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A MILP Model to Select Cutting Machines and Cutting Patterns to Minimize Paper Loss

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Abstract—**We study a paper converting process where master rolls are transformed to paper sheets with the minimal paper loss, including trim loss, set up loss, and over-production loss. Two different types of cutters are involved in the converting process – one transversal knife and two transversal knife machines. To solve the problem, various types of paper sheets specified by material type, size, and required quantity are grouped into all possible cutting patterns using a heuristic method. Then, a MILP model is proposed to assign cutting patterns to each type of cutter while satisfying the machine capacity constraints. Applying the methodology to industrial problems, we found the average total loss of 4.6% for single-product group problems and 3.1% for two-product group problems, which respectively correspond to 25.8% and 41.9% improvement over a current method based on worker experiences used by the company.**

Index Terms—**cutter selection, trim loss reduction, paper converting, heuristics, MILP**

I. Introduction

IN a paper converting process, master rolls are transformed to product paper rolls or paper sheets. Generally, when there are multiple cutters in a mill, products are preassigned to cutters based on machine condition or capacity. Then, in each cutter, products are grouped into cutting patterns and the number of cuts of each cutting pattern is determined to satisfy customer demands.

A significant amount of paper wastes result from the trim loss, which occurs when cutting patterns or combinations of cuts do not fit the size of master rolls. A problem to reduce the trim loss is referred to as a cutting stock problem, which can be classified into 2 groups – one-dimensional and two-dimensional cutting stock problems. The one-dimensional problem considers only the cutting width as a decision variable whereas the two-dimensional problem considers both cutting width and cutting length as decision variables.

One-dimensional cutting stock problems are traditionally formulated by using Mixed Integer Non-Linear Programming (MINLP). The nonlinearity is normally resolved by treating a complete list of cutting pattern vectors as known, leading to a MILP model. However, this approach can be computationally inefficient due to a large solution space. Many papers proposed different MIP models without pre-specified cutting patterns. For example, Johnston et al. [1] proposed an exact integer model. Kasimbeyli et al. [2] proposed a two-objective integer linear model related to the minimization of the total trim loss and the total number of different lengths of stock rolls in inventory. Schilling and Georgiadis [3] applied a MIP model to industrial cases with the objective of maximizing the total profit, where the profit comes from an income less cost of cutting rolls, cost of changing knife positions, and cost of disposing any trim.

Many heuristics were also proposed to minimize trim loss, e.g., a genetic algorithm based on bin-packing by Vahrenkamp [4] and weight annealing heuristics by Loh et al. [5]. Correria et al. [6] used three-step heuristics including generating all feasible cutting pattern, determining cutting lengths, and adjusting the solutions to fit an environment of real settings.

Similar to the one-dimensional problem, a non-convex MINLP can be used to formulate the two-dimensional cutting stock problem. Since no straightforward method for finding an optimal solution of this model exists, Westerlund et al. [7], [8] proposed a 2-step procedure, whereby all feasible cutting patterns are included and the global optimal cutting patterns are obtained from a MILP model. Westerlund et al. [7] assumed equal lengths of product rolls, while Westerlund et al. [8] assumed that the lengths may vary within a given range. Harjunkoski et al. [9] proposed and compared different ways to transform a non-convex MINLP model to a convex MINLP model while Harjunkoski et al. [10] proposed methods to transform a non-convex INLP model to a MILP model or a convex MINLP model. Harjunkoski et al. [11] proposed linear transformation methods to transform a non-convex MINLP model to ILP or MILP models, and convex transformation methods to transform a non-convex MINLP model to convex INLP or MINLP models. Most of two-dimensional cutting stock problems assume that products in each cutting pattern must have equal length. Puemsin et al. [12] allowed to have more than one product length in each cutting pattern and proposed a MILP model to solve the problem.

Most of the previous papers in trim loss reduction considers selecting cutting patterns to transform all products on a single machine. However, a paper mill in a real settings has one or more machines of different types. The overall loss may not be minimized if the products are not properly assigned to the machines. In this paper, we consider minimizing the total paper loss across multiple machines by jointly determining what cutting patterns to assign to which machine.

II. Problem Statement

We consider the process where different types of master rolls specified by their grades and grammages are converted to different types of paper sheets according to customer specifications via multiple cutters. The paper sheet specification includes paper grammage, the sheet width and length, and the total product weight, which is converted to the total required cutting length within its allowance. Table I shows examples

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TABLE I
PRODUCT SPECIFICATIONS

No.	Product Group	Grammage	Size (inches)		Order quantity		
			width	length	Demand (tons)	Lower demand (inches)	Upper demand (inches)
1	A	90	31	43	2.75	1680559	1781392
2	A	90	28	36	2.88	1945192	2061904
3	A	90	22	27	4.75	4090286	4335703
4	A	90	29	22.5	1.25	816573	865567
5	A	90	27	20	1.38	964765	1022651
6	A	90	37.5	40	1.00	505186	535497
7	B	85	24	35	10.18	7750000	8215000
8	B	85	35	24	1.93	1002943	1063120
9	B	85	24	34.5	4.35	3301355	3499436

of the product specification. Paper sheets in the same product group can be cut via the same cutting patterns since they use the same type of master rolls. In this example, paper sheets 1-6 can be in the same cutting patterns while paper sheets 7-9 must be in another set of cutting patterns. Normally, customers place orders in a unit of ton requirements. To order the production, demands in tons must be computed into the required product lengths. For example, 2.75 tons of paper sheet 1 is equivalent to 1,680,559 inches. However, customers are willing to accept a small amount of production over this requirement, which can be justified as the upper demand.

Generally, to convert master rolls to paper sheets, cutting patterns must be determined. Cutting patterns are possible combinations of products with the same grammage diminished by the width of a master roll. Since trim loss occurs if the cutting pattern width is less than the master roll width, the company specifies the minimum width of a cutting pattern that is allowed to be cut.

Two non-identical cutting machines are considered in this settings – the cutter with one transversal knife and the cutter with two transversal knives. Each machine has three adjustable longitudinal knives (excluding two longitudinal trim knives) as shown in Figure 1. The number of longitudinal knives limits the number of product types in each cutting pattern. For the machine with one transversal knife, the lengths of all products assigned to each cutting pattern must be equal, whereas two different lengths are also feasible for cutting patterns of the two-transversal knife machine. In other words, cutting patterns of a one-transversal knife cutter are a subset of those of a two-transversal knife cutter.

This paper proposes a methodology to allocate demands to different cutters considering machine capacity constraints while minimizing the total paper loss. The paper loss is of three types, including trim loss, set up loss, and over-production loss. First, the trim loss in the longitudinal direction occurs if the width of a cutting pattern is less than the width of a master roll, and the trim loss in the transversal direction occurs if the required lengths of products in each cutting patterns are different. Second, the set up loss arises when we start a new master roll or a new cutting patterns. The over-production loss occurs if we produce more than the acceptable demand levels.

III. SOLUTION METHODOLOGY

Pre-assigning products to cutters and optimizing cutting patterns of each cutter may lead to a suboptimal solution. We thus propose a methodology to minimize the total paper loss across all the machines as the following. First, an MINLP model of this problem is formulated. The nonlinearity constraints are relaxed by pre-determining a set of all possible cutting patterns for each cutter. Details of the heuristics to generate cutting patterns is included in Section III-A. Once the cutting patterns become model parameters, the MINLP model is transformed to a MILP model as discussed in Section III-B.

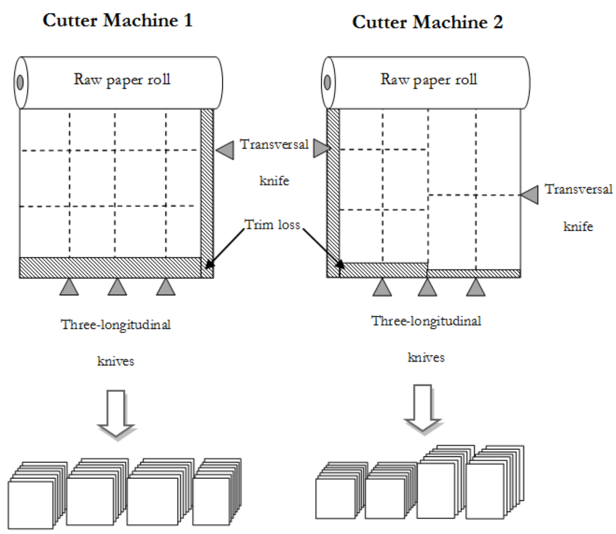

Fig. 1. 1-transversal and 2-transversal knife cutters

A. Cutting Pattern Heuristics

Since products may use various grades and grammages of master rolls. We classify them into product groups so that each group can be cut via the same cutting pattern using the same type of master roll. Then, the heuristics is used to generate all feasible cutting patterns of each product group.

Due to machine specification, the feasible cutting patterns of the one-transversal knife cutter must have equal product length, and the feasible cutting patterns of the two-transversal knife cutter may have at most two different product lengths. Therefore, the feasible cutting patterns of the one-transversal knife cutter are a subset of those of the two-transversal knife cutter.

Before illustrating the heuristics, let us first define the following notations:

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$W_{\max}$: Maximum width of cutting patterns (inches)
$W_{\min}$: Minimum width of cutting patterns (inches)
$N_{\max}$: Maximum number of longitudinal cuts in each cutting pattern
l_i : Length of product i (inches)
w_i : Width of product i (inches)
n_{ij} : Number of units of product i in pattern j, with $\boldsymbol{n}_j = \{n_{1j}, n_{2j}, \ldots, n_{pj}\}$

Each feasible cutting pattern j is a set of number of units of product i in a pattern j or n_{ij} whose combined width w_i is less than the width of a master roll, $W_{\max}$, but greater than the minimum width of a cutting pattern, $W_{\min}$. Different cutting patterns correspond to different placement positions of the longitudinal and the transversal knives are generated. The total number of units of product in the cutting pattern is limited by the number of longitudinal cuts $N_{\max}$. Obviously, each cutting pattern generates a trim loss if the total paper width in the cutting pattern is less than $W_{\max}$ or the total length of each product in the cutting pattern are not equal.

In the first step, all cutting patterns are generated with the constraints on the minimum and maximum of the cutting pattern width ($W_{\min}$ and $W_{\max}$), the maximum number of longitudinal cuts in each cutting pattern ($N_{\max}$), and the constraint of the number of transversal knives. We adopt the algorithm by Westerlund et al. [7] whose concept is summarized below. Starting from an initial empty cutting pattern,

- Step 1: Compute the maximum number of each product i for the given cutting pattern. If the number of units in all products have already reached their maximum allowed, this means all feasible cutting patterns have been generated and we stop here.
- Step 2: For each product i, if the number of units has reached the maximum allowed, reset the number of units to zero. Otherwise, add one unit to each product i. If all products have reached the maximum allowed, stop.
- Step 3: Repeat Step 2 until the total width in the cutting pattern exceeds $W_{\min}$. This step creates a cutting pattern.
- Step 4: Using the cutting pattern obtained in Step 3, go to Step 1.

From the set of cutting patterns generated above, the cutting patterns with different paper lengths are removed to satisfy the constraint of having one transversal knife as well as more than two different paper lengths are removed to satisfy the constraint of having two transversal knives. These cutting pattern are used as parameters in the MILP model.

Table II shows the feasible cutting patterns for one-transversal and two-transversal knife cutters based on the product specifications in Table I. For example, the first pattern cuts three units of product no. 1 and none of the others, and the second pattern cuts two units of product no.4 and one unit of product 6, and so on. In this example, the maximum width of cutting pattern is 96 inches, the minimum width of cutting pattern is 89 inches, and the maximum number of longitudinal cuts equals 4.

TABLE II
FEASIBLE CUTTING PATTERNS

No.	Pattern	Total width	Cutting length
1	[3, 0, 0, 0, 0, 0, 0, 0, 0]	93	1
2	[0, 0, 0, 2, 0, 1, 0, 0, 0]	95.5	2
3	[0, 0, 3, 1, 0, 0, 0, 0, 0]	95	2
4	[0, 1, 3, 0, 0, 0, 0, 0, 0]	94	2
5	[0, 2, 0, 0, 0, 1, 0, 0, 0]	93.5	2
6	[0, 0, 3, 0, 1, 0, 0, 0, 0]	93	2
7	[0, 0, 0, 0, 2, 1, 0, 0, 0]	91.5	2
8	[2, 0, 0, 1, 0, 0, 0, 0, 0]	91	2
9	[2, 1, 0, 0, 0, 0, 0, 0, 0]	90	2
10	[2, 0, 0, 0, 1, 0, 0, 0, 0]	89	2
11	[1, 0, 0, 2, 0, 0, 0, 0, 0]	89	2
12	[0, 0, 0, 0, 0, 0, 4, 0, 0]	96	1
13	[0, 0, 0, 0, 0, 0, 0, 0, 4]	96	1
14	[0, 0, 0, 0, 0, 0, 3, 0, 1]	96	2
15	[0, 0, 0, 0, 0, 0, 2, 0, 2]	96	2
16	[0, 0, 0, 0, 0, 0, 1, 0, 3]	96	2
17	[0, 0, 0, 0, 0, 0, 1, 2, 0]	94	2
18	[0, 0, 0, 0, 0, 0, 0, 2, 1]	94	2

B. MILP Model

Basically, models to minimize trim loss are formulated as MINLP models. In this paper, since cutting patterns are specified using the heuristics in the previous section, the non-linearity constraints are relaxed. Thus the MINLP model becomes a MILP model. Unlike previous models that focus on minimizing trim loss in a given machine, we propose a mathematical model to select proper cutters and cutting patterns as follows.

Index:

I : A set of products $\{1, 2, \ldots, p\}$
J : A set of cutting patterns $\{1, 2, \ldots, t, t+1, \ldots, q\}$
$\{1, 2, \ldots, t\}$ are equal length patterns
$\{t+1, t+2, \ldots, q\}$ are 2-length patterns
K : A set of master rolls $\{1, 2, \ldots, r\}$
C : A set of cutters $\{1, 2\}$
$\{1\}$ is a one-transversal knife cutter
$\{2\}$ is a two-transversal knife cutter

Parameters:

L : Length of master roll
$l_{\min,i}$: Minimum required length for product i
$l_{\max,i}$: Maximum required length for product i
$P_{\min}$: Minimum pattern length
S : Setup loss
M : a huge number
Cap_c : Capacity of cutter c

Decision variables:

p_{jkc} : Cutting length of pattern j in master roll k at cutter c
x_{ijkc} : Number of cuts of product i in pattern j, master roll k at cutter c
a_i : Total cutting length of product i

$$y_{jkc} = \begin{cases} 1 & \text{if pattern } j \text{ is used in master roll } k \text{ at cutter } c \\ 0 & \text{Otherwise} \end{cases}$$

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$$Y_{kc} = \begin{cases} 1 & \text{if master roll } k \text{ is used at cutter } c \\ 0 & \text{Otherwise} \end{cases}$$

s_i^+ : Amount of product i exceeding the maximum required length

s_i^- : Amount of product i under the minimum required length

U_c^+ : Number of rolls exceeding the capacity of cutter c

U_c^- : Number of rolls under the capacity of cutter c

$U_{\max}$: Maximum number of rolls exceeding the cutter capacity

$$\min \quad z = \sum_{k\in K}\sum_{c\in C} Y_{kc} W_{\max} L - \sum_{i\in I} a_i w_i + \sum_{i\in I} s_i^+ w_i + M\, U_{\max} \quad (1)$$

subject to

$$\sum_{k\in K} Y_{kc} + U_c^- - U_c^+ = \mathrm{Cap}_c \qquad \forall c \in C \quad (2)$$

$$U_c^+ \le U_{\max} \qquad \forall c \in C \quad (3)$$

$$\sum_{j\in J} (p_{jkc} + S\, y_{jkc}) \le Y_{kc}\, L \qquad \forall k \in K, \forall c \in C \quad (4)$$

$$\sum_{j=t+1}^{q} y_{jk1} = 0 \qquad \forall k \in K \quad (5)$$

$$p_{jkc} \le L\, y_{jkc} \qquad \forall j \in J, \forall k \in K, \forall c \in C \quad (6)$$

$$p_{jkc} \ge P_{\min}\, y_{jkc} \quad \forall j \in J, \forall k \in K, \forall c \in C \quad (7)$$

$$x_{ijkc} l_i \le p_{jkc} \qquad \forall i \in I, \forall j \in J, \forall k \in K, \forall c \in C \quad (8)$$

$$\sum_{j\in J}\sum_{k\in K}\sum_{c\in C} n_{ij} l_i x_{ijkc} = a_i \qquad \forall i \in I \quad (9)$$

$$a_i \ge l_{\min,i} \qquad \forall i \in I \quad (10)$$

$$a_i - s_i^+ + s_i^- = l_{\max,i} \qquad \forall i \in I \quad (11)$$

$$Y_{kc} \ge Y_{k+1c} \qquad \forall k \in K \setminus r, \forall c \in C \quad (12)$$

where

$$\begin{aligned} x_{ijkc} &\in \mathrm{Int} && \forall i \in I, \forall j \in J, \forall k \in K, \forall c \in C \\ Y_{kc} &\in \{0,1\} && \forall k \in K, \forall c \in C \\ y_{jkc} &\in \{0,1\} && \forall j \in J, \forall k \in K, \forall c \in C \\ p_{jkc} &\ge 0 && \forall j \in J, \forall k \in K, \forall c \in C \\ a_i, s_i^+, s_i^- &\ge 0 && \forall i \in I \\ U_c^+, U_c^- &\in \mathrm{Int} && \forall c \in C \\ U_{\max} &\in \mathrm{Int} \end{aligned}$$

A MILP model with multiple objectives is formulated. First, cutters are selected to minimize the maximum over capacities among cutters and then cutting patterns are selected to minimize paper loss as shown in (1). Paper loss include trim loss, set up loss, and over-production loss. Since we assume that the demand must be satisfied within machine capacities, constraints (2) and (3) determine if we need to produce over machine capacities and the maximum over capacity. Constraints (4) force that the length of all cutting patterns assigned to each master roll plus the set up loss must be less than or equal to the length of a master roll. Constraints (5) force that cutting patterns with 2 lengths cannot be assigned to the single transversal knife machine. Constraints (6) establish that if pattern j exists in master roll k cutter c, then y_{jkc} = 1. Then, y_{jkc} is used to calculate set up loss. Constraints (7) specify that the cutting length of every used cutting pattern must exceed the minimum length. Constraints (8) force that product length in each cutting pattern cannot exceed the cutting length. Constraints (9) are used to calculate the total cutting length of each product. Constraints (10) are the demand constraints representing that the total cutting length must satisfy the demand. In a paper sheet production, customers accept excess amount of production over the demand as long as it is not over the maximum acceptable level. Amount of production beyonds the maximum acceptable level is considered loss. Constraints (11) are used to determine if this loss is occurred. Constraints (12) force to sequentially used master rolls. These constraints are developed to reduce alternative solutions with an attempt to find the optimal solution faster. They do not affect the optimal solution.

IV. Experimental Results

The MILP model in Section III-B is used to solved industrial cases whose specifications of master rolls shown in Table III. The width of master roll is 96 inches and the length is 525,000 inches. Due to longitudinal trim loss, the company does not allow to use any patterns with the width lower than 89 inches. All cutting patterns generated from the heuristics already consider this requirement. Due to set up loss, the company does not allow to cut any patterns that are shorter than 10,000 inches in length. Additionally, the set up loss of 3,500 inches occurs once we start a new cutting pattern.

In terms of product demands, the product specifications including paper grammages, sheet widths and sheet lengths are already arranged into feasible cutting patterns. Therefore, the minimum and maximum required cutting lengths of each product along with cutting patterns are used as model parameters. Then, the MILP model selects proper cutters and cutting patterns for each product and determines an optimal number of cuts for each cutting pattern that minimizes the machine over capacities and the total loss.

TABLE III
Specifications of master rolls from real settings

Parameter	Value
Roll width ($W_{\max}$)	96 inches
Acceptable cutting pattern width ($W_{\min}$)	89 inches
Roll length (L)	525,000 inches
Set up length (S)	3,500 inches
Minimum pattern length ($P_{\min}$)	10,000 inches

To illustrate the solutions from the MILP model, the product specifications in Table I as well as the feasible cutting patterns in Table II are used as the model inputs, with the results shown in Table IV. Four master rolls are cut via cutter 1 and eight master rolls are cut via cutter 2. In this case, we limit the machine capacities to 8 rolls. Since cutter 1 has a single transversal knife, patterns 12 and 13, which have one cutting length, are assigned to this machine.

Pattern 12 is used to cut product 7 and pattern 13 is used to cut product 9. For cutter 2, any feasible cutting patterns can be assigned. The optimal solution selects pattern 4 for products 2 and 3, pattern 15 for products 7 and 9, pattern 11 for products 1 and 4, pattern 5 for products 2 and 6, pattern 10 for products 1 and 5, pattern 17 for products 7 and 8, pattern 1 for product 1, and pattern 6 for products 3 and 5. The number of cuts for each product in each pattern is in column "Cuts" in Table IV.

TABLE IV
AN EXAMPLE OF SOLUTIONS

Cutter	Roll	Product no.	Pattern	Cuts
1	1	9	13	15115
1	2	7	12	14900
1	3	7	12	14900
1	4	7	12	14900
2	1	2	4	14486
2	1	3	4	19314
2	2	9	15	5343
2	2	7	15	5267
2	2	4	11	14829
2	2	1	11	7759
2	3	2	5	14486
2	3	6	5	13037
2	4	1	10	10309
2	4	5	10	22165
2	4	1	11	1737
2	4	4	11	3320
2	5	7	17	14900
2	5	8	17	21729
2	6	7	15	14900
2	6	9	15	15115
2	7	1	1	2992
2	7	2	4	10815
2	7	3	4	14420
2	8	3	6	19314
2	8	5	6	26075

We also apply the MILP model to other industrial cases. Table V shows the experimental results of problems with single-product groups. We found that the total loss from the MILP model is 4.6% on average, which corresponds to 25.8% improvement compared to the total loss due to a current method used by the company, where the cutting patterns are assigned based on worker experiences.

Table VI shows the experimental results of problems with two-product groups. The optimal solutions cannot be found in 9 out of 10 problem sets since the MIP solver runs out of memory. In those cases, only the best found solutions and their percent gaps are shown in the table. Even though the MILP model cannot find optimal solutions, we found that the average total loss from the MILP model is 3.1%, which corresponds to 41.9% improvement over the current method used by the company.

V. CONCLUSION

Most of the previous works to minimize paper trim loss assume that products are preassigned to machines. Therefore they focused on selecting proper cutting patterns for each machine, which may provide sub-optimal solutions. This paper proposes an MILP model to select cutters and cutting patterns with the objectives to balance the machine capacities and minimize the total loss. We consider two types of cutters – one-transveral and two-transveral knife machines, and three types of losses – trim loss, set up loss, and over-production loss. Generally, the models to minimize trim loss are formulated as MINLP. In this paper, we transform the non-linearity constraints by pre-specifying cutting patterns with a heuristics. Once cutting patterns are model parameters, the MINLP model becomes a MILP model.

From the experiments of 20 industrial cases having a single product group, we found that the MILP model provides on average 4.6% loss, which corresponds to 25.8% improvement over the company current method based on worker experiences. Also, the computational time of the proposed method is less than a minute. However, for the cases of two product groups, the optimal solutions of many problem sets cannot be found due to memory exhaustion. However, compared to the company current method, the MILP model provides on average 3.1% total loss, corresponding to 41.9% improvement over the current method.

Due to the issue of computational time, the proposed MILP model may not be proper for large problems having multiple product groups. Other models or heuristics are needed to find (near) optimal solutions within an acceptable search time.

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TABLE V
TEST RESULTS OF A SINGLE PRODUCT GROUP UNDER REAL SETTINGS

Problem	Problem size		Computational	% Loss		
	Products	Cutting patterns	time (sec)	MILP	Current method	% Difference
1	2	2	0.05	3.72	3.98	6.53
2	3	9	1.81	1.61	2.3	30.00
3	3	12	53.35	2.36	3.4	30.59
4	3	15	1.06	3.23	5.67	43.03
5	4	9	96.6	5.46	6.81	19.82
6	4	10	1.57	5.98	7.33	18.42
7	4	10	7.19	4.55	6.14	25.90
8	4	10	0.05	3.93	5.89	33.28
9	4	15	1.09	3.5	4.16	15.87
10	4	16	9.67	5.72	7.4	22.70
11	4	16	18.03	3.81	4	4.75
12	5	19	45.03	2.41	3.46	30.35
13	6	23	3.35	3.26	4.58	28.82
14	6	47	4.29	7.89	8.5	7.18
15	6	48	127.06	4.97	7.66	35.12
16	7	19	6.72	2.22	3.54	37.29
17	8	31	3.59	8.54	10.34	17.41
18	9	50	41.71	4.15	4.69	11.51
19	10	40	189.08	7.11	13.2	46.14
20	13	67	108.89	6.927	14	50.57

TABLE VI
TEST RESULTS OF TWO PRODUCT GROUPS UNDER REAL SETTINGS

Problem	Problem size		Computational	%Gap	% Loss		
	Products	Cutting patterns	time (sec)		MILP	Current method	% Difference
1	6	21	13,666	10.93	1.66	2.74	39.5
2	7	25	12,710	6.24	2.19	2.98	26.6
3	7	28	13,708	3.38	3.09	3.70	16.6
4	7	28	14,692	5.35	2.57	4.79	46.4
5	8	25	2,433	2.66	3.84	6.91	44.5
6	8	26	10.28	optimum	4.39	7.35	40.3
7	9	18	110,000	5.46	2.83	7.73	63.4
8	10	57	5,972	7.52	2.80	7.47	62.5
9	12	40	8,256	4.69	5.23	8.08	35.3
10	13	66	9,177	7.15	2.82	5.07	44.3

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