Proceedings of the International MultiConference of Engineers and Computer Scientists 2012 Vol II,
IMECS 2012, March 14 - 16, 2012, Hong Kong

An Affine Projection Algorithm with Variable Projection Order using the MSE Criterion

JinWoo Yoo and PooGyeon Park

***Abstract*—This paper proposes an affine projection algorithm (APA) with variable projection order using the mean-square error (MSE) criterion. The proposed algorithm determines the projection order at every iteration to use it in the update equation of the proposed APA. The MSE criterion is employed to decide the projection order reasonably. Moreover, this algorithm checks the steady-state condition to change the MSE criterion for improving the performance of APA. The experimental results show that the proposed algorithm achieves faster convergence rate, smaller steady-state estimation error and lower computational complexity than the existing APAs.**

***Index Terms*—Affine projection algorithm (APA), projection order, mean-square error (MSE), steady-state estimation error, computational complexity.**

I. Introduction

THE affine projection algorithm (APA) is the well-known algorithm of adaptive filter in signal processing along with recursive least squares (RLS) algorithm and least mean square (LMS) algorithm [1], [2], [3]. The APA has fast convergence rate of correlated input data compared to the normalized least-mean-square (NLMS) algorithm, because it uses multiple input vectors rather than a single input vector. However, APA causes high computational complexity due to multiple input vectors and its inverse term in the update equation of the filter coefficient vector. It also results in large steady-state estimation error. In APA, a high projection order leads to a fast convergence rate but a large estimation error. Meanwhile, A low projection order gives rise to a slow convergence rate but a small estimation error. Therefore, the reasonable adjustment of the projection order is worth considering to satisfy fast convergence rate and small steady-state estimation error [8].

Recently, there are several papers related to regulating the projection order to improve the performance of APA [5], [6], [7], [8]. Among them, an APA with Evolving order (E-APA) [5] and an APA with Dynamic Selection of input vectors (DS-APA) [6] are the representative algorithms. The E-APA chooses its projection order by an evolutionary order comparing the output error with a threshold. The DS-APA selects its projection order using a criterion derived by the largest decrease of the mean-square deviation (MSD). Although these algorithms have faster convergence rate and lower estimation error than the conventional APA, there remains room for improvement by adjusting the projection order reasonably.

This paper proposes an APA with variable projection order using the mean-square error (MSE) criterion [4] to satisfy fast convergence, small steady-state error, and low computational complexity. The structure of the proposed algorithm is similar with the DS-APA, but it uses the different criterion and the steady-state condition to adjust the projection order. The proposed algorithm not only uses the MSE criterion [4] for varying the projection order, but also checks the steady-state condition to have better performance than the existing APAs. The proposed algorithm actually has fast convergence rate, small steady-state estimation error and low computational complexity compared to both E-APA and DS-APA.

This paper is organized as follows. Section II describes the conventional affine projection algorithm firstly. Section III proposes a novel APA with variable projection order using the MSE criterion. The steady-state condition is also explained in this section, which is used for improving the performance of the proposed algorithm. Section IV provides the experimental results, and Section V presents the conclusion.

II. Preliminary: Affine Projection Algorithm

Consider data d_i obtained from an unknown system [1], [2], [3]:

$$d_i = \mathbf{u}_i^T \mathbf{w}_o + v_i, \tag{1}$$

where $\mathbf{w}_o$ is an n-dimensional column vector of unknown system that is to be estimated, v_i accounts for measurement noise with variance σ_v^2, and u_i denotes an n-dimensional column vector, $\mathbf{u}_i = [u_i, ..., u_{i-n+1}]^T$. The APA update equation of the filter coefficient and the error vector at i iteration are as below:

$$\hat{\mathbf{w}_{i+1}} = \hat{\mathbf{w}}_i + \mu \mathbf{U}_i (\mathbf{U}_i^T \mathbf{U}_i)^{-1} \mathbf{e}_i, \tag{2}$$

$$\mathbf{e}_i = \mathbf{d}_i - \mathbf{U}_i^T \hat{\mathbf{w}}_i, \tag{3}$$

where

$$\mathbf{U}_i = [\mathbf{u}_i, \ldots, \mathbf{u}_{i-M+1}],$$
$$\mathbf{d}_i = [d_i, \ldots, d_{i-M+1}]^T,$$
$$\mathbf{e}_i = [e_0(i), \ldots, e_{i-M+1}(i)]^T,$$

μ is the step-size parameter, and M is the projection order which is the number of input vector. The step-size μ and the projection order M affects the performance of APA in aspect

Manuscript received December 26, 2011; revised January 7, 2012. This research was supported by the MKE(The Ministry of Knowledge Economy), Korea, under the ITRC(Information Technology Research Center) support program supervised by the NIPA(National IT Industry Promotion Agency) (NIPA-2011-(C-1090-1121-0006) & NIPA-2012-(C1090-1211-0011)). This research was supported by World Class University program funded by the Ministry of Education, Science and Technology through the National Research Foundation of Korea(R31-10100). This research was supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education, Science and Technology (2010-0009743).

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ISBN: 978-988-19251-9-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

of the convergence rate and the steady-state estimation error [1], [2].

III. APA WITH VARIABLE PROJECTION ORDER USING THE MSE CRITERION

A. Variable Projection Order using the MSE criterion

Unlike the DS-APA [6] which uses the MSD criterion, the proposed algorithm employs the MSE criterion to adjust the projection order reasonably. The MSE can be described as below [4]:

$$\begin{aligned}\text{MSE} = \lim_{t\to\infty} E|e_i|^2 &= \frac{\mu\sigma_v^2}{2-\mu} Tr(\mathbf{R}_u) E[\frac{M}{\|\mathbf{u}_i\|^2}] + \sigma_v^2 \\ &\approx \frac{\mu\sigma_v^2 M}{2-\mu} + \sigma_v^2 \end{aligned} \tag{4}$$

$P_{t(i)} = \{p_1, p_2, \ldots, p_{t(i)}\}$ is a subset with $t(i)$ members of the set $\{0, 1, \ldots, M_{max}-1\}$, which means that the proposed algorithm selects $t(i)$ number among the maximum number of input vector M_{max}. p_k is the delay of the selected input vector at iteration i, where the range of k is 1 to M_i ($0 \le M_i \le M_{max}$) [6]. The update equation of the proposed APA is given as follows:

$$\hat{\mathbf{w}_{i+1}} = \begin{cases} \hat{\mathbf{w}}_i, & \text{t(i)=0} \\ \hat{\mathbf{w}}_i + \mu_i \mathbf{U}_{i,M_i}(\mathbf{U}_{i,M_i}^T \mathbf{U}_{i,M_i})^{-1}\mathbf{e}_{i,M_i}, & \text{else} \end{cases}$$

$$where \quad \mathbf{e}_{i,M_i} = \begin{bmatrix} e_{p_1}(i) \\ e_{p_2}(i) \\ \vdots \\ e_{p_{t(i)}}(i) \end{bmatrix}, \quad \mathbf{U}_{i,M_i} = \begin{bmatrix} \mathbf{u}_{i-p_1} \\ \mathbf{u}_{i-p_2} \\ \vdots \\ \mathbf{u}_{i-p_{t(i)}} \end{bmatrix}^T, \tag{5}$$

As can be seen in the update equation, the proposed algorithm dynamically chooses the number of input vectors like DS-APA [6]. Because large number of input vectors gives rise to the large steady-state errors, varying the number of input vectors is effective to reduce the steady-state estimation errors maintaining the convergence rate of initial iteration part. The input vectors are selected when each input vector satisfies the following criterion as below:

$$|e_q(i)| > \sqrt{\frac{\mu\sigma_v^2 M_i}{2-\mu} + \sigma_v^2} \quad (q = 1, 2, \ldots, M_{max}) \tag{6}$$

$$\text{where} \quad M_i = t(i)$$

In this way, the proposed algorithm selects the input vector to be used at the update equation of the filter coefficient.

B. The steady-state condition to change the criterion

The proposed algorithm checks at every iteration in order to achieve the small steady-state estimation error conserving the convergence rate at initial iteration part. If the steady-state condition is satisfied firstly, the MSE criterion used for selecting the input vector is modified. This two-stage concept enables to attain a fast convergence rate, a small steady-state estimation error and low computational complexity [8].

Lemma1 (Steady-sate condition, Chang et al. [7]). The current state becomes steady when it satisfies the following condition:
$\gamma_i = t(i)/M_i \le 0.32$
where $t(i)$ is the number of input vectors satisfying the transition criterion (6) and γ_i is the ratio between $t(i)$ and M_i.

The proposed algorithm determines the optimal point that the modified MSE criterion must be applied to select the input vector dynamically. The reason why this steady-state condition should be checked at every iteration is that the proposed algorithm have to satisfy the fast convergence rate and the small steady-state estimation error.

C. After the steady-state condition is satisfied

After the steady-state condition is satisfied for the first time, the MSE criterion which is used for selecting the input vector will be modified. It is essential that before the steady-state condition is satisfied, the proposed algorithm concentrates on having the fast convergence rate. On the other hand, after the steady-state condition is satisfied, it is important to secure the small steady-state estimation error. Therefore, after the steady-state condition is satisfied, the proposed algorithm must use the strict criterion to select the input vector for the APA filter coefficient equation. In this concept, the proposed algorithm uses the modified MSE criterion as below:

$$|e_q(i)| > \sqrt{\frac{\mu\sigma_v^2 M_{max}}{2-\mu} + \sigma_v^2} \quad (q = 1, 2, \ldots, M_{max}) \tag{7}$$

More strict criterion means the value of the MSE criterion is high. Because the large MSE criterion value makes the small selected input vectors, the proposed algorithm has the low projection order, which means the low input vector number is used for update equation. Due to the characteristic of APA related to the projection order, after the steady-state condition is satisfied, the low projection order enables to achieve the small steady-state estimation error.

IV. EXPERIMENTAL RESULTS

We illustrate the performance of the proposed algorithm by performing computer experiments in channel estimation. The channel of the unknown system is generated by a moving average model with 16 taps (n=16). We assume that the adaptive filter and the unknown channel have the same number of taps. We also assume that the noise variance, ${\sigma_v}^2$, is known a priory, because it is easy to be estimated. The input signal u_i is generated by filtering a white, zero-mean, Gaussian random sequence through $G_1(z) = 1/(1-0.9z^{-1})$, $G_2(z) = (1+0.6z^{-1})/(1+1.0z^{-1}+0.21z^{-2})$. The SNR is set to 30dB which is defined by $SNR = 10log_{10}(E[y^2(i)/E[v^2(i)]$ with $y(i) = \mathbf{u}_i{}^T\mathbf{w}_o$. The mean square deviation (MSD) is defined as $MSD = (\mathbf{w}_o - \hat{\mathbf{w}}_i)^T(\mathbf{w}_o - \hat{\mathbf{w}}_i)/\mathbf{w}_o{}^T\mathbf{w}_o$. The simulation results are obtained by ensemble averaging over 200 independent trials.

Fig. 1 and 2 show the MSD of the conventional APA, E-APA, DS-APA, and proposed APA when the input vector is generated by G1 and G2 with high SNR situation. As can

ISBN: 978-988-19251-9-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

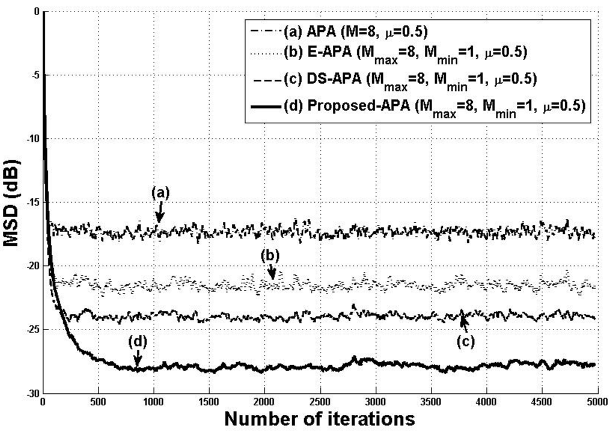

Fig. 1. Mean-square deviation of APA, E-APA, DS-APA, and the proposed APA (the input signal is generated by G1, SNR=30dB, n=16).

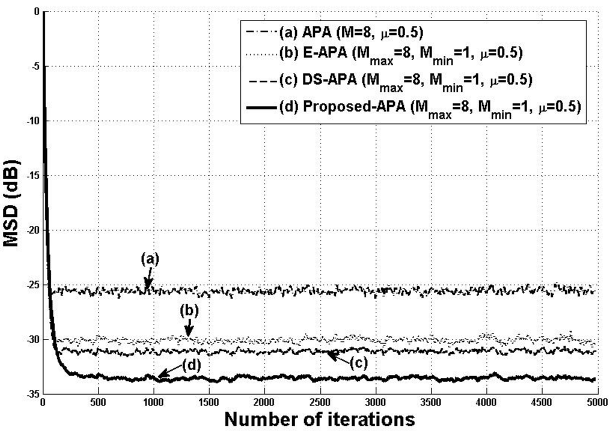

Fig. 2. Mean-square deviation of APA, E-APA, DS-APA, and the proposed APA (the input signal is generated by G2, SNR=30dB, n=16).

be seen, these simulation results verify that the proposed APA has faster convergence rate and smaller steady-state estimation error than the conventional APA, E-APA, DS-APA.

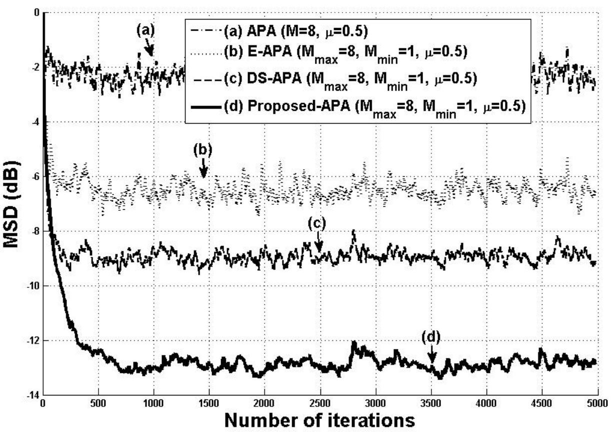

Fig. 3. Mean-square deviation of APA, E-APA, DS-APA, and the proposed APA (the input signal is generated by G1, SNR=15dB, n=16).

Fig. 3 and 4 show the MSD of the conventional APA, E-

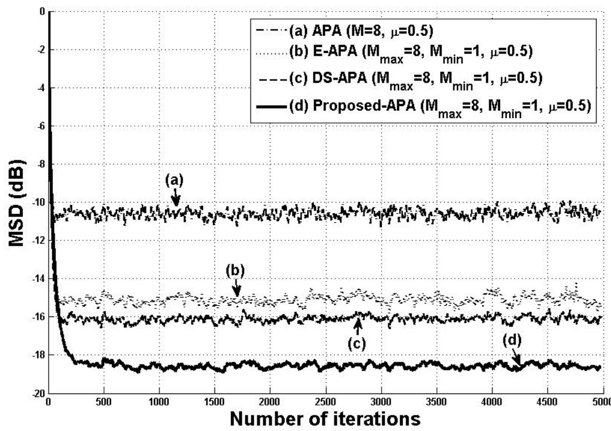

Fig. 4. Mean-square deviation of APA, E-APA, DS-APA, and the proposed APA (the input signal is generated by G2, SNR=15dB, n=16).

APA, DS-APA, and proposed APA when the input vector is generated by G1 and G2 with low SNR situation. As can be seen, these simulation results verify that the proposed APA has faster convergence rate and smaller steady-state estimation error than the conventional APA, E-APA, DS-APA.

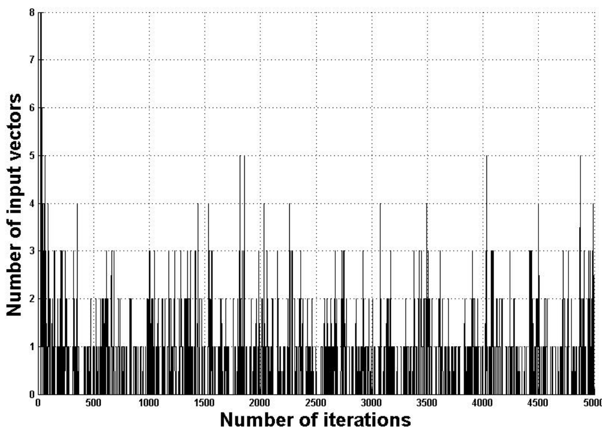

Fig. 5. Number of input vetors of the proposed APA over 1 trial (the input signal is generated by G1, SNR=30dB, n=16, M_{max}=8, $M_{min} = 1$).

Fig. 5 shows the number of input vectors of the proposed APA over 1 trial when the input signal is generated by G1. In the proposed APA, the maximum number of input vectors, M_{max}, is 8, and the minimum number of input vectors, M_{min}, is 1. The proposed APA determines the projection order dynamically using the MSE criterion with two stage concept. As can be seen, the number of input vectors varies dynamically, which is similar with the number of input vector in DS-APA [6]. However, since the proposed APA chooses the MSE criterion at every iteration and checks the steady-state condition for applying the strict MSE criterion to reduce the projection order after the steady-state condition, its average number of input vectors is smaller than DS-APA. Table I represents the average projection order during 5000 iterations of APA, E-APA, DS-APA, and the proposed APA. As can be seen, the proposed APA uses the smaller input vector number than both E-APA and DS-APA.

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TABLE I
AVERAGE PROJECTION ORDER DURING 5000 ITERATIONS OF APA, E-APA, DS-APA, AND THE PROPOSED APA.

Algorithm	Average projection order
E-APA	2.05
DS-APA	1.53
Proposed-APA	0.43

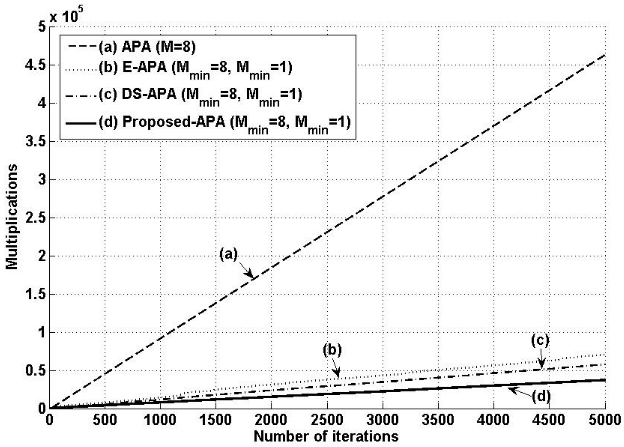

Fig. 6. Accumulated sum of multiplications in the conventional APA, E-APA, DS-APA, and the proposed APA (the input signal is generated by G1, SNR=30dB, n=16).

Table II shows the computational complexity of the conventional APA, E-APA, DS-APA, the proposed APA at every iteration. As can be seen, the dominant term of complexity is the number of input vectors, not constant term. Fig. 6 shows the accumulated sum of multiplications in the conventional APA, E-APA, DS-APA, and the proposed APA. Due to adjust the number of input vectors, it is reasonable that the accumulated sum of multiplications in the proposed APA is smaller than those of the conventional APA, which has the fixed number of input vectors (M=8). Since the average number of input vectors in the proposed APA is smaller than that in both E-APA and DS-APA, the computational complexity of the proposed APA is lower than that of both E-APA and DS-APA.

V. CONCLUSION

This paper has proposed an affine projection algorithm with variable projection order using the mean-square error (MSE) criterion. The proposed algorithm reasonably determines the projection order of APA at every iteration using the MSE criterion. Moreover, this algorithm checks the steady-state condition to change the MSE criterion appropriately for upgrading the performance of the proposed APA in aspect of convergence rate, steady-state estimation error and computation complexity. The experimental results verified that the proposed algorithm accomplished faster convergence rate, smaller steady-state estimation errors and lower computational complexity than the existing affine projection algorithms.

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TABLE II
COMPUTATIONAL COMPLEXITY OF APA, E-APA, DS-APA, AND THE PROPOSED APA.

Algorithm	Number of Mulplications
APA	$(M^2 + 2M)n + M^3 + M^2$
E-APA	$(M_i^2 + M_i + M)n + M_i^3 + M_i^2 + M_i + 1$
DS-APA	$(M_i^2 + M_i + M)n + M_i^3 + M_i^2$
Proposed-APA	$(M_i^2 + M_i + M)n + M_i^3 + M_i^2$+2

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