Revenue Management in E-Commerce: A Case Study

Başak Demirci, S. Emre Alptekin

***Abstract*—The price is one of the most important aspects of marketing activities. Establishing the appropriate price is the ultimate requirement for revenue maximization. Dynamic pricing, which is simply a price change whenever there is a change in demand or other parameters, could prove useful to determine the appropriate price. Dynamic pricing could be used when demand, customer sensitivity and other factors are measurable. E-commerce is a commercial activity, where each order, transaction and delivery is made by electronic processes online. Having demand data and the sensitivity to price changes, it is possible to build models for the pricing decision. Moreover, e-commerce is very suitable for dynamic pricing especially with the help of the current technological advances which allow frequent price changes because it is easy to collect data and customers can be easily informed of price changes. In this paper, we used models based on game theory. Our proposed model takes into account parameters such as the level of quality (which is very important for an e-commerce company) and the sensitivity of the customer on price levels and quality. The applicability of the proposed methodology is demonstrated via a case study**

***Index Terms*— E-commerce, dynamic pricing, revenue management, game theory.**

I. Introduction

The question of what the determinants of price are is an important one in economic theory. In fact, pricing of goods is one of the main factors contributing to the revenue of the firm with significant effect to a firm's profit [1].

Revenue management in e-commerce along with dynamic pricing activities aims to maximize sales and also increase revenues. The goal is to set a price for goods or services and observing the dynamics of the market with the inclusion of time in the analysis change the price accordingly.

Electronic commerce can be defined as all exchanges related to electronic business. It covers the entire process of selling goods and services via an electronic channel. Electronic commerce provides companies with considerable growth prospects. But it also represents a major challenge, requiring significant changes in terms of internal organization, operation and strategy.

Replacing the traditional marketing methods, Internet marketing is a growing field since the 1990s. The increasing use of computers and the Internet contributed to the development of Internet marketing. This type of marketing offers advantages such as cost and time. In addition, there are other conveniences such as establishing a database, which could effectively be used to segment the market and sales for each consumer group. Internet marketing could be used to reach the consumers and as well as businesses. Internet marketing with its dynamism actually gave businesses the opportunity to compare and analyze the effectiveness of different pricing strategies.

One of the most important benefits of Internet marketing for buyers and sellers is saving the high costs of distribution, storage and transport. Thanks to this advantage, it is possible to offer reasonable prices without sacrificing profits [2]. On the other hand, as the information regarding the purchases on the Internet could be obtained rapidly, customers are more price sensitive compared with the traditional channel. That is why the price is very important in electronic commerce [3].

The price has become an important competitive tool for marketing on the Internet because of the similarity between the products offered. As in traditional marketing, it is also important on the Internet to differentiate offerings in order to satisfy the demands and expectations of consumers. Therefore, the price becomes a very critical parameter that must be dynamic and flexible.

There are different methods of pricing used by businesses that accept transactions over the Internet and price differentiation is one of them. This approach could be found in the B2C model which simply is based on "business-to-customer" activities. The price differentiation means that the same product will have different prices depending on the type of buyer, current demand, etc..

Companies that perform e-commerce have significant advantage against their competitors which are using just the classic channels, with their ability to effectively gather information on their customers. This information actually builds the foundation of dynamic pricing. Dynamic pricing approaches are widely used by airlines and car galleries and it brought innovation to their businesses and made it easier to read the client's response to different pricing strategies and collect information above all. The main objective is to make use of the advantage of information systems: increased service levels, reduced costs, and the reduction in costs reflected to consumers.

Many online shopping search engines allows consumers to find retailers that sell a specific product and compare product prices. Offering the lowest price does not always guarantee that consumers are buying on your site. Other non-price attributes, such as service quality and brand recognition of a merchant, also play an important role in helping online retailers to build a competitive advantage.

In this paper, we present a model of price competition that assumes e-tailers can mainly differentiate themselves by offering different levels of service and establishing a different online recognition. The purpose of this study is to propose a pricing strategy for e-tailers. The proposed

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framework is based on mathematical models of game theory.

The case study used in this paper will enable to explore different scenarios with different parameter settings. The aim is to examine the effect of some parameters on the pricing and suggest settings that should maximize revenue of the company.

The remaining part of the paper is organized as follows: in Section 2 we give related literature related to dynamic pricing. Section 3 briefly describes the methodology that constitutes the proposed framework. The steps and details of the proposed framework and its implementation into e-commerce is given in Section 4. Finally, Section 5 concludes the study.

II. Literature review

Product or service providers face a number of fundamental challenges. First of all, decisions related to marketing activities usually have uncertainties attached. Secondly, everybody wants to sell at a time when market conditions are more favorable, but nobody knows what the future might hold. The price must be fair - not set too high to cause providers to lose potential buyers and not too low to cause providers to lose potential profits. It is important to know how many buyers are appreciating the products or services, but usually this number is just a guess. In fact, it is difficult to find providers who are completely satisfied with their price and related decisions.

The idea of management of revenue actually has its origins in the airline industry. There is not any other business practice whose origin is so closely tied to a single industry [4]. In the 60s, the revenue management appeared first at American Airlines with the segmentation and analysis of consumers. The company tried to sell seats to the appropriate client at the right time and at the right price, in order to obtain maximum customer satisfaction and economic benefits. These systematic management concepts and methods combined computer technology and modern scientific prediction and optimization techniques [5].

The starting point for revenue management was the Act in airline deregulation in 1978. With this act, the Civil Aviation Council of the United States released the price control of airlines, which had been strictly regulated on standardized prices and standardized profitability objectives [6]. The passage of the law has led to rapid changes and a rash of innovation in the industry. Major airlines have accelerated their development of computerized reservation systems (CRS) and global distribution systems (GDS) and the GDS concept became profitable.

At the same time, the new low cost charter airlines have entered the market. Many of these entrants - because of their lower labor costs, simple operation, and service-frills managed to offer much lower prices than the major airlines. These new entrants have tapped into an entirely new and large market consisting of families, couples who travel for weekends, students visiting their homes, several of which could have used their cars or did not travel at all. It turned out (quite surprisingly) that the air was quite price elastic, with prices low enough, consumers preferred air travel over ground options. American Airlines actually was able to "compete on cost" with new entrants using its surplus capacity. The revenue management practices used by American Airlines generated $1.4 billion in additional revenue over a period of three years from 1988.

The basic parameters of the theory of conventional pricing are a demand curve that describes the related market with sales volume V at a price P and a scale of costs to define the unit cost C changing with the rate of production. A major shortcoming of the conventional price theory is its short term focus. It assumes static market and production environments and uses instantaneous profit flow as a key parameter to value judgments [7]. One of the reasons for its absence in the real world cases today is the fact that managers are well aware that their market and production environments are changing over time. They use pricing rules which they feel will produce a better long-term performance. Classical marginal pricing became suboptimal for a rapidly evolving business; today dynamic models that suite better can be formulated and planning based on dynamic models can lead to a significant improvement in long-term earnings performance.

Electronic markets have significantly reduced the cost of making price changes [8]. For the first time providers are able to make immediate adjustments to prices quickly and realistically. As a proof of this, many online businesses today make automated adjustments to the prices, even on hourly basis. An example of such a business online is Buy.com. As described by Smith [8] Buy.com uses software agents to search for websites competing for competitive prices, and in response, Buy.com lowers the price to match or beat the price. Their pricing strategy is based on the assumption that their customers are extremely price sensitive and choose to buy from the seller offering the lowest price. Not surprisingly, Buy.com has managed to collect huge sales, but profits are very low or even negative. The example of Buy.com highlights two things. Firstly, automated dynamic pricing is an option for businesses today. Secondly, a model too simplistic or erroneous can produce undesirable results.

In recent years, we have seen an increased adoption of dynamic pricing strategies in existing and developing retail and other industries [9].

Three factors contributed to this phenomenon: (1) increased availability of demand data, (2) ease of varying prices using new technologies, and (3) an availability of "decision support tools" to analyze demand data and pricing dynamics. Determining the "fair" price to charge a customer for a product is a complex task, requiring that the company knows not only its own operating costs and the availability of supply, but also how the current client values the product and what future demand will be. Therefore, to charge a customer the right price, a company must have a wealth of information about its customers and be able to set and adjust prices at minimal cost [10].

Today, in both Internet and brick and mortar stores, new technology enables retailers to gather information not only on sales but also on the demographics and preferences of customers. With the ease of making price changes on the Internet, dynamic pricing strategies, especially in the form of price markdowns are now frequently used in business-to-consumer (B2C) and business-to-business (B2B) transactions by many companies.

The advance of information technology (IT) and e-commerce have played an important role in the management of inventory. For example, programs such as collaborative planning, forecasting and replenishment (CPFR), quick response (QR) and Vendor Managed Inventory (VMI) allow the sharing of information and collaboration between

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partners in the chain supply, reducing inventory costs while simultaneously increasing service levels [10]. However, despite significant improvements in reducing costs in the supply chain through better inventory management, a large proportion of retailers still lost millions of dollars per year as a result of the lost sales and excess inventory. Therefore, many are now ready to consider the application of the supply-demand equation, review their pricing policies, and explore dynamic pricing for better management of demand.

In addition to the analysis and learning of buying behavior of a customer, the Internet also provides online merchants with the ability to proactively test pricing to better characterize demand market [11]. The cost is perhaps the biggest factor preventing the widespread use of dynamic pricing in many markets, because in traditional markets, it is expensive to continually change product prices. But in the digital markets, the costs associated with changes to the prices are greatly diminished [8]. While the price tests offline can cost hundreds of thousands of dollars and take months to complete, the Web allows you to test in real time virtually at no cost. While some companies, such as Amazon.com, have received a lot of negative reactions from their customers on price tests, other companies, including GE, Penske, Hotwire, and DHL Worldwide Express, were able to use them in developing their pricing policies [10].

Companies seek to maximize their profit under the constraint of capacity by offering prices to attract potential customers. Pricing strategy of a firm affects the demand of other companies. There is an interaction between strategic pricing decisions of firms, so the game theory is applied to analyze this problem [12].

III. The Methodology

The essence of competition is interdependence. Interdependence means that the consequences of an action for a company not only depend on the action of the firm, but also on the actions of its competitors. How can we decide what action to choose in a competitive environment? On the surface, the problem seems insoluble, because what is optimal for a firm depends on what other companies are doing. Game theory offers a way out of this mess by imposing restrictions on what answer a "rational, intelligent" company can give to the question, "What are other companies doing?" [13].

Game theory is the formal study of conflict and cooperation. Game theoretical concepts apply whenever the actions of several agents are interdependent. These agents may be individuals, groups, companies, or any combination of these. The concepts of game theory provide a language to formulate, structure, analyze and understand strategic scenarios [14].

The major development of the theory began in the 1920s with the work of mathematician Emile Borel (1871-1956) and polymath John von Neumann (1903-1957). A crucial event in the development of the theory was the publication in 1944 of the book "Game theory and economic behavior" by Von Neumann and Oskar Morgenstern, who laid the foundations of the field. Game theory is generally considered to have begun with this publication. Although very little of the theory mentioned in this large volume is relevant to the current works in the area, it introduced the idea that the conflict could be mathematically analyzed and disseminated [15].

In the early 1950s, John F. Nash has developed a key concept (Nash equilibrium) and initiated the study of negotiation in game theory. Shortly after the work of Nash models of game theory began to be used in economic theory and political science, and psychologists have begun to study how humans behave in experimental games. In the 1970s, game theory was first used as a tool in evolutionary biology. Subsequently, theoretical methods started to dominate games microeconomic theory and are also used in many other areas of the economy and in a range of other social and behavioral sciences. [16].

The object of study in game theory is the game, which is a formal model of an interactive situation. It usually involves several players; a game with one player is usually called a decision problem. The formal definition states players, their preferences, their information, strategic actions available to them, and how they influence the outcome.

The games can be described formally at different levels of detail. A cooperative game is a high-level description, stating only that each group or coalition can be potentially achieved through cooperation with its members. What is not made explicit is the process by which the coalition is formed.

Branches of game theory also differ in their assumptions. A central assumption in many variations of game theory is that players are rational. A rational player is the one who always chooses an action that gives the result that is most preferred considering the expected reactions of its opponents.

Companies are smart; they will recognize that other firms are rational. Smart companies can put themselves in the positions of other firms and look from their point of view. The central concept of the theory of non-cooperative games is based on the intelligence present in businesses [13]. This paper focuses mainly on the theory of non-cooperative games with rational actors.

Nash's theory of non-cooperative games is recognized to be one of the outstanding intellectual advances of the twentieth century. The formulation of the Nash equilibrium has had a fundamental and omnipresent impact to the economy and the social sciences that are comparable to the discovery of the DNA double helix in the biological sciences [17].

In game theory, Nash equilibrium (named after John Forbes Nash) is a solution concept of a game involving two or more players, in which each player is supposed to know the strategies of other players, and no player has anything to gain by changing only his own strategy unilaterally. If each player has chosen a strategy and no player can benefit by changing his strategy while the other players keep all their strategies unchanged, then the current set of strategic choices and corresponding gains constitute Nash equilibrium.

Game theorists use the concept of Nash equilibrium to analyze the outcome of the strategic interaction of several decision makers. In other words, it provides a way to predict what will happen if several persons or institutions are making decisions at the same time, and if the result depends on the decisions of others.

IV. E-commerce Dynamic Pricing Framework

A. *Proposed Model*

We model our pricing problem based on a research of Işıklar Alptekin and Bener [18], who used their non-cooperative game model for pricing and transmission power control in a cognitive radio environment. The model's properties are given below:

Players: e-tailers
Actions and strategies: The choice of the price offering

We are interested in determining if there is a focal point, from which no player would deviate, i.e. a Nash equilibrium.

The pricing strategy set consists of a set of N companies, F_i, designated by $I = \{1,2, ..., N\}$. Each company has two operating parameters: $(p,q) \in \mathcal{R}_+^{2N}$. $\boldsymbol{p} = \{\boldsymbol{p_1}, \boldsymbol{p_2}, \ldots, \boldsymbol{p_N}\}$ is the price vector and p_i is the price that F_i takes for each unit of demand and $\boldsymbol{q} = \{\boldsymbol{q_1}, \boldsymbol{q_2}, \ldots, \boldsymbol{q_N}\}$, where q_i measures the quality offered by F_i.

F_i experiences a demand which is symbolized by $D_i(p,q)$: $\mathcal{R}_+^{2N} \to \mathcal{R}_+$. The important aspect of this model is that the demand F_i depends not only on its own parameters p_i and q_i, but also on the prices offered by its competitors. The utility function is given $U_i(p,q)$: $\mathcal{R}_+^{2N} \to \mathcal{R}_+$ while the strategy space is given by $S_i \in \mathcal{R}^2$ [18].

$$S_i = \{(p_i, q_i): 0 \le p_i^{min} \le p_i \le p^{max}\ ; 0 \le q_i^{min} \le q_i \le q_i^{max}\} \quad (4.1)$$

In this model, we assume that the average demand is linear in prices and has the following properties [18]:

$$\frac{\partial D_i(p,q)}{\partial p_i} \le 0, \frac{\partial D_i(p,q)}{\partial q_i} \ge 0\ , \frac{\partial D_i(p,q)}{\partial p_j} \ge 0, \frac{\partial D_i(p,q)}{\partial q_j} \le 0,\ \ j \ne i \quad (4.2)$$

$$D_i(p,q) = a_i - b_i.p_i + \sum_{j\in I, j\ne i} c_{ij}.p_j\ + \beta_i.q_i - \sum_{j\in I, j\ne i} \gamma_{ij}.q_j \ge 0 \quad (4.3)$$

with a_i as the base demand and $b_i, c_{ij}, \beta_i, \gamma_{ij}$ are positive constants representing the extent to which customers are affected by changes in the price and quality. For example c_{12} is the measure that shows how the first customer is influenced by the price offers of F_1 to the second client. The constants b and c satisfy:

$$b_i > \sum_{j\ne i} c_{ij}, i \in I \quad (4.4)$$

It implies that the influence of an enterprise's own price is larger on its own demand than the prices of its competitors. This may be due to the presence of loyalty or the imperfect knowledge of competitors' prices.

Customers must be able to differentiate between online sellers on the market according to certain parameters of quality of service. To calculate the parameters and q_i and q_j we constructed a method to score the quality levels of some online vendors. We evaluated the companies based on three main factors. Scores are given from the consumer perspective empirically using criteria presented in Table 4.1.

After the e-tailers were scored, q_i and q_j are calculated by obtaining arithmetic mean of the scores of the three criteria. The revenue of a company can be calculated by multiplying its price by its demand.

$$U_i(p,q) = p_i.D_i(p,q) \quad (4.5)$$

$$U_i\ (p,q) = p_i\ (a_i - b_i.p_i + \sum_{j\in I, j\ne i} c_{ij}.p_j\ + \beta_i.q_i - \sum_{j\in I, j\ne i} \gamma_{ij}.q_j\ ) \quad (4.6)$$

TABLE I. CRITERIA TO JUDGE THE QUALITY OF AN ONLINE SELLER

Criteria	**Maximum Score Possible**
Interaction with the website	10
Ease of use	
Confidentiality	
Simple design	
Consistency and flexibility	
Good information	
Delivery of the product	10
The timeliness of the order	
The accuracy of the order	
The condition of the order	
Addressing Problems When Occurred	10
The opportunity to talk to someone	
Fairness of policies and procedures	
Compensation and apology	

Now, we have to prove the supermodularity of the game. Supermodular games are those that are characterized by "strategic complementarities" - basically, this means that when a player takes additional actions, others want to do the same. The game G is supermodular if the following conditions are met [18]:

- S_n is an interval of $\mathcal{R}^N$, where $S_n = [\underline{y_n}, \overline{y_n}] = \{x | \underline{y_n} \le x \le \overline{y_n}\}$;
- f_n is twice continuously differentiable on S_n ;
- $\frac{\partial^2 f_n}{\partial x_{ni} \partial x_{mj}} \ge 0$ for all n and all $1 \le i < j \le N$;
- $\frac{\partial^2 f_n}{\partial x_{ni} \partial x_{mj}} \ge 0$ for all $n \ne m$, $1 \le i \le N$ and $1 \le j \le M$.

A pure Nash equilibrium is a strategy set of $x = (x_n; n \in N)$ such that each x_n maximise $f(\hat{x}_n, x_{-n})$ over S_n. The strategic feasible set of the game $S_i = \{p_i: 0 \le p^{min} \le p_i \le p^{max}; i = 1,2, \ldots, N\}$is a subnet of R^N [18].

$$\frac{\partial U_i(p,q)}{\partial p_i} = D_i(p,q) - b_i.p_i \quad (4.7)$$

$$\frac{\partial^2 U_i(p,q)}{\partial p_i^2} = -2b_i \le 0 \quad (4.8)$$

$$\frac{\partial U_i(p,q)}{\partial q_i} = \beta_i.p_i \quad (4.9)$$

$$\frac{\partial^2 U_i(p,q)}{\partial q_i^2} = 0 \le 0 \quad (4.10)$$

$$\frac{\partial^2 U_i(p,q)}{\partial p_i \partial p_j} = \sum_{i\ne j} c_{ij} \ge 0 \quad (4.11)$$

To find the q_i that maximizes revenue, we must take the derivative of the utility function and set it equal to zero.

$$\frac{\partial U_i\ (p,q)}{\partial p_i} = D_i(p,q) + p_i(-b_i) = 0 \quad (4.12)$$

$$\frac{\partial U_i\ (p,q)}{\partial p_i} = a_i - b_i.p_i + \sum_{j\in I, j\ne i} c_{ij}.p_j\ + \beta_i.q_i - \sum_{j\in I, j\ne i} \gamma_{ij}.q_j - b_i.p_i = 0 \quad (4.13)$$

$$\frac{\partial U_i\ (p,q)}{\partial p_i} = a_i - 2.b_i.p_i + \sum_{j\in I, j\ne i} c_{ij}.p_j\ + \beta_i.q_i - \sum_{j\in I, j\ne i} \gamma_{ij}.q_j = 0 \quad (4.14)$$

$$2.b_i.p_i - \sum_{j\in I, j\ne i} c_{ij}.p_j\ = a_i + \beta_i.q_i - \sum_{j\in I, j\ne i} \gamma_{ij}.q_j \quad (4.15)$$

This is a linear system of equation in p, which can be written in matrix form.

$$\boldsymbol{Ap} = \left[\boldsymbol{a_i} + \boldsymbol{\beta_i}.\boldsymbol{q_i} - \textstyle\sum_{j\in I, j\neq i} \boldsymbol{\gamma_{ij}}.\boldsymbol{q_j}\right] \tag{4.16}$$

$$\boldsymbol{A} = \begin{pmatrix} \boldsymbol{2b_1} & -\boldsymbol{c_{12}} \cdots & -\boldsymbol{c_{1N}} \\ -\boldsymbol{c_{(N-1)1}} \vdots & \ddots & -\boldsymbol{c_{(N-1)N}} \vdots \\ -\boldsymbol{c_{N1}} & -\boldsymbol{c_{N2}} \cdots & \boldsymbol{2b_N} \end{pmatrix} = \Phi(1 - T) \tag{4.17}$$

$$\boldsymbol{\Phi} = \boldsymbol{diag}(\boldsymbol{2b_1}, \boldsymbol{2b_2}, \ldots, \boldsymbol{2b_N}) \tag{4.18}$$

$$\boldsymbol{T} = \begin{pmatrix} \boldsymbol{0} & \cdots & \frac{\boldsymbol{c_{1N}}}{\boldsymbol{2b_1}} \\ \vdots & \ddots & \vdots \\ \frac{\boldsymbol{c_{N1}}}{\boldsymbol{2b_N}} & \cdots & \boldsymbol{0} \end{pmatrix} \tag{4.19}$$

Therefore, $A^{-1} = (I - T)^{-1}.\Phi^{-1}$ and p*=$A^{-1}.X = (I - T)^{-1}.\Phi^{-1}.X$ with $X = a_i + \beta_i.q_i - \sum_{j\in I, j\neq i} \gamma_{ij}.q_j$

$$p_i^* = \sum_{j=1}^{N} A_{ij}^{-1}.a_i + (A_{ii}^{-1}.\beta_i - \sum_{i\neq j} A_{ij}^{-1}.\gamma_{ji}).q_i + \sum_{j\neq i}(A_{ij}^{-1}.\beta_j - \sum_{l\neq j} A_{il}^{-1}.\gamma_{li}).q_j \tag{4.20}$$

To prove the uniqueness of the equilibrium point, we used the approach of contraction. A sufficient condition for a contraction is given as below [18]:

$$\frac{\partial^2 U_i(p,q)}{\partial p_i^2} + \sum_{i\neq j}\left|\frac{\partial^2 U_i(p,q)}{\partial p_i \partial p_j}\right| < 0 \tag{4.21}$$

$$-2b_i + \sum_{i\neq j} c_{ij} < 0 \tag{4.22}$$

B. Numerical Application of the Proposed Framework

The applicability of the proposed model will be demonstrated via a numerical application where two e-tailers, each with a different level of quality are competing in the same market. To make the comparison based on quality levels, we conducted research on social media to see the perceived quality of online shoppers about these two companies. 35 people participated in the survey ranking the quality levels of the two companies on a scale of 0 to 10 relative to the criteria listed in Table I. The final results we obtained from this survey are given in Table II.

TABLE II. LEVELS OF QUALITY OF SERVICE ACCORDING TO THE SURVEY

$q_{e-tailer\_1}$	0,68
$q_{e-tailer\_2}$	0,84

We used the parameters given in Table III, which represent the responsiveness of customers to price levels and quality [18]. F denotes the e-tailers, A denotes the clients.

The parameters used in the proposed model try to measure a customer's sensitivity to the quality and price of the company and the quality and price of the competitor. Solving the formula given in (4.20), the Nash equilibrium price is p^* obtained.

The parameters used in the case study, based on the research of Alptekin and Bener [18], suggest that two customer profiles are present, a high profile customer (A_1) and a low profile customer (A_2). The base demand (a) is assumed to be the same for both customer profiles and is set at 12 for each. In order to calculate p^*, a set of equations had to be solved simultaneously.

TABLE III. THE VALUES OF THE PARAMETERS OF THE DEMAND FUNCTION

	$\boldsymbol{F_1}$		$\boldsymbol{F_2}$	
	$\boldsymbol{A_1}$	$\boldsymbol{A_2}$	$\boldsymbol{A_1}$	$\boldsymbol{A_2}$
$\boldsymbol{\beta}$	6	6	4	4
$\boldsymbol{\gamma_{F_1\to A_1}}$	0	2.2	1.2	0.9
$\boldsymbol{\gamma_{F_2\to A_1}}$	2.2	0	0.9	1.2
$\boldsymbol{\gamma_{F_1\to A_2}}$	1.9	2	0	1.4
$\boldsymbol{\gamma_{F_2\to A_2}}$	2	1.9	1.4	0
$\boldsymbol{b}$	4	4	6.8	6.8
$\boldsymbol{c_{F_1\to A_1}}$	0	1.2	2	1.7
$\boldsymbol{c_{F_2\to A_1}}$	1.2	0	1.7	2
$\boldsymbol{c_{F_1\to A_2}}$	0.9	1.4	0	1.3
$\boldsymbol{c_{F_2\to A_2}}$	1.4	0.9	1.3	0
$\boldsymbol{q}$	0.68	0.68	0.84	0.84

The following matrix has been build to calculate the values of demand (D), the price (p^*), and the utility (U^*).

$$\boldsymbol{A} = \begin{pmatrix} 8 & -1.2 & -0.9 & -1.4 \\ -1.2 & 8 & -1.4 & -0.9 \\ -2 & -1.7 & 13.6 & -1.3 \\ -1.7 & -2 & -1.3 & 13.6 \end{pmatrix}$$

The formula given in (4.20) is used to obtain the results presenten in Table IV.

TABLE IV. OPTIMUM RESULTS FOR PRICE, DEMAND AND UTILITY

	$\boldsymbol{F_1 \to A_1}$	$\boldsymbol{F_2 \to A_1}$	$\boldsymbol{F_1 \to A_2}$	$\boldsymbol{F_2 \to A_2}$
$\boldsymbol{D}$	9.04	9.61	11.42	11.81
$\boldsymbol{p^*}$	2.26	2.40	1.68	1.74
$\boldsymbol{U^*}$	20.44	23.07	19.17	20.51

We started experimenting. When we increased the quality of the first firm, which had a score of 68% to 100% and decreased the quality of the second firm, which had a score of 84% to 1%, the optimum results changed as given in Table V.

TABLE V. OPTIMUM RESULTS WITH THE CHANGE OF QUALITY LEVEL OF THE FIRST FIRM

	$\boldsymbol{F_1 \to A_1}$	$\boldsymbol{F_2 \to A_1}$	$\boldsymbol{F_1 \to A_2}$	$\boldsymbol{F_2 \to A_2}$
$\boldsymbol{D}$	11.07	7.58	12.75	10.33
$\boldsymbol{p^*}$	2.77	1.89	1.86	1.52
$\boldsymbol{U^*}$	30.65	14.36	23.92	15.69

Now we varied the sensitivity for quality of the low profile customer. If they are as sensitive as high profile customers, the following results are obtained (Table VI).

TABLE VI. OPTIMUM RESULTS AFTER VARIATION OF THE SENSITIVITY FOR QUALITY OF THE LOW PROFILE CUSTOMER

	$\boldsymbol{F_1 \to A_1}$	$\boldsymbol{F_2 \to A_1}$	$\boldsymbol{F_1 \to A_2}$	$\boldsymbol{F_2 \to A_2}$
$\boldsymbol{D}$	9.23	9.79	12.28	12.82
$\boldsymbol{p^*}$	2.31	2.45	1.81	1.88
$\boldsymbol{U^*}$	21.30	23.97	22.74	24.16

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In the opposite case where high profile customer is no longer sensitive to the quality, the following optimum results are obtained (Table VII).

TABLE VII. OPTIMUM RESULTS AFTER VARIATION OF THE SENSITIVITY FOR QUALITY OF THE HIGH PROFILE CUSTOMER

	$F_1 \rightarrow A_1$	$F_2 \rightarrow A_1$	$F_1 \rightarrow A_2$	$F_2 \rightarrow A_2$
D	8.12	8.54	10.91	11.30
p^*	2.03	2.14	1.60	1.66
U^*	16.47	18.24	17.50	18.77

The results obtained demonstrate that obvious changes in demand and prices can be observed when different sensibility values for quality are used. The proposed model could be used to obtain more realistic results because the demand is no longer linear and the model could incorporate different customer profiles, which is the case with e-commerce customers. In order to provide tailored services to different customers, their profile should be included in the decision process, especially the sensibility to price changes and quality levels.

Now when we inspect the data that we have acquired and given in Table V, we varied the level of quality of companies. We have increased the quality of the first company from 68% to 100% and reduced the quality of the second company from 84% - 1%. It could be seen that customer demand of high profile customers to the first company has increased and the demand of the same client to the second firm has decreased. In addition, the price of the first firm is increased for all customers. Consequently, the first firm was able to increase its total utility, whereas the second company lost revenue.

Finally, we varied the sensitivity levels of customer profiles in Tables 6 and 7. The change has made high profile customer price insensitive. Here, the most interesting thing is when a customer sacrifices his desire for quality, prices fall sharply. Similarly, if all clients are becoming more sensitive to the quality, prices increase. The revenues of the companies are increasing when all customers want a high quality service and accept paying more money.

V. CONCLUSION

Our paper gives a clear view on possible pricing methods to be used in e-commerce. Pricing in the online world is not like pricing in the real world. The online market has a translucent structure with many different business models and different types of clients. The amount of competition in electronic commerce is forcing companies to make intelligent decisions on pricing. There is no standard approach in electronic commerce. Income can be maximized by treating each customer in a unique way in terms of price. Pricing models related to time, related to demand or related to sensitivity/loyalty may be applied.

When we look to the airline industry, they applied this pricing model for years and it is clear that this is the one of the best ways to maximize their profits. The electronic retail industry tries to adapt the same strategy. Online businesses are gradually becoming very responsive to pricing decisions of each other because their clients are aware of pricing policies and have a greater power of choice now. Companies that are aware of this trend follow each other so they can catch the customer with the lowest price. But as we have seen in our numerical application, it is possible to apply different prices for similar products with different quality levels. Therefore, setting the right price for the different consumer profiles to increase the total utility will be possible.

In order to apply dynamic pricing, we have to know the market very well. Especially, in order to know the client, we have to collect all the necessary data about their buying habits, their sensitivity on certain factors and what they seeks in the market. This process requires extensive data mining, which should produce the data needed to create efficient algorithms for pricing activities.

Our work provides a basis for understanding the logic behind dynamic pricing. Further study could be required to form more accurate models that could reveal the dynamic structure of pricing mechanism better. In this project, we used certain assumptions for demand and added other parameters to make it more realistic. In the real world it is difficult to predict the demand level. Therefore, a demand model more suitable for dynamic pricing should include probability functions also.

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