Modified Harmony Search Algorithm for the Capacitated Vehicle Routing Problem

Tantikorn Pichpibul, Ruengsak Kawtummachai

***Abstract*—This paper presents the modification of a harmony search algorithm (HS) for the capacitated vehicle routing problem (CVRP). The objective is to find a feasible set of vehicle routes that minimizes the total traveling distance and the number of vehicles used. The modified HS has two stages. First, the probabilistic Clarke-Wright savings algorithm was incorporated into harmony memory mechanism to improve its initial solution. Second, the roulette wheel selection procedure was employed into new harmony improvisation mechanism to improve its selection process. Computational results on the well-known CVRP benchmark problems show that the modified HS is competitive to the best existing algorithms.**

***Index Terms*—Vehicle Routing Problem, Harmony Search Algorithm, Clarke-Wright Savings Algorithm, Optimization.**

I. Introduction

THE purpose of this paper is to develop a new approach for harmony search algorithm (HS) to solve the capacitated vehicle routing problem (CVRP) which is the first variant of the well-known vehicle routing problem introduced by Dantzing and Ramser [1]. It is known to be NP-hard problem [2] which is the combination between the traveling salesman problem and the bin packing problem. In Fig. 1, an example of the CVRP consists of one depot which is assigned to be number 0, five customers which are assigned to be number 1 to 5 and two vehicle routes. Each route represents a sequence of customers served by a vehicle. Both vehicles are homogeneous fleet and always depart from the depot. After completing the service, both vehicles must return to the depot as shown in Fig. 1 that the routes are 0-1-2-0 and 0-3-4-5-0. The objective of this paper is to construct the vehicle routes that minimize the total traveling distance and the number of vehicles used. The constraints of the CVRP are as follows.

--Each customer must be served once by one vehicle.

--All vehicles start and end at the depot.

--The customer demand in each vehicle cannot exceed the capacity.

--The vehicles used in each solution cannot exceed the number of available vehicles.

Since the CVRP was proposed in 1959, many algorithms were developed to solve the problem consisting of an exact algorithm (branch-and-bound algorithm [3], branch-and-cut algorithm [4], branch-and-cut-and-price algorithm [5]), a heuristic algorithm (Clarke-Wright savings algorithm [6], sweep algorithm [7]), and a metaheuristic algorithm (simulated annealing [8], tabu search [9], genetic algorithm [10], ant colony optimization algorithm [11], particle swarm optimization algorithm [12]).

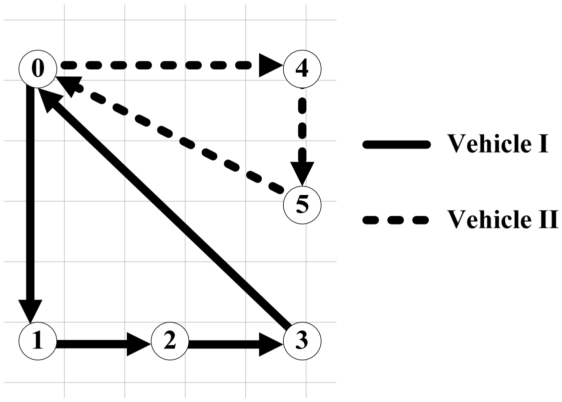

Fig. 1. An example of the CVRP.

Although the algorithms that we have mentioned above can find optimal or near-optimal solutions, the development of new algorithms, which can produce better solutions in less iteration, has still received attention from the researchers. Due to the successful applications to various optimization problems [13][14], we therefore studied the HS which is one of recent algorithms. In addition, we also modified the HS to suitably solve the CVRP in this paper.

II. Harmony Search Algorithm

In 2001, the harmony search algorithm (HS) was originally proposed by Geem et al. [15]. It is simulated by the process of music improvisation that an example is shown in Figs. 2 to 4. The illustration in Fig. 2 presents a group of musicians playing musical instruments together. Each musician sounds any pitch within the possible range of notes (Do, Re, Mi, Fa, Sol, La, Ti) in three times. The composition of notes in each time including (Sol, Mi, Do), (La, Fa, Re), and (Do, Sol, Mi) makes three harmony vectors which are kept in the harmony memory (HM). Each harmony vector is estimated by an aesthetic standard. In the improvisation process, a new harmony vector is made by using memory consideration, pitch adjustment and random selection. Fig. 3 shows an example of the new harmony improvisation. In Fig. 3 (a) the notes in the first instrument (Cello) of three harmony vectors from the HM are chosen to be new note (La) with equal probability (33.3%) by the

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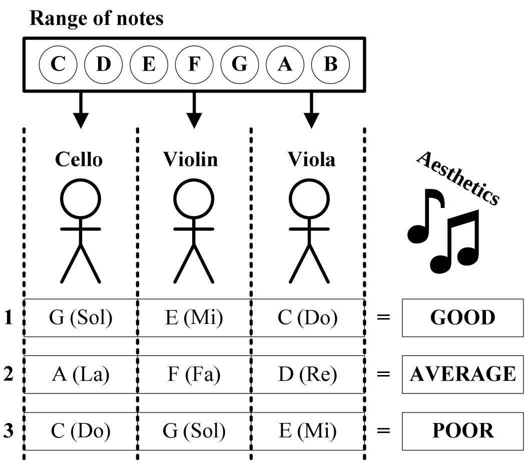
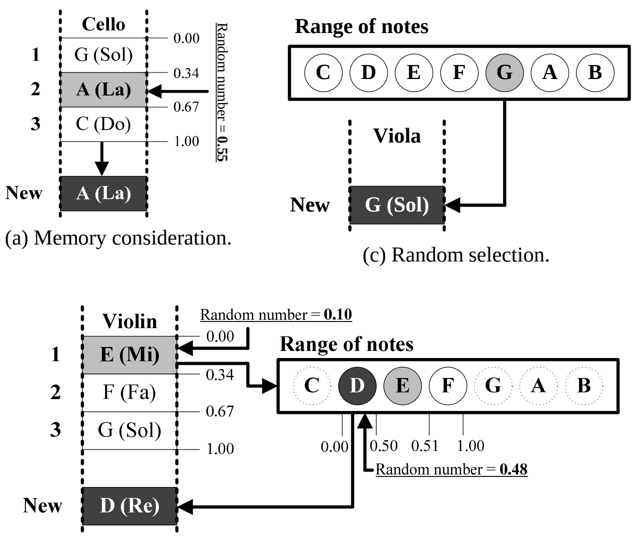


Fig. 2. An example of the harmony memory.

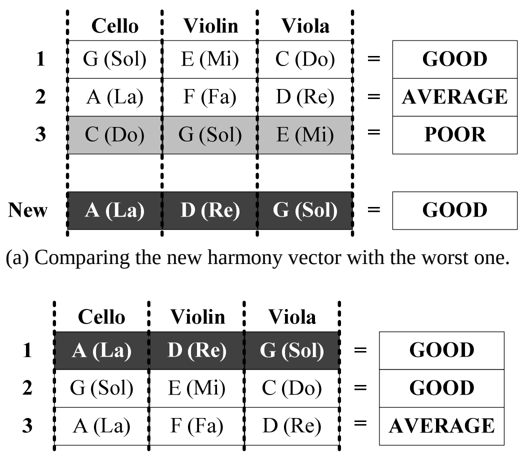


(a) Memory consideration.

(c) Random selection.

(b) Pitch adjustment.

Fig. 3. An example of the new harmony improvisation.



(a) Comparing the new harmony vector with the worst one.

(b) Updating the new harmony vector instead of the worst one.

Fig. 4. An example of the harmony memory update.

memory consideration with a probability of harmony memory considering rate (HMCR). In Fig. 3 (b), after the new note (Mi) in the second instrument (Violin) is chosen from the memory consideration, it is shifted to be new note (Re), which is neighboring notes composed of an upper note (Re) and a lower note (Fa), within the possible range of notes (Do, Re, Mi, Fa, Sol, La, Ti) with equal probability (50%) by the pitch adjustment with a probability of pitch adjusting rate (PAR). In Fig. 3 (c), new note (Sol) in the third instrument (Viola) is chosen within the possible range of notes (Do, Re, Mi, Fa, Sol, La, Ti) by the random selection with a probability of 1-HMCR. If the new harmony vector (La, Re, Sol) is better than the worst harmony vector (Do, Sol, Mi) in the HM as shown in Fig. 4 (a), the new harmony vector is updated to the HM instead of the worst one as shown in Fig. 4 (b). This process is repeated until a perfect harmony is found by the musicians.

III. Modified Harmony Search Algorithm for the Capacitated Vehicle Routing Problem

The process of music improvisation described in Section II can be modified to solve the capacitated vehicle routing problem (CVRP) that the objective is to find the optimum or near-optimum solution of the vehicle routes iteration by iteration. In this paper, the procedures of the HS are explained and applied to the CVRP as following Subsections.

A. Initializing the HS parameters

The HS includes some parameters which are described as follows:

--Harmony memory (HM) is a set of harmony vectors that the best harmony vector is ranked at the first vector and the others are sorted in the increasing order based on the aesthetic values.

--Harmony memory size (HMS) is the number of harmony vectors in the HM.

--Harmony vector is a set of notes of all musical instruments.

--Range of notes is a set of customers who require delivery of goods from the depot.

--Note is customer's identity to serve by the vehicle.

--Musical instrument is the sequence of customer for delivery.

--Aesthetic value is the total traveling distance between notes in each harmony vector.

--Harmony memory considering rate (HMCR) is a probability number to choose a note from the HM for the memory consideration as shown in Fig. 3 (a). For the random selection as shown in Fig. 3 (c), the probability of 1-HMCR is used to choose a random note within the possible range of notes.

--Pitch adjusting rate (PAR) is a probability number to choose a neighboring notes from note of the memory consideration within the possible range of notes as shown in Fig. 3 (b).

--Practice time is the number of iterations which is set as the stopping condition.

B. Initializing the HM

In order to suitably solve the CVRP by using the HS, we therefore modify the structures of the HM according to the HS parameters as shown in Fig. 5. The CVRP routes from Fig. 1 are presented as an example solution by using the customers' data from Tables I and II including location, demand, and Euclidean distance matrix. We assume five

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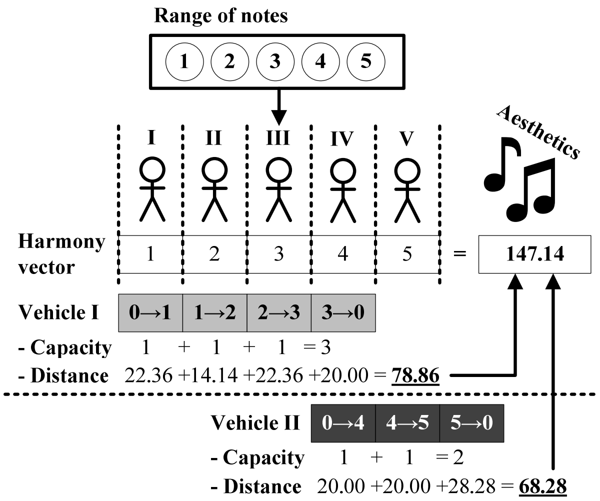

Fig. 5. An example of the harmony memory for the CVRP.

TABLE I
CUSTOMERS' LOCATION AND DEMAND

Customer	Location (X, Y)	Demand
0	(0, 0)	0
1	(10, -20)	1
2	(20, -10)	1
3	(0, -20)	1
4	(20, 0)	1
5	(20, -20)	1

TABLE II
EUCLIDEAN DISTANCE MATRIX

Customer	0	1	2	3	4	5
0	-	22.36	22.36	20.00	20.00	28.28
1	22.36	-	14.14	10.00	22.36	10.00
2	22.36	14.14	-	22.36	10.00	10.00
3	20.00	10.00	22.36	-	28.28	20.00
4	20.00	22.36	10.00	28.28	-	20.00
5	28.28	10.00	10.00	20.00	20.00	-

customers which are served by two vehicles. The number of notes in each harmony vector is also equal to five based on the number of customers. In each harmony vector, there are five integer numbers between one and five in order to identify each customer. Notice that, the customer's identity in each note must be distinct due to the CVRP constraint that each customer must be served once by one vehicle. As shown in Fig. 5, if we suppose each customer demand to be equal to one and each vehicle capacity to be equal to three, the aesthetic standard of the HS for the CVRP is calculated by considering the composition of notes in each harmony vector. For the harmony vector (1, 2, 3, 4, 5), we assign the first place to start from the depot (represented by customer 0), and then assign the customer 1 in the first note to be the next place. The Euclidean distance between depot and customer 1, which is equal to 22.36, is chosen from Table II, and the demand of customer 1, which is equal to one, is added to the first vehicle. We repeat this process until the remaining capacity of the first vehicle is equal to zero or less than the demand of the next remaining customer due to the CVRP constraint that the customer demand in each vehicle cannot exceed the capacity. For the first vehicle, the customer 3 in the third note is the last customer that we assign to return to the depot due to the CVRP constraint that all vehicles start and end at the depot. In the fourth note, we start the second vehicle from the depot and then follow by the customer 4. Finally, the customer 5 is assigned to be the last customer for the second vehicle before return to the depot. Hence, both routes are constructed as (0-1-2-3-0) in which the traveling distance is equal to 78.86, and (0-4-5-0) in which the traveling distance is equal to 68.28. The total traveling distance of both routes is calculated as the aesthetic value for this harmony vector.

In general, the HS always randomly generates each harmony vector to collect in the HM. In addition, we generate one more harmony vector by using an improved Clarke and Wright savings algorithm proposed by Pichpibul and Kawtummachai [16] in order to make a better chance to reach the best harmony vector.

C. Improvising a new harmony vector

A new harmony vector is generated by using memory consideration, pitch adjustment and random selection that we applied the roulette wheel selection which is one of an operator in the genetic algorithm [17] to improve the selection process. First, in the general memory consideration, the customers in each sequence of all harmony vectors in the HM are chosen to be a new customer with equal probability. In contrast, we use the probabilities created by the roulette wheel selection process. For harmony vector number h with total traveling distance td_h, its selection probability p_h and cumulative probability q_h are calculated as:

$$p_h = \frac{td_h}{\sum_{i \in H} td_i} \quad \text{for } h \in H \tag{1}$$

$$q_h = \sum_{i \in h} p_i \quad \text{for } h \in H \tag{2}$$

Here, H is the HMS. The selection process starts by spinning the roulette wheel with a random number r from the range between 0 and 1. If $r \leq q_1$, then choose the customer from the first harmony vector td_1; otherwise, choose the customer from the hth harmony vector td_h ($2 \leq h \leq H$). Note that, the customers who are chosen by the roulette wheel selection process are discarded from the next selection process in order to avoid the duplicate customer. Second, in the general pitch adjustment, after the new customer is chosen from the memory consideration, it is shifted to be new customer, which is neighboring customers composed of an upper customer and a lower customer with equal probability. In contrast, we consider the neighboring customers by ranking the distances between the new customer and the others. Finally, in the general random selection, the new customer is chosen within the possible range of customers. In contrast, we also use the probabilities created by the roulette wheel selection process. For customer number c with distance from the new customer d_c, its selection probability p_c and cumulative probability

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q_c are calculated as:

$$p_c = \frac{d_c}{\sum_{i \in H} d_i} \text{ for } c \in C \quad (3)$$

$$q_c = \sum_{i \in c} p_i \text{ for } c \in C \quad (4)$$

Here, C is the number of available customers (which is unassigned to be the new customer in the new harmony vector). The selection process starts by spinning the roulette wheel with a random number r from the range between 0 and 1. If $r \le q_1$, then choose the customer from the first customer d_1; otherwise, choose the customer from the c th customer d_c ($2 \le c \le C$).

D. Updating the HM

The new harmony vector replaces the worst harmony vector in the HM if the total traveling distance of the new harmony vector is less than the worst one. After that, all harmony vectors in the HM are sorted in the increasing order based on the total traveling distance. Note that, the best harmony vector which represents the best CVRP solution is always ranked at the first position in the HM.

E. Stopping condition

The procedures of the HS are continued until the stopping condition represented by the number of iterations is satisfied.

IV. Computational Results

The modified HS was coded in Visual Basic 6.0 on an Intel® Core™ i7 CPU 860 clocked at 2.80 GHz with 1.99 GB of RAM under Windows XP platform. Before the execution, we have preset the HS parameters (including HMS = 50, HMCR = 0.95, PAR = 0.5, Practice time = 1000). The numerical experiment used well-known CVRP benchmark problems composed of 71 instances [18][19][20] which are available online at http://www.coin-or.org/symphony/branchandcut/vrp/data. In each instance, we use the same nomenclature, consisting of a data set identifier, followed by n which represents the number of customers (including depot), and k which represents the number of available vehicles. We discuss each CVRP benchmark problem in which the percentage deviation between the optimal solution (OPT) and the obtained solution (OBT) is calculated as follows:

$$\text{Percentage deviation} = \left(\frac{obt - opt}{opt} \right) \times 100 \quad (5)$$

The numerical results are shown in Table III. Out of 71 problems, we found the optimal solutions, which are highlighted by using bold type, for all problems with up to 135 customers. These indicate that the proposed HS is very effective and efficient in producing high quality solutions for well-known CVRP benchmark problems.

V. Conclusions

In this paper, we have presented a modified harmony search algorithm to solve the capacitated vehicle routing problem (CVRP). We have modified the harmony search algorithm (HS) by two procedures consisting of the probabilistic Clarke and Wright savings algorithm and the roulette wheel selection procedure. We also have done the experiments by using the well-known CVRP benchmark problems composed of 71 instances obtained from the literatures, and have compared them with the optimal solutions.

The computational results show that the proposed HS is competitive to the best existing algorithms in terms of solution quality. It generates the optimal solution in all of the tested instances. Therefore, we can conclude that the performance of our algorithm is excellent while comparing with other algorithms in each instance.

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TABLE III
COMPUTATIONAL RESULTS

No	Instance	Optimal solution	Percentage deviation with the optimal solution (%)						
			Ganesh and Narendran [21]	Juan et al. [22]	Groër et al. [23]	Ai and Kachitvichyanukul [24]	Kim and Son [25]	Moghaddam et al. [26]	Pichpibul and Kawtummachai
1	A-n32-k5	784	**0.000**	**0.000**	-	**0.000**	-	**0.000**	**0.000**
2	A-n33-k5	661	**0.000**	**0.000**	-	**0.000**	**0.000**	**0.000**	**0.000**
3	A-n33-k6	742	**0.000**	**0.000**	-	**0.000**	-	**0.000**	**0.000**
4	A-n34-k5	778	**0.000**	-	-	**0.000**	-	**0.000**	**0.000**
5	A-n36-k5	799	**0.000**	-	-	**0.000**	-	**0.000**	**0.000**
6	A-n37-k5	669	**0.000**	**0.000**	-	**0.000**	-	**0.000**	**0.000**
7	A-n37-k6	949	**0.000**	-	-	**0.000**	-	**0.000**	**0.000**
8	A-n38-k5	730	**0.000**	**0.000**	-	**0.000**	-	**0.000**	**0.000**
9	A-n39-k5	822	**0.000**	-	-	**0.000**	-	**0.000**	**0.000**
10	A-n39-k6	831	**0.000**	**0.000**	-	-	-	-	**0.000**
11	A-n44-k6	937	**0.000**	-	0.427	0.320	-	**0.000**	**0.000**
12	A-n45-k6	944	**0.000**	**0.000**	0.424	-	-	-	**0.000**
13	A-n45-k7	1146	**0.000**	**0.000**	-	-	-	-	**0.000**
14	A-n46-k7	914	**0.000**	-	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
15	A-n48-k7	1073	**0.000**	-	**0.000**	-	-	-	**0.000**
16	A-n53-k7	1010	0.693	-	**0.000**	-	-	-	**0.000**
17	A-n55-k9	1073	**0.000**	**0.000**	-	-	-	-	**0.000**
18	A-n60-k9	1354	0.295	**0.000**	-	0.074	**0.000**	**0.000**	**0.000**
19	A-n61-k9	1034	0.387	**0.000**	-	-	-	-	**0.000**
20	A-n63-k9	1616	0.681	**0.000**	-	-	-	-	**0.000**
21	A-n65-k9	1174	0.256	**0.000**	-	-	-	-	**0.000**
22	A-n80-k10	1763	0.964	**0.000**	-	-	-	-	**0.000**
23	B-n31-k5	672	**0.000**	**0.000**	-	**0.000**	-	**0.000**	**0.000**
24	B-n34-k5	788	**0.000**	-	-	**0.000**	-	**0.000**	**0.000**
25	B-n35-k5	955	**0.000**	**0.000**	-	**0.000**	**0.000**	**0.000**	**0.000**
26	B-n38-k6	805	**0.000**	-	-	**0.000**	-	**0.000**	**0.000**
27	B-n39-k5	549	**0.000**	**0.000**	-	**0.000**	-	**0.000**	**0.000**
28	B-n41-k6	829	**0.000**	**0.000**	1.206	**0.000**	-	**0.000**	**0.000**
29	B-n43-k6	742	**0.000**	-	**0.000**	**0.000**	-	**0.000**	**0.000**
30	B-n44-k7	909	**0.000**	-	**0.000**	0.330	-	**0.000**	**0.000**
31	B-n45-k5	751	**0.000**	**0.000**	-	**0.000**	**0.000**	**0.000**	**0.000**
32	B-n45-k6	678	**0.000**	-	-	**0.000**	-	**0.000**	**0.000**
33	B-n50-k7	741	**0.000**	**0.000**	**0.000**	0.675	-	**0.000**	**0.000**
34	B-n50-k8	1312	0.457	-	-	**0.000**	-	**0.000**	**0.000**
35	B-n51-k7	1032	**0.000**	-	**0.000**	**0.000**	-	**0.000**	**0.000**
36	B-n52-k7	747	**0.000**	**0.000**	**0.000**	**0.000**	-	**0.000**	**0.000**
37	B-n56-k7	707	0.424	**0.000**	**0.000**	**0.000**	-	**0.000**	**0.000**
38	B-n57-k7	1153	3.290	-	-	**0.000**	-	**0.000**	**0.000**
39	B-n57-k9	1598	0.063	**0.000**	-	**0.000**	-	**0.000**	**0.000**
40	B-n63-k10	1496	0.936	-	-	0.201	-	**0.000**	**0.000**
41	B-n64-k9	861	0.348	**0.000**	2.671	0.232	-	0.232	**0.000**
42	B-n66-k9	1316	-	-	-	**0.000**	-	**0.000**	**0.000**
43	B-n67-k10	1032	0.484	**0.000**	-	0.194	-	0.291	**0.000**
44	B-n68-k9	1272	0.236	**0.000**	-	0.157	0.236	**0.000**	**0.000**
45	B-n78-k10	1221	3.194	**0.000**	-	0.164	0.164	**0.000**	**0.000**
46	P-n23-k8	529	**0.000**	-	-	**0.000**	-	**0.000**	**0.000**
47	P-n40-k5	458	**0.000**	**0.000**	-	**0.000**	-	**0.000**	**0.000**
48	P-n45-k5	510	**0.000**	-	-	**0.000**	-	**0.000**	**0.000**
49	P-n50-k7	554	**0.000**	-	-	**0.000**	-	**0.000**	**0.000**
50	P-n50-k8	631	1.902	**0.000**	-	**0.000**	-	**0.000**	**0.000**
51	P-n50-k10	696	**0.000**	**0.000**	-	**0.000**	-	**0.000**	**0.000**
52	P-n51-k10	741	**0.000**	**0.000**	-	**0.000**	-	**0.000**	**0.000**
53	P-n55-k7	568	**0.000**	-	-	**0.000**	-	**0.000**	**0.000**
54	P-n55-k8	588	-	-	-	**0.000**	-	**0.000**	**0.000**
55	P-n55-k10	694	0.576	-	-	**0.000**	-	**0.000**	**0.000**
56	P-n55-k15	989	**0.000**	**0.000**	-	0.607	-	**0.000**	**0.000**
57	P-n60-k10	744	**0.000**	**0.000**	-	**0.000**	-	**0.000**	**0.000**
58	P-n60-k15	968	**0.000**	-	-	**0.000**	-	**0.000**	**0.000**
59	P-n65-k10	792	1.010	**0.000**	-	0.379	-	**0.000**	**0.000**
60	P-n70-k10	827	**0.000**	**0.000**	-	**0.000**	-	**0.000**	**0.000**
61	P-n76-k4	593	**0.000**	**0.000**	-	0.169	0.169	**0.000**	**0.000**
62	P-n76-k5	627	**0.000**	**0.000**	-	**0.000**	-	0.319	**0.000**
63	P-n101-k4	681	0.881	**0.000**	-	0.294	0.294	1.028	**0.000**
64	E-n30-k3	534	**0.000**	**0.000**	-	**0.000**	**0.000**	**0.000**	**0.000**
65	E-n51-k5	521	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
66	E-n76-k7	682	1.173	**0.000**	-	0.733	0.733	**0.000**	**0.000**
67	E-n76-k10	830	4.458	**0.000**	-	-	-	-	**0.000**
68	E-n76-k14	1021	1.077	**0.000**	-	-	-	-	**0.000**
69	F-n45-k4	721	-	**0.000**	-	-	-	-	**0.000**
70	F-n72-k4	237	-	**0.000**	-	**0.000**	**0.000**	**0.000**	**0.000**
71	F-n135-k7	1159	-	**0.000**	-	0.518	0.949	0.690	**0.000**

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