A Study of Modeling the Simultaneous Berth and Crane Allocation Problem

Sheng-Yuan Shen, and Fang-Shen Ko

***Abstract*—How to efficiently assign vessels to berths and how to allocate limited cranes to serve each vessel at each berth are vital decisions to be made in operating container terminals. This paper same as previous works assumes that quay cranes to be transferred are operated in a push-in and pull-out way, and can not cross each other. However, previous works do not consider all possible ways of transferring cranes in their model and algorithms. This paper first discusses conditions for feasibly transferring cranes between berths, followed by providing a complete mixed integer programming model. Though to optimally solve the MIP model directly is almost impossible for large size problems, the fundamental insights revealed in modeling the problem can provide valuable thinking in developing approximate solution method.**

***Index Terms*—berth allocation, crane transfer, mixed integer programming, transfer path**

I. Introduction

Due to the economic efficiency of containerization using a unit-load concept, sea container service has dramatically grown in an average of 7-9% per year during the past two decades [1] and nowadays it has been a primary means for international transport of sea freight. As an ever severe competitiveness among major hub seaports like Singapore, Hong Kong, Shanghai in Asia, Rotterdam and Hamburg in Europe is still ongoing, how to enhance the effectiveness and efficiency of relevant container terminal operations at the quayside area, the yard and the landside area has been the core theme of port authorities [2].

Many factors such as turn-around time for ship liners, crane utilization, crane productivity, and berth utilization can affect the success of container terminal operations [3]. Reducing turn-around time in a multi-user terminal is particularly crucial for both ship liners and terminal operators. After an arrival of a container ship at the port, the ship is moored at an assigned berth using quay cranes to unload and load containers. Thus, availability of berths and quay cranes has direct impact on the performance of a container terminal. However, berths and quay cranes are the two most expensive investments in a container terminal. Hence, for an existing infrastructure system of a container terminal, how to optimally allocate vessels to available berths and how to optimally assign limited number of quay cranes to vessels are the two central problems to be solved such that total turnaround time for serving all container ships is minimized.

A quay crane is a large quayside gantry crane having a supporting framework and a moving spreader. Because of its heaviness and size, a quay crane is mounted on a rail track alongside the quay to facilitate the movement among berths. Quay cranes to be transferred are operated in a push-in and pull-out way, and can not cross each other. This spatial limitation tends to constrain the use of limited cranes to efficiently serve ships in a less turn-around time.

Research on relevant issues in a container terminal has been growing quickly during the last decade. Regarding a general overview of container terminal operations and their optimization planning, we refer the readers to [2], [4], and [5]. As to review concentrated on the quayside operational decisions, Bierwirth and Meisel [6] recently gave a comprehensive classification on berth allocation problem (BAP), quay crane assignment problem (QCAP), quay crane scheduling problem (QCSP) and their respected partial or full integration problems, e.g. the integration of BAP and QCAP (denoted as BAP+QCAP). The BAP is to determine best berthing positions and berthing times for serving ships at berths (discrete BAP) or at arbitrary positions along the quay length if empty space is enough (continuous BAP). To avoid ambiguity, we remark that, as used by [6], the QCAP considers the allocation of specific quay cranes to each vessel while the QCSP is to determine the sequence of tasks (e.g. individual container, container group, ship-bay) to be operated for each crane assigned to a vessel. Moreover, static and dynamic version of the above problems can be differentiated respectively according to that all ships are ready at the start of planning and ships may arrive during the planning horizon. We focus on discussing issues resulting from the integration of discrete dynamic BAP and QCAP.

This paper considers the BAP and the QCAP simultaneously in a container terminal so that a berth schedule with the least total turnaround time, which includes the waiting and processing times for each ship, can be determined. In particular, we focus on discussing the non-crossing requirements among cranes. In the next Section, we formally describe our research problem. Section 3 discusses feasible conditions for moving quay cranes among berths/vessels. A mixed integer programming model is then formulated in Section 4. Finally, some conclusions are given in Section 5.

II. Problem Description

Because the duration of serving a ship depends on the number of cranes assigned to the ship, and the cranes can not move freely, investigating an integrated plan of the BAP and

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the QCAP is essential in order to lessen the probability of developing an infeasible berthing plan. Recently, Imai et al. [7] investigated the discrete dynamic BAP+QCAP, where the number of quay cranes required for each vessel is a given constant. Scheduling was on a continuous time axis too. In particular, Imai et al. [7] explicitly considered the non-crossing constraints on quay cranes in their mixed integer program (MIP) formulation, and proposed a genetic solution algorithm. However, their MIP model has several mistakes, and this paper addresses these issues. The objective is to minimize the total turnaround time, the sum of the difference between the departure time and the arrival time of each vessel. We first state the assumptions same as those considered by [7] below.

A1. The number of berths is given, and the service beginning time of each berth is not the same; a rolling berth schedule can thus be built from a practical viewpoint.
A2. The arrival time of each vessel during the planning horizon is given, and without interruption once a vessel is in service.
A3. Each berth can serve at most one ship anytime.
A4. Every vessel can be served at any one berth and has a favorable berthing position.
A5. Every vessel requires a specific number of cranes to start its service.
A6. Cranes are on the same rail track and can not cross each other. That is, cranes are moved in a push-in and pus-out way.
A7. Cranes can not move from berth *i* to berth *j* if any intermediate berth between *i* and *j* is busy in handling a vessel. Once cranes are assigned to a vessel, they can not be transferred to other berths until the vessel has finished all its workload.

Assumptions of A6-A7 impose non-crossing restrictions on transferring cranes among berths. On one hand, cranes can not be freely moved from berth *i* to berth *j* through intermediate berths between berths *i* and *j* except that no any other cranes are held in the intermediate berths. On the other hand, time invariant assignment of cranes is considered in our study; that is a vessel uses a fixed number of cranes throughout its service duration. Thus, certain conditions have to be satisfied in order to transfer cranes among berths. Imai et al.[7] discussed conditions to guarantee the feasibility of transferring cranes for building a mixed integer program model. However, they ignored several possible ways for transferring cranes. In the next Section, we give a detailed analysis about the feasibility conditions for transferring cranes.

III. Feasibility Conditions for Transferring Cranes

Fig. 1 (a) illustrates a situation for any two consecutive ships of each ordered pair of adjacent berths along a continuous time axis. For clearance and latter use, we add a node between two consecutive ship service positions, say *k* and *k*+1, of each berth as shown in Fig. 1(b); the node constructed between positions of *k* and *k*+1 will be labeled *k*. Our purpose is to determine certain conditions such that a feasible transfer arc can be built from node *k* to node k'. In other words, the existence of a feasible transfer arc from node *k* to node k' says that some of the cranes after finishing the service of the k^{th} ship of berth *i* may be feasibly moved to berth i' before the beginning time of serving the $(k'+1)^{th}$ ship at the berth, where i' can be $i-1$ or $i+1$. Due to the assumption of disallowing crane transfer from a berth which is currently serving a ship, we first discuss conditions to ensure the feasibility of moving cranes between two adjacent ordered berths, followed by discussing the case of between nonadjacent ordered berths. Due to limited space, we therefore skipped the proof for propositions 2 and 3 stated below.

A. The Case of Adjacent Ordered Berths

Let $t^s_{i,k}$ and $t^f_{i,k}$ be respectively the start and finish time for the ship served at the k^{th} position of berth *i*. Imai et al. [7] have discussed conditions required for moving cranes between two adjacent ordered berths using illustration shown in Figure 2; that is a feasible transfer arc from node *k* to node k' exists if one of the following conditions (a) $t^s_{i',k'} \le t^f_{i,k}$ & $t^f_{i',k'} > t^f_{i.k}$ or (b) $t^s_{i',k'+1} \ge t^f_{i,k}$ & $t^f_{i',k'} \le t^f_{i.k}$ is satisfied. Imai et al. [7] then formulated corresponding constraints for each above case separately in their mixed integer programming model. However, the above two cases is indeed a single case as stated in Proposition 1.

Proposition 1:

The conditions of either (a) $t^s_{i',k'} \le t^f_{i,k}$ & $t^f_{i',k'} > t^f_{i.k}$ or (b) $t^s_{i',k'+1} \ge t^f_{i,k}$ & $t^f_{i',k'} \le t^f_{i.k}$ is equivalent to $t^s_{i',k'+1} \ge t^f_{i,k} \ge t^s_{i',k'}$.

Proof:

$(\Rightarrow)$

From (a) $\left.\begin{matrix} t^s_{i',k'} \le t^f_{i,k} \\ t^f_{i',k'} > t^f_{i,k} \end{matrix}\right\} \Rightarrow t^s_{i',k'+1} \ge t^f_{i',k'} > t^f_{i,k} \ge t^s_{i',k'} \Rightarrow t^s_{i',k'+1} \ge t^f_{i,k} \ge t^s_{i',k'}$,

From (b) $\left.\begin{matrix} t^s_{i',k'+1} \ge t^f_{i,k} \\ t^f_{i',k'} \le t^f_{i,k} \end{matrix}\right\} \Rightarrow t^s_{i',k'+1} \ge t^f_{i,k} \ge t^f_{i',k'} \ge t^s_{i',k'} \Rightarrow t^s_{i',k'+1} \ge t^f_{i,k} \ge t^s_{i',k'}$,

$\therefore$ either (a) or (b) $\Rightarrow t^s_{i',k'+1} \ge t^f_{i,k} \ge t^s_{i',k'}$

$(\Leftarrow)$

$$t^s_{i',k'+1} \ge t^f_{i,k} \ge t^s_{i',k'} \Rightarrow \text{either } t^s_{i',k'+1} \ge t^f_{i,k} > t^f_{i',k'} \ge t^s_{i',k'}$$
$$\text{or} \quad t^s_{i',k'+1} \ge t^f_{i',k'} \ge t^f_{i,k} \ge t^s_{i',k'}$$

Thus $t^s_{i',k'} \le t^f_{i,k}$ & $t^f_{i',k'} > t^f_{i,k}$ or $t^s_{i',k'+1} \ge t^f_{i,k}$ & $t^f_{i',k'} \le t^f_{i,k}$
$\Leftrightarrow t^s_{i',k'+1} \ge t^f_{i,k} \ge t^s_{i',k'}$ QED

Therefore, condition of $t^s_{i',k'+1} \ge t^f_{i,k} \ge t^s_{i',k'}$ can guarantee the feasibility of transfer arc (k,k'). However, as shown in Fig. 3, feasible arc (k,k') is clearly existed but conditions considered by [7] will not allow this construction. Fig. 4 is an example of 3 berths for illustrating the case ignored by [7]. Note that berth service times are not equal. Considering only the sufficient conditions of Proposition 1, the optimal objective value in terms of the total time stayed in the container terminal is *123* minutes, as shown in Fig. 4a. The left vertical bar and right vertical bar of each red thick line represent respectively the start and completion service time of a ship, and the number along each red thick line is the number of cranes available for the ship. For each berth, a dummy ship with arrival time same as the berth ready time and zero service time is constructed for building possible transferring arcs; otherwise, the first ship of each berth can not use cranes transferred from other adjacent berths. The number with underline along the arrowed line is the number of cranes transferred. But a feasible solution with the objective value of *119* can be found as shown in Fig. 4b. This is because no feasible transfer arcs from ship 1 to ship 4 can be built due to the violation of sufficient conditions of $t^s_{i',k'+1} \geq t^f_{i,k} \geq t^s_{i',k'}$. However, it is obvious to see that the cranes after serving ship 1 can be feasibly moved for serving ship 4 if required.

In fact, a necessary and sufficient condition for ensuring the feasibility of transfer arc connected to node k' is that $t^s_{i',k'+1} \geq t^f_{i,k}$ is satisfied for at least one k as stated in Proposition 2(a). However, this loose condition will allow a bunch of transfer arcs to be built, in which lots of them are useless but complicating the problem to be solved. In the example of Fig. 5, cranes through transfer arcs of $(k-2,k')$, $(k-1,k')$ and (k,k') are all feasible, but cranes may be delayed and held in berth *i*, and then finally moved through arc (k,k') if necessary. Therefore, if feasible transfer arcs from an adjacent berth for supporting cranes of the $(k'+1)^{th}$ ship of berth i' do exist, we perhaps would like to construct at least one feasible arc for node k', otherwise feasible transfer between non-adjacent berths may not be allowed. Proposition 2(b) gives the sufficient conditions.

Proposition 2:

(a) There exists at least a feasible transfer arc from berth i to node k' of berth i' if and only if there exist a k such that $t^s_{i',k'+1} \geq t^f_{i,k}$.

(b) Conditions of (i) $t^s_{i',k'+1} \geq t^f_{i,k}$ & $t^f_{i,k} \geq t^s_{i',k'}$ or (ii) $t^s_{i',k'+1} \geq t^f_{i,k}, t^f_{i,k} < t^s_{i',k'}$ & $t^f_{i,k+1} > t^s_{i',k'+1}$ guarantee the existence of feasible transfer arc (k,k').

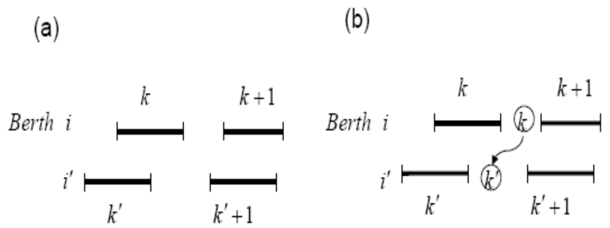

Fig. 1. An illustration of how nodes are constructed

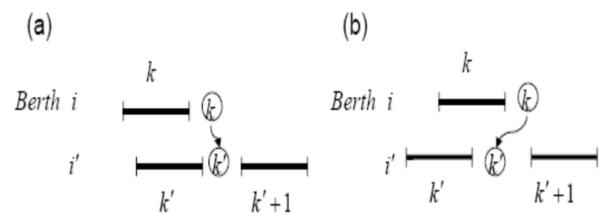

Fig. 2. Possible transfer cases considered by Imai et al

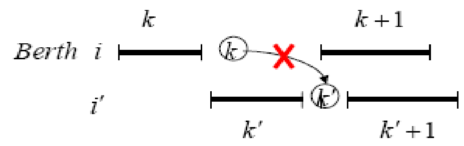

Fig. 3. A transfer arc not allowed by proposition 1

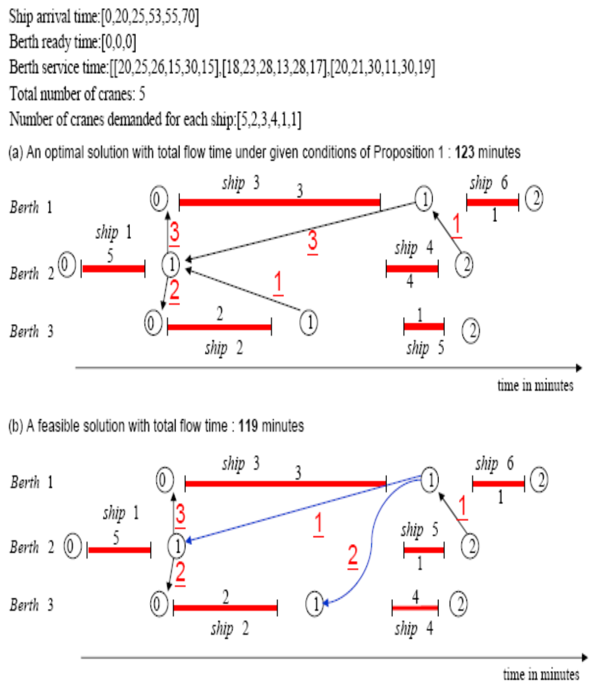

Fig. 4. A counter example

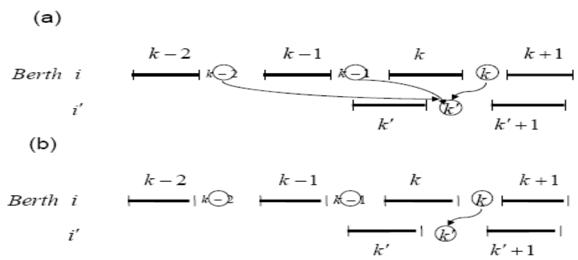

Fig. 5. An illustration of how transfer arcs are built

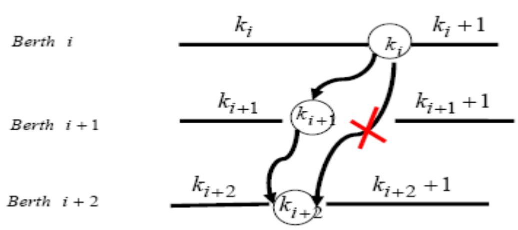

Fig. 6. An infeasible case for transferring cranes

B. The Case of Adjacent Ordered Berths

Is every pair of adjacent ordered berths satisfying $t^s_{i',k'+1} \geq t^f_{i,k}$ also feasible for moving cranes between non-adjacent ordered berths? An infeasible example is illustrated in Fig. 6, where transferring cranes through arcs (k_i, k_{i+1}) and (k_{i+1}, k_{i+2}) are both feasible while cranes can not be transferred from the k_i^{th} ship of berth i to

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the $(k_{i+2}+1)^{th}$ ship of berth i+2 due to $t^f_{i,k_i} > t^s_{i+2,(k_{i+2}+1)}$. In this example, a transfer path from node k_i to node k_{i+2} is feasible if transfer arcs of (k_i, k_{i+1}), (k_{i+1}, k_{i+2}) and (k_i, k_{i+2}) are all feasible. In addition, all cranes after serving the $k_{i+1}{}^{th}$ ship of berth $i+1$ have to be moved to berth i+2 or beyond, otherwise those cranes will block the movement path.

Proposition 3:
Let $t^s_{i,k}$ and $t^f_{i,k}$ be the start time and the finish time for the k^{th} ship served at the berth i, respectively. Let $B_r = \{0_r\} \cup \{1_r, \ldots, k_r, \ldots \mathrm{B}_r\}$, where 0_r is a dummy node of berth r, and $\{1,2,\ldots,\mathrm{B}_r\}$ is the set of nodes constructed according to the service order for the ships assigned to berth r. Adding node 0_r at each berth is to allow that cranes used for the first ship of a berth may also be used for the first ship of another berth. Without loss of generality, we assume $i \le i'-2$ below. Given that cranes are not allowed to crossover a busy berth, and cranes bypass through an idle berth in a push-in and pull-out way, then transferring cranes from node k_i of berth i to node $k_{i'}$ of berth i' is feasible if there exists a path of $(k_i, \ldots, k_r, \ldots, k_{i'}) \in B_i \times \ldots \times B_r \times \ldots \times B_{i'}$, such that the following conditions are satisfied:

(i) $t^s_{r',k_{r'}+1} \ge t^f_{r,k_r}$ for $i \le r < i', r < r' \le i'$.

(ii) All cranes after serving the k_r^{th} ship of berth r have to be transferred to berth q, where $i+1 \le r \le i'-1$ and $q \ge i'$.

It is clearly that cranes used for the $k_i{}^{th}$ ship of berth i can be feasibly transferred to the $k_{i'}{}^{th}$ ship of berth i' if there exist a feasible path bypassing cranes through berths of $(i+1, i+2, \ldots, i'-2, i'-1)$. However, to consider all such kind of feasible transfer paths would result in a rather complex and impractical optimization model described in next section. Thus, from practical viewpoints, we may restrict that cranes used for berth i can be moved to berth i' satisfying $|i'-i| \le \beta$, where β is a given parameter.

IV. The Formulation

Since the mixed integer programming (MIP) model given by Imai et al. [7] considers partial transfer possibilities and has some errors, we formulate a general MIP in this section. To construct possible transfer arcs for each pair of berths, say from berth i to berth i' , we check condition of $t^s_{i',k'+1} \ge t^f_{i,k}$ as stated in proposition 2(a); which is expressed as constraints (12). Moreover, to check whether a feasible transfer path exists from berth i to berth i' , we require that each pair of berths of the transfer path considered is feasible, that is, we examine a total of $0.5(i'-i)(i'-i-1)$ pairs of berths of the transfer path as stated in proposition 3, which is expressed as constraints (17) ~ (19) and (21) ~(22). Indices and variables used in the mixed integer programming model are first described below, followed by a complete MIP model that considers scheduling each vessel at a berth with sufficient cranes through possible transfer mechanism.

Parameters, sets, and indices:

T	total number of berths
N	total number of vessels
B	set of berths with indices i or i', $B = \{1, \ldots, \mathrm{T}\}$
V	set of vessels with indices j or j', $V = \{1, \ldots, \mathrm{N}\}$
U	set of service orders with indices k or k', $U = \{1, \ldots, \mathrm{N}\}$
W	set of service orders, $W = U \cup \{0\}$
S_i	ready time for serving vessels at berth i
A_j	arrival time of vessel j
TQ	total number of quay cranes
F_j	number of quay cranes required for vessel j
$\mathrm{C}_{i,j}$	service time of vessel j at berth i
$W^{i,i',k,k'}$	all possible paths from the k th order of berth i to the k' th order of berth i' through intermediate berths between i and i'
$p(\hat{i})$	the corresponding node of berth $\hat{i}$ on path $p \in W^{i,i',k,k'}$
$\Omega^{i,i',k,k',p,\hat{i}}$	set of all possible paths that start from node $p(\hat{i})$ of an intermediate berth $\hat{i}$ of path p in $W^{i,i',k,k'}$, and have the same subpath of p while visiting berth i'
M	a big constant

Variables:

$x_{i,j,k}$	= 1, if vessel j is assigned to the kth order of berth i; =0, otherwise
$b_{i,j,k}$	start time of servicing vessel j at the kth order of berth i
$f_{i,j,k}$	completion time of servicing vessel j at the kth order of berth i
$z_{i,k}$	number of quay cranes held at berth i after servicing a vessel at the kth order
$m_{i,i',k,k'}$	number of quay cranes transferred from node k of berth i to node k' of berth i'
$\delta_{i,i',k,k'}$	=1, if there exists a feasible transfer arc from node k of berth i to node k' of adjacent berth i' ; =0, otherwise
$\phi^p_{i,i',k,k'}$	=1, if there exists a feasible transfer path p from node k of berth i to node k' of adjacent berth i' ; =0, otherwise

Mixed integer programming model:

Minimize

$$\sum_{j \in V} \left(\sum_{i \in B} \sum_{k \in U} f_{i,j,k} - \mathrm{A}_j \right) \quad (1)$$

Subject to

$$\sum_{i \in B} \sum_{k \in U} x_{i,j,k} = 1 \qquad \forall j \in V \quad (2)$$

$$\sum_{j \in V} x_{i,j,k} \le 1 \qquad \forall i \in B, k \in U \quad (3)$$

$$\sum_{i\in B}\sum_{k\in U} b_{i,j,k} \ge \mathrm{A}_j \quad \forall j \in V \tag{4}$$

$$\sum_{j\in V} b_{i,j,k} + \mathrm{M}(1-\sum_{j\in V} x_{i,j,k}) \ge \sum_{j\in V} f_{i,j,k-1} \quad \forall i \in B, k \in U \tag{5}$$

$$b_{i,j,k} \le \mathrm{M}x_{i,j,k} \qquad \forall i \in B, j \in V, k \in U \tag{6}$$

$$b_{i,j,k} + C_{i,j}x_{i,j,k} = f_{i,j,k} \qquad \forall i \in B, j \in V, k \in U \tag{7}$$

$$\sum_{j\in V} f_{i,j,0} = S_i \qquad \forall i \in B \tag{8}$$

$$\sum_{i\in B} z_{i,0} = \mathrm{TQ} \tag{9}$$

$$\sum_{j\in V} \mathrm{F}_j x_{i,j,k} \le z_{i,k} \quad \forall i \in B, k \in U \tag{10}$$

$$z_{i,k} + \sum_{i'\ne i\in B}\sum_{k'\in W} m_{i',i,k',k} = z_{i,k+1} + \sum_{i'\ne i\in B}\sum_{k'\in W} m_{i,i',k,k'} \quad \forall i \in B, k \in W \tag{11}$$

$$\sum_{j'\ne j\in V} b_{i',j',k'+1} - \mathrm{M}(\delta_{i,i',k,k'} - 1) \ge \sum_{j\in V} f_{i,j,k} \qquad \forall i \in B, i' \in B, i' \ne i, k \in W, k' \in W \tag{12}$$

$$m_{i,i',k,k'} \le \mathrm{M}\delta_{i,i',k,k'} \qquad \forall i \in B, i' \in B, i' \ne i, k \in W, k' \in W \tag{13}$$

$$\sum_{i'\ne i\in B}\sum_{k'\in W} m_{i,i',k,k'} \le \mathrm{M}\sum_{j\in V} x_{i,j,k} \quad \forall i \in B, k \in U \tag{14}$$

$$\sum_{i'\ne i\in B}\sum_{k'\in W} m_{i,i',k,k'} \le z_{i,k} \qquad \forall i \in B, k \in W \tag{15}$$

$$\sum_{i\in B}\sum_{i'\ne i\in B} m_{i,i',0,0} = 0 \tag{16}$$

$$\sum_{i\le r<i'}\sum_{r<r'\le i'} \delta_{r,r',p(r),p(r')} + \mathrm{M}(1-\phi^{p}_{i,i',k,k'}) \ge 0.5(i'-i)(i'-i-1) \qquad \forall i \in B, i' \in B, i' \ge i+2, k \in W, k' \in W, p \in W^{i,i',k,k'} \tag{17}$$

$$\sum_{i'<r\le i}\sum_{i'<r'<r} \delta_{r,r',p(r),p(r')} + \mathrm{M}(1-\phi^{p}_{i,i',k,k'}) \ge 0.5(i-i')(i-i'-1) \qquad \forall i \in B, i' \in B, i \ge i'+2, k \in W, k' \in W, p \in W^{i,i',k,k'} \tag{18}$$

$$\sum_{p\in W^{i,i',k,k'}} \phi^{p}_{i,i',k,k'} \le 1 \qquad \forall i \in B, i' \in B, i' \ne i, k \in W, k' \in W \tag{19}$$

$$m_{i,i',k,k'} \le \mathrm{M}\sum_{p\in W^{i,i',k,k'}} \phi^{p}_{i,i',k,k'} \quad \forall i \in B, i' \in B, i' \ne i, k \in W, k' \in W \tag{20}$$

$$\sum_{i'\le j\le \mathrm{T}}\sum_{p'\in\Omega^{i,i',k,k',p,\hat{i}}} m_{\hat{i},j,p(\hat{i}),p'(j)} + \mathrm{M}(1\text{-}\phi^{p}_{i,i',k,k'}) \ge z_{\hat{i},p(\hat{i})}$$
$$\forall i \in B, i' \in B, i' \ge i+2, i+1 \le \hat{i} \le i'-1, k \in W, k' \in W, p \in W^{i,i',k,k'} \tag{21}$$

$$\sum_{0\le j\le i'}\sum_{p'\in\Omega^{i,i',k,k',p,\hat{i}}} m_{\hat{i},j,p(\hat{i}),p'(j)} + \mathrm{M}(1\text{-}\phi^{p}_{i,i',k,k'}) \ge z_{\hat{i},p(\hat{i})}$$
$$\forall i \in B, i' \in B, i \ge i'+2, i-1 \le \hat{i} \le i'+1, k \in V, k' \in V, p \in W^{i,i',k,k'} \tag{22}$$

$$x_{i,j,k} = 0 \text{ or } 1 \qquad \forall i \in B, j \in V, k \in U \tag{23}$$

$$b_{i,j,k} \ge 0 \qquad \forall i \in B, j \in V, k \in U \tag{24}$$

$$f_{i,j,k} \ge 0 \qquad \forall i \in B, j \in V, k \in U \tag{25}$$

$$z_{i,k} \ge 0 \qquad \forall i \in B, k \in W \tag{26}$$

$$m_{i,i',k,k'} \ge 0 \qquad \forall i \in B, i' \in B, i' \ne i, k \in W, k' \in W \tag{27}$$

$$\delta_{i,i',k,k'} = 0 \text{ or } 1 \qquad \forall i \in B, i' \in B, i' \ne i, k \in W, k' \in W \tag{28}$$

$$\phi^{p}_{i,i',k,k'} = 0 \text{ or } 1 \qquad \forall i \in B, i' \in B, i' \ne i, k \in U, k' \in U, p \in W^{i,i',k,k'} \tag{29}$$

The goal of objective function (1) is to minimize the total turnaround time. Constraints (2) ensure that each vessel has to be served. Constraints (3) say that at anytime a berth can be occupied at most by a vessel. Constraints (4) require that the start time for serving a vessel at a berth must be not smaller than its arrival time. Constraints (5) ensure that the start time of serving a vessel in larger order can not be less than the completion time of serving a ship in smaller order. If no vessels are assigned to a service position of a berth, constraints (6) set zero to the corresponding service start time. The relationship for serving two consecutive vessels at a berth is specified in constraints (7). Constraints (8) say that the start time for serving the first vessel at each berth can not smaller than the corresponding berth ready time. The total number of cranes available is given in constraint (9). Constraints (10) force that the number of cranes required for each vessel must be satisfied. Constraints (11) maintain the flow conservation of cranes. Constraints (12) ensure whether the feasible transfer arc exists. Constraints (13) ask that transferring cranes is not allowed if no feasible transfer arcs exist. If no vessel is assigned in a service order of a berth, constraints (14) do not consider the action of transferring cranes. Constraints (15) restrict that after serving a vessel the maximum number of cranes can be transferred to other berths. No transfer considered between dummy nodes is stated in constraint (16). Whether the existence of a feasible transfer path is what constraints (17) and (18) addressed. Constraints (19) restrict that using at most one path to transfer cranes from source to destination. If no feasible transfer paths exist, constraints (20) forbid the transfer. Constraints (21) and (22) require that if a feasible transfer path exists between source and destination, then all cranes for those intermediate berths of the path considered have to be emptied and moved to other berths. Constraints (23) ~ (29) declare the domain of variables.

V. Conclusions

This paper considers how to simultaneously allocation each vessel to a berth and how to transfer cranes to satisfy the requirement of each vessel at each berth. A very complicate MIP model that attempts to modify the one described by Imai et al. [7] is developed. We discuss conditions for feasibly transferring cranes between berths. Though to optimally solve the MIP model directly is almost impossible for large size problems, the fundamental insights revealed in modeling the problem can provide valuable thoughts for developing approximate solution methods. For instance, you may restrict neighboring berths as candidates for crane transfer. Studying efficient and effective heuristic algorithms for the problem is our current job.

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