# Adaptive Control System for Solution of Fault Tolerance Problem for MIMO Systems

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***Abstract*-Adaptive control can provide desirable behaviour of a process even though the process parameters are unknown or may vary with time. Conventional adaptive control requires that the speed of adaptation must be more rapid than that of the parameter changes. In practice, however, problems do arise when this is not the case. For example, when fault occurs in a process, the parameters may change very dramatically. A new approach based on simultaneous identification and adaptation of unknown parameters is suggested for compensation of rapidly changing parameters of a multivariable process.**

***Index Terms*-Adaptive control, fault tolerance, multivariable process, time-varying parameters.**

## I. THE APPROACH

Consider a multivariable process with time-varying parameters. Assume that the process is described by the following equations:

$$\begin{aligned}\dot{x}_1 &= a_{11}(t)x_1 + a_{12}(t)x_2 + g_1 \\ \dot{x}_2 &= a_{21}(t)x_1 + a_{22}(t)x_2 + g_2\end{aligned} \tag{1}$$

where parameters $a_{ij}(t)$ ($i, j = 1, 2$) are considered as the sum of a constant term and a time varying term:

$$a_{ij}(t) = a_{ij}^0 + \Delta a_{ij}(t)$$

where: $g_i$ ($i = 1, 2$) are the input signals to each channel of the Multi-Input-Multi-Output (MIMO) process.

The desirable behavior of the process (plant) can be specified by a reference model in the form:

$$\begin{aligned}\dot{x}_{1m} &= a_{1m}x_{1m} + g_1 \\ \dot{x}_{2m} &= a_{2m}x_{2m} + g_2\end{aligned} \tag{2}$$

In order to provide the desirable behavior of the process (1) according to model (2) a feedback controller can be used. The overall system is described by the following equations:

$$\begin{aligned}\dot{x}_1 &= a_{11}(t)x_1 + a_{12}(t)x_2 + k_{11}(t)x_1 + k_{12}(t)x_2 + g_1 \\ \dot{x}_2 &= a_{21}(t)x_1 + a_{22}(t)x_2 + k_{21}(t)x_1 + k_{22}(t)x_2 + g_2\end{aligned} \tag{3}$$

where

$k_{ij}(t) = k_{ij}^0 + \Delta k_{ij}(t)$,

$k_{ij}^0$ are constant parameters of the controller,

$\Delta k_{ij}(t)$ are time-varying parameters of the controller.

The following error equations can be obtained from the closed loop process (3) and the reference model (2):

$$\begin{aligned}\dot{e}_1 &= a_{1m}e_1 + (\Delta a_{11}(t) + \Delta k_{11}(t))x_1 + (\Delta a_{12}(t) + \Delta k_{12}(t))x_2 \\ \dot{e}_2 &= a_{2m}e_2 + (\Delta a_{21}(t) + \Delta k_{21}(t))x_1 + (\Delta a_{22}(t) + \Delta k_{22}(t))x_2\end{aligned} \tag{4}$$

where

$e_i = x_i - x_{im}$.

It can be seen from the error equations (4) that in order to obtain a stable closed loop system it is necessary to provide the following conditions:

$$\Delta a_{ij}(t) + \Delta k_{ij}(t) = 0\,. \tag{5}$$

With reference to the model reference adaptive control theory based on a Lyapunov function [1] and [2], the adaptive control laws for the system (3) are obtained in the following form:

$$\begin{aligned}\Delta\dot{k}_{11}(t) &= \sigma_1 x_1 \\ \Delta\dot{k}_{12}(t) &= \sigma_1 x_2 \\ \Delta\dot{k}_{21}(t) &= \sigma_2 x_1 \\ \Delta\dot{k}_{22}(t) &= \sigma_2 x_2\end{aligned} \tag{6}$$

where: $\sigma_i = p_{ij}e_i$.

Positive definite symmetric matrices $p_{ij}$ can be found from the solution of the matrix Lyapunov equation:

$$e^T(A_m^T P + PA_m)e = -e^T Q e$$

where

$e^T = [e_1, e_2]$,

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$$A_m = \begin{bmatrix} a_{1m} & 0 \\ 0 & a_{2m} \end{bmatrix},$$

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix},$$

$$Q = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

It can be seen from equations (6) that the adaptive contours are linked to the dynamics of the plant. Therefore, the approach is to decouple the adaptive contours in order to achieve a higher dynamic precision of the system behavior. It can be shown [3], [4] that the adaptive contours can be decoupled from the plant dynamics if $\sigma_i$ ($i = 1,2$) can be formed such that:

$$\sigma_i^* = (s + a_{im})e_i$$

where $s$ denotes a Laplace variable.

Consider the first channel ($i = 1$) of the MIMO plant. For this case the following relationship can be obtained:

$$\sigma_1^* = (\Delta a_{11}(t) + \Delta k_{11}(t))x_1 + (\Delta a_{12}(t) + \Delta k_{12}(t))x_2 \qquad (7)$$

In order to solve equation (7) with two variable parameters, the following approach is suggested. Multiply both parts of equation (7) by state variables $x_i$ ($i = 1,2$) and integrate the new equations between the time interval $(t_1, t_2)$, where: $t_2 = t_1 + \Delta t$. Taking the initial conditions as $t_1 = 0$, $\Delta k_{ij} = 0$, $(\mathrm{i, j} = 1,2)$, the following equations are obtained:

$$\begin{aligned} \int_{t_1}^{t_1+\Delta t} \sigma_1^* x_1 dt &= \Delta a_{11} \int_{t_1}^{t_1+\Delta t} x_1^2 dt + \Delta a_{12} \int_{t_1}^{t_1+\Delta t} x_1 x_2 dt \\ \int_{t_1}^{t_1+\Delta t} \sigma_1^* x_2 dt &= \Delta a_{11} \int_{t_1}^{t_1+\Delta t} x_1 x_2 dt + \Delta a_{12} \int_{t_1}^{t_1+\Delta t} x_2^2 dt \end{aligned} \qquad (8)$$

Denote:

$$\int_{t_1}^{t_1+\Delta t} \sigma_1^* x_1 dt = c_1 \qquad \int_{t_1}^{t_1+\Delta t} x_1^2 dt = l_{11} \qquad \int_{t_1}^{t_1+\Delta t} x_1 x_2 dt = l_{12}$$

$$\int_{t_1}^{t_1+\Delta t} \sigma_1^* x_2 dt = c_2 \qquad \int_{t_1}^{t_1+\Delta t} x_1 x_2 dt = l_{21} \qquad \int_{t_1}^{t_1+\Delta t} x_2^2 dt = l_{22}$$

Equations (8) can now be represented in the form:

$$\begin{aligned} c_1 &= \Delta a_{11} l_{11} + \Delta a_{12} l_{12} \\ c_2 &= \Delta a_{11} l_{21} + \Delta a_{12} l_{22} \end{aligned} \qquad (9)$$

From the solution of equations (9) the bias of the plant parameters $\Delta a_{ij}, (i, j = 1,2)$ for each channel can be determined. The controller can be adjusted according to the estimated parameter bias as:

$$\Delta k_{ij} = -\Delta a_{ij}.$$

Therefore, conditions (5) are satisfied, which means that the behavior of the system (3) follows the desirable trajectories of the model reference (2) even if the parameters of the plant are changing dramatically.

## II. EXPERIMENTAL RESULTS

Consider a MIMO process with 2 inputs and 2 outputs described by equations (1). The nominal parameters are: $a_{11}^0 = a_{12}^0 = -1$; $a_{21}^0 = a_{22}^0 = -2$. The reference model is described by equations (2) with parameters: $a_{1m} = -1$, $a_{2m} = -2$.

Fig. 1 shows that the biases from nominal parameters are deviated 100% from nominal parameters at time t =1 sec.: $\Delta a_{11} = \Delta a_{12} = 1$, $\Delta a_{21} = \Delta a_{22} = 2$. The adaptation is switched off. It can be seen that the system is unstable ($x_1$ and $x_2$) at time t =1sec.

Fig. 2 shows that the biases are the same as in Fig. 1, but the adaptation is switched on after time t =1sec. for parameters $a_{ij}(t)$, $(i, j)$=1,2. It can be seen that all parameters are adjusted at time t =10 sec. The state variables $x_1$ and $x_2$ of the process coincide with the state variables $x_{1m}$ and $x_{2m}$ of the reference model.

Fig. 3. The bias for the parameter $\Delta a_{11}$ (=100% from $a_{11}^0$) is shown. It can be seen that adaptive control compensates the variation of the parameter $a_{11}$. States $x_1$ and $x_2$ coincide exactly with states $x_{1m}$ and $x_{2m}$.

Fig. 4 illustrates an example when the parameter $\Delta a_{12}$ is deviated 100% from $a_{12}^0$.

Fig. 5 illustrates an example when the parameter $\Delta a_{21}$ is deviated 100% from $a_{21}^0$.

Fig. 6 illustrates an example when the parameter $\Delta a_{22}$ is deviated 100% from $a_{22}^0$.

Figures 1 - 6 show that the adaptive control system gives a close tracking of the rapidly changing process parameters.

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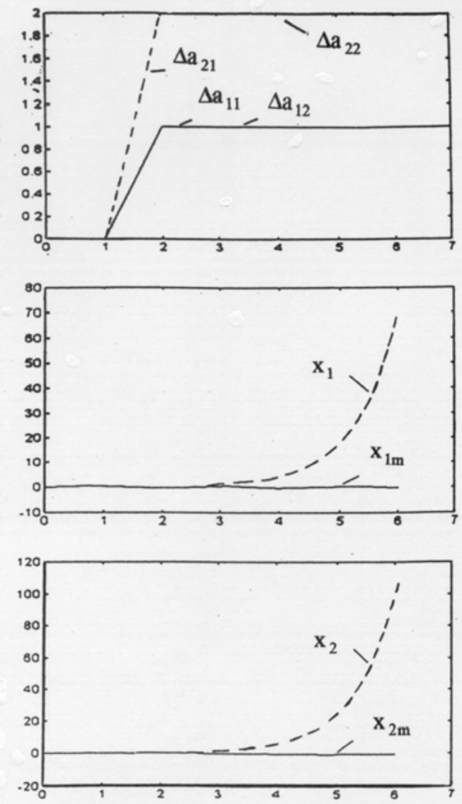


Fig. 1. Bias: $\Delta a_{11} = \Delta a_{12} = 1,\ \Delta a_{21} = \Delta a_{22} = 2.$
The adaptation is switched off

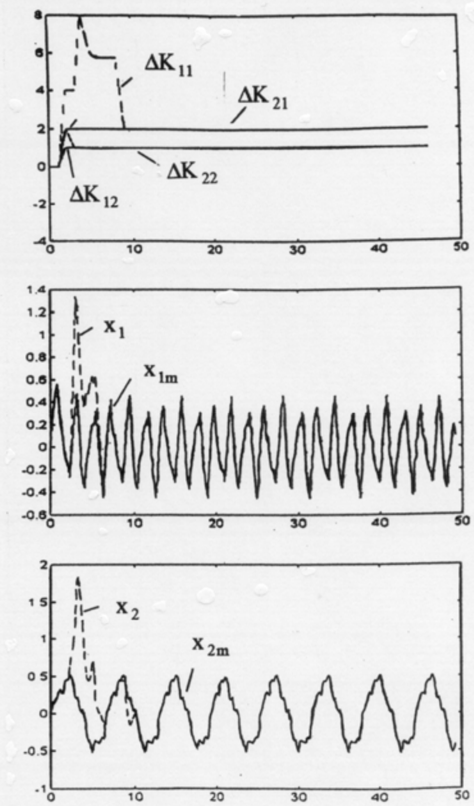


Fig. 2. Bias: $\Delta a_{11} = \Delta a_{12} = 1,\ \Delta a_{21} = \Delta a_{22} = 2.$
The adaptation is switched on.

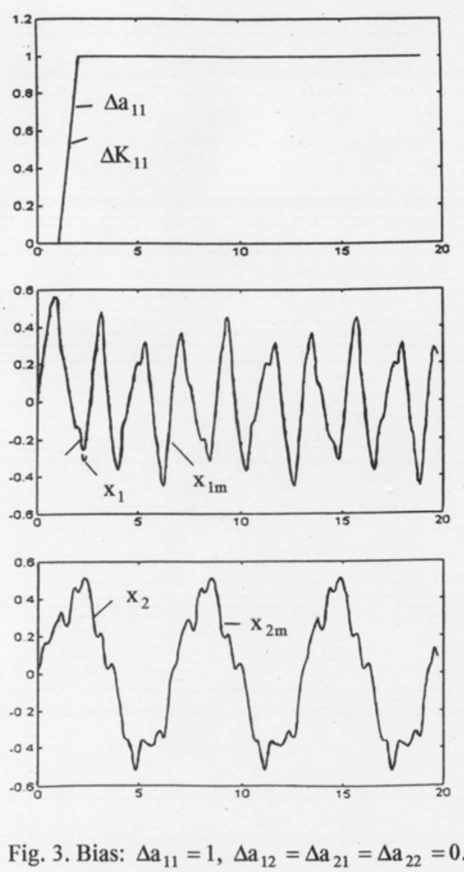


Fig. 3. Bias: $\Delta a_{11} = 1,\ \Delta a_{12} = \Delta a_{21} = \Delta a_{22} = 0.$
The adaptation is switched on.

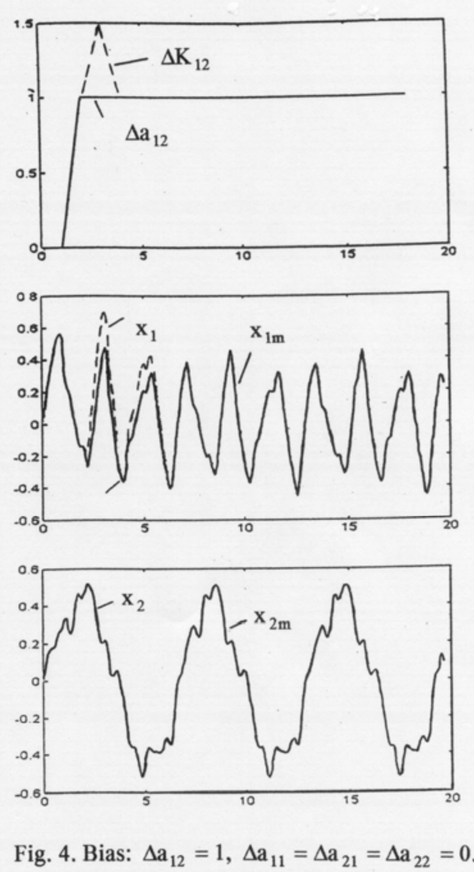


Fig. 4. Bias: $\Delta a_{12} = 1,\ \Delta a_{11} = \Delta a_{21} = \Delta a_{22} = 0.$
The adaptation is switched on.

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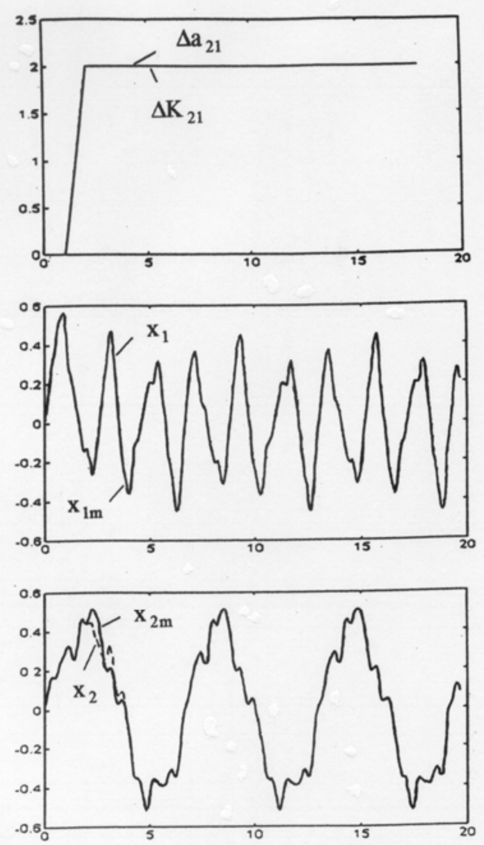


Fig. 5. Bias: $\Delta a_{21} = 2$, $\Delta a_{11} = \Delta a_{12} = \Delta a_{22} = 0$.
The adaptation is switched on.

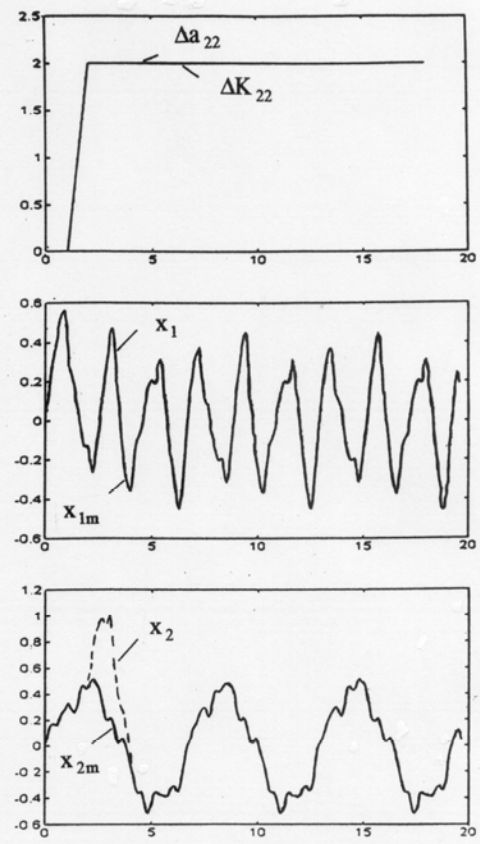


Fig. 6. Bias: $\Delta a_{22} = 2$, $\Delta a_{11} = \Delta a_{12} = \Delta a_{21} = 0$.
The adaptation is switched on.

## III. CONCLUSIONS

The adaptive control system for multivariable processes is described in this paper. The method is based on simultaneous identification and adaptation of unknown process parameters. When fault occurs, the rapidly changing parameters in the MIMO process can be compensated by applying this method.

One needs to take into account that for the solution of equations (9), according to the hypothesis of quasi-stationary of the process, the time interval $\Delta t$ is selected such that the bias of parameters $\Delta a_{ij}$ must be constant at this interval. On the other hand, the interval $\Delta t$ should be sufficiently large to accumulate values of variables $x_i$ in order to avoid singularity of the solution of equation (9) and to provide the desirable accuracy of the adaptation.

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