system. Therefore, a suitable low pass filter for this plant is proposed in this paper. After the IMC controller is designed, Maclaurin expansion is utilized to find the corresponding approximated PID controller.

II. MATHEMATICAL MODEL OF THE TWO-MASS SYSTEM

A. Two-mass system

The two-mass system for which two motors linked with flexible shaft always produces a torsional vibration in speed of a motor. This paper, such vibration will be reduced by the IMC-Based PID in order to achieve high tracking performance and excellent load-change effect the speed[2].

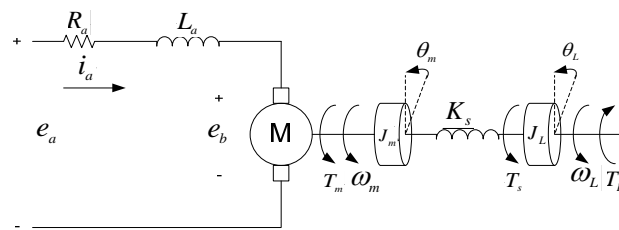


Fig. 1. Two-mass system.

B. Model of the two-mass system

The system shown in Fig.1 can be represented by the linear dynamical equation.

$$\begin{aligned}
 J_m \frac{d\omega_m}{dt} &= K_m i_a - T_s \\
 J_L \frac{d\omega_L}{dt} &= T_s - T_L \\
 L_a \frac{di_a}{dt} &= -K_e \omega_m - R_a i_a + e_a \\
 \frac{dT_s}{dt} &= K_s (\omega_m - \omega_L)
 \end{aligned} \tag{1}$$

Where the input variable is defined by e_a and the transfer function, while T_L denotes the unmeasured load disturbance. All parameters and their value are given in Table.1.

$$P(s) = \frac{\omega_m(s)}{E_a(s)} = \frac{a_2 s^2 + a_0}{b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0} \quad (2)$$

$$a_0 = \frac{K_m K_s}{J_m J_L L_a}, \quad a_2 = \frac{K_m}{J_m L_a},$$

$$b_0 = \frac{K_m K_e K_s}{J_m J_L J_a}, \quad b_1 = \frac{K_s R_a (J_m + J_L)}{J_m J_L J_a},$$

$$b_2 = \frac{K_s L_a (J_m + J_L) + K_m K_e J_L}{J_m J_L L_a}, \quad b_3 = \frac{R_a}{L_a}, \quad b_4 = 1$$

Parameter two-mass system

Symbol	Description	Unit
J_m	Motor inertia	1 kg-cm ²
J_L	Load inertia	0.9 kg-cm ²
K_s	Torsional stiffness of drive shaft	10 Nm/rad
K_m	Motor torque constant	2.9 Nm/A
K_e	Back electromotive force constant	3.5V/krpm
R_a	Armature resistance	0.26 Ω
L_a	Armature inductance	0.2 mH

Table 1 Parameter two-mass system

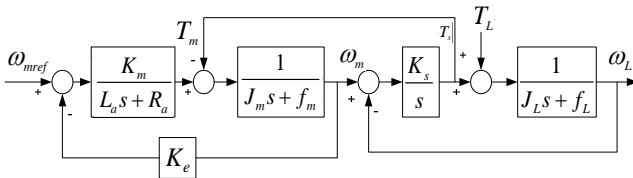


Fig. 2. Proposed control system structure.

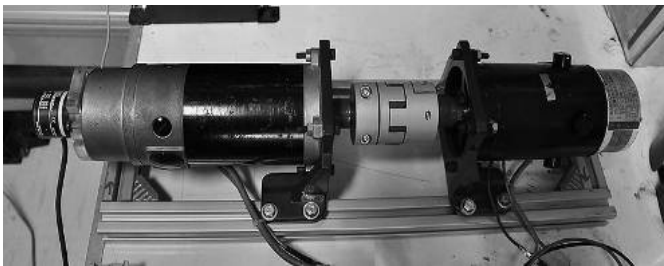


Fig. 3 Structure of Two-mass system.

III. IMC-BASED PID CONTROLLER DESIGN

The structure of IMC is shown in Fig. 4, in which G is a controlled object, \hat{G} is the model of the controlled object, C is an IMC controller [4].

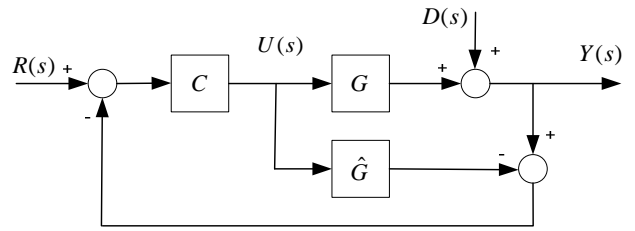


Fig. 4. Structure of IMC system.

In general, the IMC controller can be designed by following two steps [1].

Step 1 Divide the model of the controlled object $\hat{G}(s)$ into two parts as $\hat{G}(s) = \hat{G}_+(s)\hat{G}_-(s)$, where $\hat{G}_+(s)$ contains all the time delays and right-half-plane zeros. While $\hat{G}_-(s)$ is the transfer function with minimum phase characteristic and contains no predictive item[6].

Step 2 To design the internal model controller, a low-pass filter $f(s)$ must be added to $\hat{G}_-(s)$ to ensure the stability and robustness of the control system. The internal model controller is then chosen as (4)

$$C(s) = \hat{G}_-^{-1}(s)f(s). \quad (3)$$

In order to design a PID controller for a general feedback control system[5], the structure of IMC system in Fig. 2 is rearranged as Fig. 3. Where

$$K(s) = \frac{C(s)}{1 - \hat{G}(s)C(s)}, \quad (4)$$

and reversely,

$$C(s) = \frac{K(s)}{1 + \hat{G}(s)K(s)}. \quad (5)$$

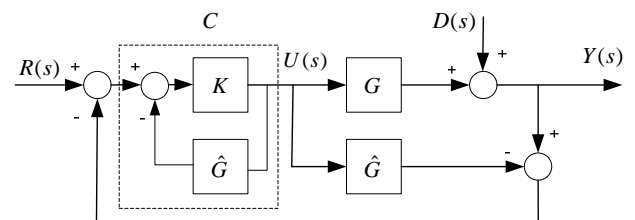


Fig. 5 Equivalent structure of IMC system.

By analyzing the input and output relation of the equivalent structure of IMC system in Fig. 3 the two models of the controlled object $\hat{G}(s)$ can be offset.[6] The equivalent structure of IMC system in Fig. 3 is then equal to a general feedback control system in Fig. 4.

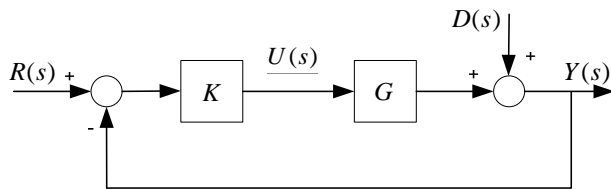


Fig. 6 General feedback control.

From (6) and (7), the controller $K(s)$ for a general feedback control system can be obtained as (9)

$$K(s) = \frac{\hat{G}_-^{-1}(s)}{f^{-1}(s) - \hat{G}_+^{-1}(s)}. \quad (6)$$

With a suitable choice of $f(s)$, the controller $K(s)$ can possess the integral action and appear in the following form

$$K(s) = \frac{1}{s} Q(s). \quad (7)$$

By performing the Maclaurin series expansion for $Q(s)$, the controller $K(s)$ can be expressed as

$$K(s) \approx \frac{1}{s} \left[Q(0) + \dot{Q}(0)s + \frac{\ddot{Q}(0)s^2}{2!} + \dots \right]. \quad (8)$$

Comparing the structure of $K(s)$ in (8) to the general transfer function of the PID controller in (9)

$$K_{PID}(s) \approx K_P + \frac{K_i}{s} + K_d s, \quad (9)$$

the PID parameters can be obtained by ignoring the higher order terms in $K(s)$ and setting

$$K_P = \dot{Q}(0), K_i = Q(0) \text{ and } K_d = \frac{\ddot{Q}(0)}{2}. \quad (10)$$

Denote that the filter $f(s)$ must be selected properly so that $Q(s)$ has no zero at the origin to ensure non-zero integral gain and acceptable performance.

IV. IMC-BASED PID SIMULATIONS OF THE TWO-MASS SYSTEM

All system parameters are taken from Table.1. After solving the optimal control problem [3].

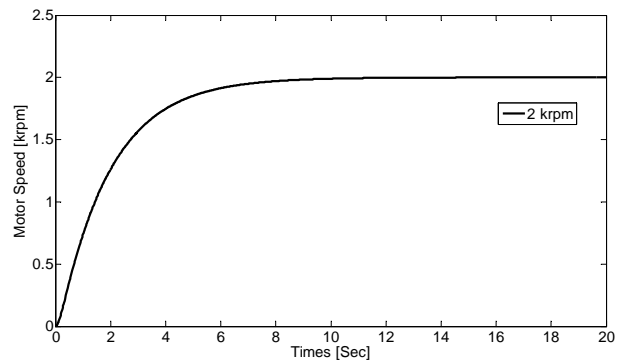


Fig.7 Open-loop responses of Two-mass system.

The simulated open-loop response of the two-mass system is shown in Fig.7. It improved the responses speed. The proposed of IMC-Based PID controller will be used with this parameter as following : $K_p = 3.9$, $K_d = 0.01$, $K_i = 4.3$.

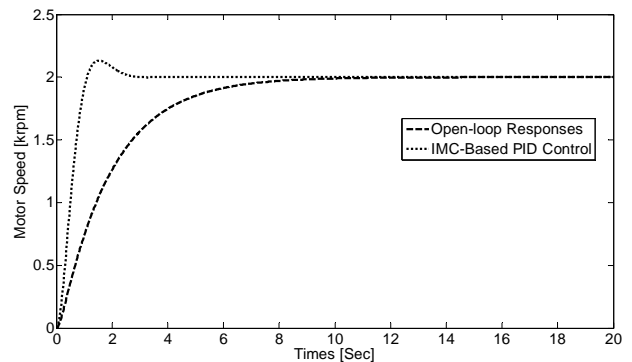


Fig.8 Responses of Two-mass system at 2000 rpm.

The simulation results in controlling the speed of two-mass system at 2000 rpm by IMC-Based PID and PID controllers show in Fig.8. It seen that the response of the IMC-Based PID control system is faster than the response of the PID control systems can reduce the vibration of the two-mass system.

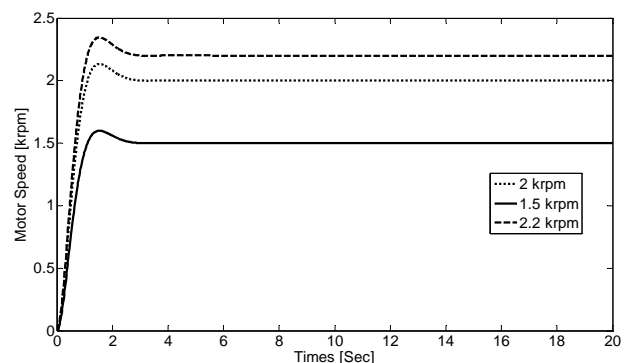


Fig.9 Speed changed due to motor responses.

In this paper the IMC-Based PID controller is used to control the speed of the two-mass system at 1500 rpm and 2200 rpm in order to show the effectiveness of the controller.

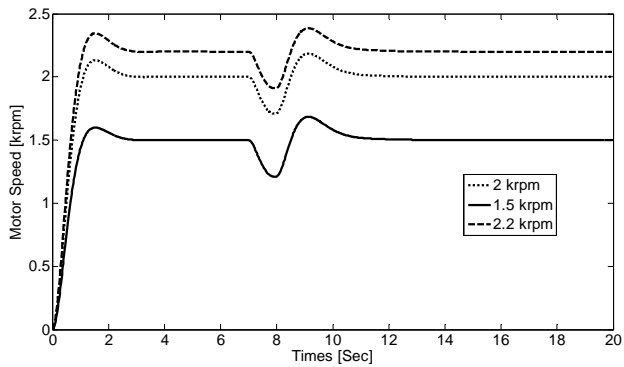


Fig.10 Speed 1500, 2000, 2200 rpm and disturbance of motor responses.

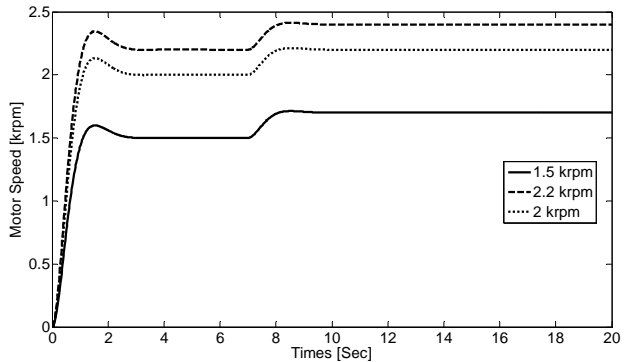


Fig.11 Increase the motor speed responses from 1500, 2000, 2200 to 1700, 2200, 2400 rpm

V.CONCLUSION

The IMC-Based PID controller design for two-mass system is proposed and used to control the motor speed of the two-mass system. The speed of responses can be improved to meet the desired system performance despite its simplicity in design. The simulation results have also shown the effectiveness of the proposed controller.

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