Optimal Pricing and Lot Sizing for a Single Period Problem

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***Abstract*--There has been significant interest in single period newsvendor problem where besides determining lot size, the decision maker has to set the selling price. The model is applicable when the demand for the product is a stochastic function of the selling price; i.e., the retailer is facing price sensitive uncertain demand. Although there have been several models, the problem needs exploration when the retailer finds it hard to estimate the shortage cost and rather use the notion of service level to set his/her lot sizing and pricing policy.**

Index Terms : Newsvendor problem, lot sizing, pricing, service level.

I. INTODUCTION

Newsvendor model has been used to model supply demand balance for seasonal, perishable as well as products with short life cycle. Pricing is a natural extension. Petruzzi and Dada [1] formulated a joint pricing inventory model where they model economic consequence of stockout using shortage cost. They allow demand to be described by additive or multiplicative demand but could not show joint concavity of the expected profit when the random error is distributed according to a general distribution. They provide certain condition on the density function under which profit is jointly concave. Recently, Propescu [2] has revisited the problem assuming shortage cost to be zero . He provides additional conditions for profit to be jointly concave using a new construct called as lost sales elasticity.

Whereas shortage costs are widely used in literature to formulate inventory models, practitioners seem to prefer using service level as a measure of economic consequence of stock outs. For example, service level approach is widely used in practice to set up reorder point or the order-up-to-level. In fact the most popular enterprise resource planning system SAP does not use shortage costs in determining reorder points or dynamic order up to level [3].

Manuscript received January 10, 2013; revised January 28, 2013.
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II. MODEL FORMULATION

Let

TABLE I
NOTATION

Q = lot size of the perishable good.

c = purchase cost per unit.

r = selling price (exogenous) in period 1.

$D(r,\varepsilon)$ = the demand function such that $D(r,\varepsilon) = f(r) + \varepsilon$, where $f(r)$ is the riskless demand when selling price is is set to r.

ε = the random error defined over a range $[A,B]$

u = the realized error.

SL = the service level specified by the retailer for the second period.

We assume that

1. The random error ε is distributed with mean 0 and standard deviation σ and its probability density and cumulative functions are denoted by $g(.), G(.)$, respectively. One special case will be when ε is distributed according to truncated Normal; i.e.,

$$g(u) = \frac{1}{G(B) - G(A)} \; \frac{e^{-\frac{1}{2}\left(\frac{u}{\sigma}\right)^2}}{\sqrt{2\pi}\sigma} \qquad (1)$$

2. The selling price, r, does not change during the season/period.

3. Without loss of generality, the salvage value is zero and there is no carrying cost.

4. The starting inventory level is zero; the order up to level and order quantity is the same.

The revenue function is given as

$$R(Q,r,u) = \begin{cases} rD(r,u) & \text{if } D(r,u) \le Q \\ rQ & \text{if } D(r,u) > Q \end{cases}$$

ISBN: 978-988-19252-6-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

After rearranging,

$$R(Q,r,u)=\begin{cases} r[f(r)+u] & \text{if } u\le Q-f(r)\\ rQ & \text{if } u>Q-f(r)\end{cases}\qquad (2)$$

Given that $E(\varepsilon)=\mu=0,$ the expected profit can be shown to be

$$\Pi(Q,r):=rf(r)+r\int_{Q-f(r)}^{B1}(Q-f(r)-u)g(u)\,\mathrm{d}u \\ -cQ \qquad (3)$$

In general, $\Pi(Q,r)$ is not jointly concave in Q and r. Procedures involving enumeration can be used to maximize $\Pi(Q,r)$. However, maximizing $\Pi(Q,r)$ may result in poor service to the customers. Beside cost, service level is the most important criterion for the practitioners. Suppose the decision maker specifies the service level to be SL. Then

$$P(f(r)+\varepsilon\le Q)=P(\varepsilon\le Q-f(r))$$

$$=G(Q-f(r))=SL.$$

Now let $z=\dfrac{Q-f(r)}{\sigma}$ or $z\sigma=Q-f(r)$.

Then $z\sigma=G^{-1}(SL)$ and

$$Q=f(r)+z\sigma$$

$$\text{Or}, \quad Q=Q^*(r)=f(r)+z\sigma \qquad (4)$$

Given the above, (3) can be rewritten as

$$\Pi(Q^*(r),r)=(r-c)f(r)-cz\sigma \\ -r\int_{z\sigma}^{B}(u-z\sigma)g(u)du \qquad (5)$$

Clearly, the convexity of $\Pi(Q^*(r),r)$ is assured if $(r-c)f(r)$ is convex. This is much simpler since $(r-c)f(r)$ is independent of the density function $g(u)$. For example $(r-c)f(r)$ is convex for linear demand function $f(r)=a-br$ or iso-elasticity (i.e., power) function $f(r)=\alpha r^{-\xi}$ for $r\le\dfrac{c(\xi+1)}{\xi-1}$ where ξ is the constant price elasticity.

III. LINEAR DEMAND

Let $f(r)=a-br$. Here

$$\Pi(Q^*(r),r)=(r-c)(a-br)-cz\sigma \\ -r\int_{z\sigma}^{B}(u-z\sigma)g(u)du$$

In this case, $\Pi(Q^*(r),r)$ is a concave function of r and the first order condition gives,

$$r^*=\frac{c}{2}+\frac{a}{2b}-\frac{\int_{z\sigma}^{B}(u-z\sigma)g(u)\,\mathrm{d}u}{2b}$$

and given (4),

$$Q^*=a-br+z\sigma \qquad (6)$$

Given above closed form results, the following properties of the optimal policy are seen for the linear demand case. Assuming that all other parameters are held constant.

1. The optimal price r increases as purchase cost c increases. The seller tends to pass the increased purchase cost to the end consumers.
2. The optimal price r increases as the market size a increases. Given that there are more potential customers, the seller can improve his margin by charging more.
3. The optimal price r decreases as the elasticity increases text. Given that fewer customers parameter are willing to pay more, the seller has to reduce her price.
4. The optimal price increases at a slight rate as z or service level SL increases.
5. The optimal price r increases as demand volatility σ increases.
6. The optimal lot size Q increases with market potential a. When market potential is large the seller will reach a more customers at a given price.

ISBN: 978-988-19252-6-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

7. The optimal lot size Q decreases as price elasticity parameter b increases.
8. The optimal lot size Q increases as safety factor constant z or service level SL increases. As z increases, the seller is carrying larger safety stock to provide the increased amount of service.
9. The optimal lot size Q increases as demand volatility σ increases. When σ increases, to provide the same amount of service, the seller has to carry more safety stock.

3.1 Example 1

Suppose,

$f(r) = 1500 - 50r, c = 6, \mathrm{SL} = 95\%$ or

$z = 1.645$. Also, ε is distributed according to a truncated Normal distribution

with μ=0,σ=33,$A = -100$ and $B = 100$.

With the above values, conditions (6) give

$r^* = 17.994, Q^* = 654.44$ and

$\Pi(Q(r^*), r^*) = 6863.91$.

IV. CONSTANT PRICE ELASTICITY OR POWER FUNCTION DEMAND

Now suppose

$f(r) = \alpha r^{-\xi}$ and

$$\Pi(Q^*(r), r) = (r - c)\alpha r^{-\xi} - cz\sigma - r\int_{z\sigma}^{B} (u - z\sigma) g(u) du$$

Denote $\pi(r) = (r - c) f(r)$. Then

$$\frac{d\pi}{dr} = -(r - c)\xi\alpha r^{-\xi-1} + \alpha r^{-\xi}$$

$$\frac{d^2\pi}{dr^2} = (r - c)\xi(\xi + 1)\alpha r^{-\xi-2} - 2\xi\alpha r^{-\xi-1}$$

It is easy to see that

$$\frac{d^2\pi}{dr^2} \begin{cases} > 0 & \text{for} \quad r > c\dfrac{\xi+1}{\xi-1} \\ = 0 & \text{for} \quad r = c\dfrac{\xi+1}{\xi-1} \\ < 0 & \text{for} \quad r < c\dfrac{\xi+1}{\xi-1} \end{cases}$$

Now $\dfrac{d\Pi}{dr} = -(r - c)\xi\alpha r^{-\xi-1} + \alpha r^{-\xi}$

$$-\int_{z\sigma}^{B} (u - z\sigma) g(u) du = 0 \rightarrow r^* < \mathrm{c}\frac{\xi}{\xi - 1}$$

Clearly, $r^* < \mathrm{c}\dfrac{\xi}{\xi-1} < c\dfrac{\xi+1}{\xi-1}$. Hence

$\pi(r)$ and therefore $\Pi(Q^*(r), r)$ is concave at r^*. Thus r^* is the global maximum of $\Pi(Q^*(r), r)$ and the optimal solution to the original problem in this case

is $r = r^*$ and $Q = Q(r^*) = \alpha r^{*^{-\xi}} + z\sigma$.

4.1 Example 2

Suppose

$\alpha = 100000, \xi = 2.5$ or $f(r) = 100000r^{-2.5}$.

Also, random error ε is distributed according to truncated Normal distribution with

μ=0, $\sigma = 33, A = -100$ and $B = 100$.

Let $SL = 95\%$ or z=1.645.

With the above values, $r^* = 9.987$,

$Q^* = 371.40$ and $\Pi(Q(r^*), r^*) = 933.88$.

V. ADDITIONAL APPROACHES TO FIND GLOBAL OPTIMUM

Since $\Pi(Q^*(r), r)$ is a function of a single variable, there are following other approaches to find the global optimum.

One could solve the first order condition

$$\frac{d\Pi(Q^*(r), r)}{dr} = (r - c)\frac{df(r)}{dr} + f(r) - \int_{z\sigma}^{B1} (u - z\sigma) g(u)\, \mathrm{d}u = 0$$

and check if $\Pi(Q^*(r), r)$ is concave at that solution. Similarly, one could show that $\Pi(Q^*(r), r)$ is unimodal.

ISBN: 978-988-19252-6-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

VI. CONCLUSIONS

In this paper, we revisit the single period problem of determining the selling price and lot size when demand is a stochastic function of the price. We used the notion of service level to determine the price and order quantity. The construct of service level is preferred by practitioners in setting reorder points and order up to level. For example ERP system SAP ECC 6.0 does not use per unit shortage cost when it comes to setting reorder points or dynamic order-up-to-levels.

Our model is parsimonious: for the linear demand case, we have derived the optimal selling price and lot size in a closed form. Even for other demand functions, the procedure should be easy to apply since the optimal lot size is a closed form function of the selling price.

REFERENCES

[1] Dada, M. and Petruzzi, N., " Pricing and the Newsvendor Problem: A Review with Extensions", *Operations Research*, 47 , pp. 183-194, March-April 1999.

[2] Popescu, I. and Kocabiyikoglu, A.," An Elasticity Approach to the Newsvendor with Price-Sensitive Demand', *Operations Research*, 59 , pp. 301-312, March-April 2011.

[3] Rawe, H., *Sap for retail,* Boston, SAP Press: Galileo Press Inc., 2009.

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