Simulation of Water Networks
Remodeling-Linear-case

Nagib G. N. Mohammed, Adel Abdulrahman, Member, IAENG

Abstract- The problem of the management of water resources is more and more important on a world scale. In particular, there is a requirement for novel concepts helping to solve the water management problem i.e. numerically efficient tools supporting optimal design (redesign) process for water networks.

The paper deals with the simulation of network remodeling and demonstrate how virtual distortion generated a chosen branch (e.g. in the branch No.4) can simulate the network modification due to total blocking flow in this branch. To this end, the condition of flow vanishing in the branch under remodelling should be postulated, where the resultant state of flow redistribution is calculated from the formulas superposing linear response of the original network configuration and the component induced by unknown virtual distortion. Then making use of the analytical network model (cf. Refs. 3, 1) of this installation and using presented below, the so-called Virtual Distortion Method (VDM), simulation of network remodeling can be performed.

Key Words: Water networks, simulation, remodeling, VDM based design.

I. INTRODUCTION

Global demand for water is continuously increasing due to population growth, industrial development, and improvements of economic conditions, while accessible sources keep decreasing in number and capacity, moreover, the applications involving manipulation and transport of water and fluids in general demand high power consumption. The optimal use of such water supply networks seems to be the best solution for the present and thus it is necessary to carefully manage water transfer [9, 10].

The proposed approach is based on continuous observation of the pressure distribution in nodes of the water network. Having a reliable (verified versus field tests) numerical model of the network and its responses for determined inlet and outlet conditions, any modifications to the normal network response (pressure distribution) can be detected. Then, applying proposed bellow numerical procedure, the correction of water supply can be determined.


II. DEFINITION AND LINEAR ANALYSIS

Let us describe the network analysis (cf. Ref.4) based approach to modelling of water systems using oriented graf of small example shown in Fig. 1, with topology defined by the following incidence matrix:

\[
L = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 \\
-1 & 0 & 1 & 1 & 0 \\
0 & -1 & 0 & -1 & 1 \\
0 & 0 & -1 & 0 & -1 \\
\end{bmatrix}
\]

(1)

where rows correspond to the network’s nodes while columns correspond to the branches.
Defining the following quantities describing the state of the water network:

\( H \) – the vector of water head in network’s nodes

\( \varepsilon \) - the vector of pressure head in network’s branches

\( Q \) – the vector of water flow in network’s branches

\( R \) – the vector of hydraulic compliance in network’s branches (depends on pipes’ cross-sections, length, material, etc.)

\[ R = \frac{K^2}{L}, \]

where:

- \( K \) - the characteristic of the element,
- \( L \) - the element’s length,
- \( H \) – denotes the water pressure in the node (height of water)
- \( q \) – denotes the flow in the branch,

and it was assumed that the network is supplied only through the node No.1 (inlet with intensity \( q_1 \)) and the only outlet is through the node No.4 (the coefficient \( R_4 \) = 1).

\( R_4 = R_5 = 0 \), what means, that the outlets in nodes No.2 and 3 vanish.

### III. VDM-BASED SIMULATION OF PARAMETER MODIFICATION

Analogously to the Virtual Distortion Method (VDM) applicable to the truss structures (cf.[3]) let us postulate that local modification of a network parameter can be introduced into the system through the virtual distortion \( \varepsilon^0 \), incorporated into the formula (4):

\[ LR(L^TH - \varepsilon^0) = q \]  

The virtual distortion \( \varepsilon^0 \) is of the same character as the pressure head \( \varepsilon_i \) (see Fig. 2) and its physical meaning is an additional pressure head externally forced in branch “i” (e.g. due to a locally installed pump).
The influence of virtual distortions on the resultant flow redistribution can be calculated using the so-called influence matrix $D_0$ collecting $i$ responses (row-wise) in terms of water heads $H_{i}^{\varepsilon=0}$ induced in the network by imposing the unit virtual distortion $\varepsilon_{j}^{0} = 1$ generated consecutively in each network branch $j$. Thus each influence vector $H_{i}^{\varepsilon=0}$ can be calculated on the basis of the following equation obtained from Eq. (7):

$$L \cdot R \cdot L^T \cdot H_{i}^{\varepsilon=0} = q^* + LRI$$  \hspace{1cm} (8)

The vector $q^*$ disregards the external inlet and outlet (the flow is now provided by the imposition of virtual distortion), and it accounts for the water flow distribution in the closed network (cf. Eq. (6)). There is a set of $j$ (i.e. outlet in node No.4), takes the following form:

$$H_{i} = H_{i}^{I} + H_{i}^{R} = H_{i}^{I} + \sum_{j} D_{ij}^{\varepsilon=0} \varepsilon_{j}^{0}$$  \hspace{1cm} (9)

and the resultant water flow as:

$$Q_{j} = Q_{j}^{I} + Q_{j}^{R} = Q_{j}^{I} + R_{j} \cdot L_{j}^T \cdot \sum_{j} (D_{ij}^{\varepsilon=0} - \delta_{ij}) \varepsilon_{j}^{0}$$  \hspace{1cm} (10)

The analogous set of relations governs the VDM based approach to modifications of truss structure system [3].

Coming back to the example shown in Fig. 2, let us generate the unit virtual distortion in branch No. 4, connecting the nodes Nos. 2 & 3. The corresponding set of equations (8), accounting for boundary conditions (i.e. outlet in node No.4), takes the following form:

$$\begin{bmatrix}
R + R_{4} & -R_{4} & -R_{4} & 0 \\
-R_{4} & R + R_{4} + R_{4} & -R_{4} & -R_{4} \\
-R_{4} & -R_{4} & R + R_{4} + R_{4} & -R_{4} \\
0 & -R_{4} & -R_{4} & R + R_{4} + R_{4}
\end{bmatrix} \begin{bmatrix}
H_{i}^{\varepsilon=0} \\
H_{i}^{\varepsilon=0} \\
H_{i}^{\varepsilon=0} \\
H_{i}^{\varepsilon=0}
\end{bmatrix} = \begin{bmatrix}
0 \\
-R_{4}^{\varepsilon=0} \\
R_{4}^{\varepsilon=0} \\
0
\end{bmatrix}$$  \hspace{1cm} (11)

where $\varepsilon_{4}^{0} = 1$. Assuming the following data: $K_1 = 0.2 \text{ m}^3/\text{s}$, $K_2 = K_3 = K_4 = K_5 = 0.4 \text{ m}^3/\text{s}$, $l_1 = l_2 = l_3 = l_5 = 10.000 \text{ m}$, $l_4 = 14.142 \text{ m}$, $q_1 = 0.050 \text{ m}^3/\text{s}$, $H_0 = 0.000 \text{ m}$, we get the following set of equations for the water head distribution:

$$\begin{bmatrix}
0.02 & -0.004 & -0.016 & 0 \\
-0.004 & 0.031 & -0.011 & -0.016 \\
-0.016 & -0.011 & 0.043 & -0.016 \\
0 & -0.016 & -0.016 & 10.032
\end{bmatrix} \begin{bmatrix}
H_{4}^{\varepsilon=0} \\
H_{4}^{\varepsilon=0} \\
H_{4}^{\varepsilon=0} \\
H_{4}^{\varepsilon=0}
\end{bmatrix} = \begin{bmatrix}
0 \\
-0.011 \\
0.0011 \\
0
\end{bmatrix}$$  \hspace{1cm} (11a)

The resulting distribution of water heads $H_{4}^{\varepsilon=0} = [0.151, -0.251, 0.251, 0.000]^T$ constitutes the 4th column of the influence matrix $D$. Continuing this procedure for virtual distortions generated in other branches, the full influence matrix can be determined as:

Taking into account relation (3) and applying it consecutively to each influence vector $H_{j}^{\varepsilon=0}$, another influence matrix $D'$ can be created, collecting the response to unit virtual distortions in terms of the pressure head $\varepsilon_{j}^{0}$:

$$D' = \begin{bmatrix}
0.314 & 0.686 & -0.284 & -0.402 & 0.284 \\
0.172 & 0.828 & 0.071 & 0.101 & -0.071 \\
0.142 & -0.142 & -0.355 & 0.503 & 0.355 \\
0.071 & -0.071 & 0.322 & -0.251 & 0.678
\end{bmatrix}$$  \hspace{1cm} (12)

VI. SIMULATION OF NETWORK REMODELLING (ELEMINATION OF BRANCH)

First, let us demonstrate how virtual distortion generated a chosen branch (e.g. in the branch No.4) can simulate the network modification due to total blocking flow in this branch. To this end, the condition of flow vanishing in the branch under remodelling ($Q_4=0$) should be postulated, where resultant state of flow redistribution is calculated from the formulas superposing linear response of the original network configuration and the component induced by unknown virtual distortion:

$$\varepsilon_{i} = \varepsilon_{i}^{L} + \sum_{j} D_{ij}^{\varepsilon=0} \varepsilon_{j}^{0}$$

$$Q_{i} = Q_{i}^{L} + R_{j} \cdot \sum_{j} (D_{ij}^{\varepsilon=0} - \delta_{ij}) \varepsilon_{j}^{0}$$  \hspace{1cm} (13)

Therefore, the virtual distortion to be generated in branch No.4 to simulate complete blocking of local flow can be calculated from the following condition:

$$Q_{4} = Q_{4}^{L} + R_{4} \cdot (D_{44}^{\varepsilon=0} - 1) \varepsilon_{4}^{0} = 0$$

or making use of (4)

$$\varepsilon_{4}^{L} + D_{44}^{\varepsilon=0} \varepsilon_{4}^{0} = \varepsilon_{4}^{0}$$

what leads to:

$$\varepsilon_{4}^{L} = -\frac{\varepsilon_{4}^{0}}{D_{44}^{\varepsilon=0} - 1} = 1.34 \text{ m}$$  \hspace{1cm} (14)
Finally the pressure head as well as the flow in modified network is (after substitution value (14) to relations (6)) as the following:

\[
\begin{align*}
\epsilon_1 &= \epsilon_4^1 + D_4^1 \epsilon_4^0 = 3.04\, \text{m}, \quad \epsilon_2 = 2.365\, \text{m}, \quad \epsilon_3 = 1.225\, \text{m}, \\
\epsilon_4 &= 1.9 + 0.248\, \text{m}, \\
\epsilon_5 &= 0.0143\, \text{m}, \\
\end{align*}
\]

and the flows:

\[
\begin{align*}
Q_1 &= Q_4^1 + R_1 D_4^1 \epsilon_4^0 = 0.01216 + 0.004\times 0.396\times 1.34 = 0.0143\, \text{m}^3/\text{s}, \\
Q_2 &= Q_4^2 + R_2 D_4^2 \epsilon_4^0 = 0.03784 + 0.016\times (-0.099)\times 1.34 = 0.0357\, \text{m}^3/\text{s}, \\
Q_3 &= Q_4^3 + R_3 D_4^3 \epsilon_4^0 = 0.0196 + 0.016\times (-0.247)\times 1.34 = 0.0143\, \text{m}^3/\text{s}, \\
Q_5 &= Q_4^5 + R_5 D_4^5 \epsilon_4^0 = 0.0304 + 0.016\times 0.248\times 1.34 = 0.0357\, \text{m}^3/\text{s},
\end{align*}
\]

For comparison, let us solve the set of equations (13)
taking into consideration excluding the element No. 4 (i.e. assuming \(R_4 = 0\) and disregarding column 4 in the matrix \(L\)) one can get the following set of equations:

\[
\begin{bmatrix}
0.020 & -0.004 & -0.016 & 0.000 & H_1 \\
-0.004 & 0.02 & 0.000 & -0.016 & H_2 \\
-0.016 & 0.000 & 0.032 & -0.016 & H_3 \\
0.000 & -0.016 & -0.016 & 1.032 & H_4
\end{bmatrix} = \begin{bmatrix}
0.05 \\
0.00 \\
0.00 \\
0.00
\end{bmatrix}
\]

The resulting distribution of water head is: \(H' = [4.514 \ 0.943 \ 2.282 \ 0.05]^T\), which leads to the following state of pressure head as well as the flow in modified network (after substitution \(H'\) to (3) and (13)):

\[
\begin{align*}
\epsilon_1 &= H'_1 - H'_2 = 4.514 - 0.943 = 3.57\, \text{m}, \\
\epsilon_2 &= H'_1 - H'_3 = 4.514 - 2.282 = 2.23\, \text{m}, \\
\epsilon_3 &= H'_2 - H'_4 = 0.943 - 0.05 = 0.89\, \text{m}, \\
\epsilon_4 &= H'_1 - H'_4 = 2.282 - 0.05 = 2.23\, \text{m},
\end{align*}
\]

and the flows:

\[
\begin{align*}
Q_1 &= Q_4^1 + R_1 D_4^1 \epsilon_4^0 = 0.01216 + 0.004\times 0.396\times 1.34 = 0.0143\, \text{m}^3/\text{s}, \\
Q_2 &= Q_4^2 + R_2 D_4^2 \epsilon_4^0 = 0.03784 + 0.016\times (-0.099)\times 1.34 = 0.0357\, \text{m}^3/\text{s}, \\
Q_3 &= Q_4^3 + R_3 D_4^3 \epsilon_4^0 = 0.0196 + 0.016\times (-0.247)\times 1.34 = 0.0143\, \text{m}^3/\text{s}, \\
Q_5 &= Q_4^5 + R_5 D_4^5 \epsilon_4^0 = 0.0304 + 0.016\times 0.248\times 1.34 = 0.0357\, \text{m}^3/\text{s},
\end{align*}
\]

States (3) as well as (4) are the same, what demonstrates that virtual distortion (14) models properly the assumed modification.

Multiplying the network response \(H' = [-0.151 \ 0.251 \ -0.251 \ 0.000]^T\) for the unit virtual distortion \(\epsilon_4^0 = 1\) by the determined above value, \(\epsilon_4^0\) the searched correction to the linear response distribution in the original network can be calculated (Fig.3c) and the resultant pressure distribution for the modified network: \(H = H' + 1.34\) \(H'\) can be also determined (Fig.3b). Similarly, other, various types of the network modifications can be simulated through virtual distortions using determined once the initial system matrix, the linear response \(H'\) and the influence matrix \(D'\).

Fig.3 Pressure distributions for the original (a), locally distorted (b) and modified (c) networks

For large water networks with small, local modifications the above VDM based approach is much cheaper numerically than the classical way through recomposing and solving the modified system.

In the case of nonlinear problem formulation a superposition of two virtual distortion fields has to be taken into consideration. The first one, \(\epsilon^0\) modeling system redesign and the second, \(\beta^0\) modeling physical nonlinearity of the system (cf. § 3).

V. CONCLUSION

The main advantages of the VDM based approach to the water network analysis WATNET-M are the reduction of numerical costs and avoidance of iteration due to incremental approach in the analysis of water network. The numerical cost of linear analysis consists (in WATNET-M) of:

- Solving the linear problem (7).
- Composing the influence matrix \(D'\) (12).
REFERENCES