Abstract—The purpose of this communication is to investigate the effectiveness of a spatial indexed guidance law to perform longitudinal trajectory tracking with overly time constraints. The aircraft flight dynamics are expressed in a space indexed frame and a nonlinear control structure is adopted. The influence of wind and navigation estimation errors on the 2D+T guidance performance is analyzed while simulation results are displayed.

Index Terms—Trajectory optimization, trajectory tracking, guidance dynamics, nonlinear control, space reference.

I. INTRODUCTION

World air transportation traffic has known a sustained increase over the last decades leading to airspace near saturation in large areas of developed and emerging countries. Then safety and environmental considerations urge today for the development of new guidance systems with improved accuracy for spatial and temporal trajectory tracking. Available infrastructure of current ATM (Air Traffic Management) will no longer be able to stand this growing demand unless breakthrough improvements are made. In the future air traffic management environment, in addition to following the trajectory cleared by ATC, aircraft will progress in four dimensions, sharing accurate airborne predictions with the ground systems, and being able to meet time constraints at specific waypoints with high precision when the traffic density requires it [1] and [2]. This will allow better separation and sequencing of traffic flows while green climb/descent trajectories will be feasible in terminal areas.

In this study we adopt a spatial reference [3] to ease the way of these necessary developments in air traffic control and management by providing new capabilities for aircraft by coping more efficiently with trajectory tracking accuracy and simultaneously with overly time constraints. Then we consider the influence of wind and position estimation error scenarios. First order approximation tracking errors are established and full nonlinear simulations are performed with different wind and position estimation error scenarios.

II. AIRCRAFT LONGITUDINAL FLIGHT GUIDANCE DYNAMICS

Here we introduce the longitudinal flight guidance dynamics of a transportation aircraft. These equations are extracted from the full flight dynamics [4] by deleting the fast longitudinal rotational dynamics composed of the pitch Euler equation and the pitch moment. The basic inputs considered in this study for the vertical guidance dynamics are the pitch angle $\theta$ and the total engines thrust $T$. The flight guidance dynamics are first introduced with respect to time as the independent parameter and then they are rewritten to take the horizontal abscissa as the independent parameter.

A. Time Indexed Longitudinal Guidance Dynamics

Let $x$ and $z$ represent in a vertical plane the coordinates of the center of gravity of an aircraft at time $t$. Let $V_{air}$ be the modulus of the airspeed of the aircraft at time $t$, let $\gamma_{air}$ be the airspeed path angle, then the rate of change with respect to time of the aircraft altitude is given by:

$$\dot{z} = V_{air} \sin \gamma_{air} + w_z$$  \hspace{1cm} (1.a)

where $w_z$ is the vertical wind speed component. The airspeed and the airspeed path angle obey to equations:

$$\dot{V}_{air} = \frac{1}{m} \left( T \cos \alpha - D(z, V_{air}, \alpha) - mg \sin \gamma_{air} \right)$$  \hspace{1cm} (2.a)

$$\dot{\gamma}_{air} = \frac{1}{m V_{air}} \left( T \sin \alpha + L(z, V_{air}, \alpha) - mg \cos \gamma_{air} \right)$$  \hspace{1cm} (3.a)

where $T$ and $D(z, V_{air}, \alpha)$ are respectively the engine thrust and the drag force. Here $m$ is the mass of the aircraft. The longitudinal position of the aircraft is given by:

$$\dot{x} = V_{air} \cos \gamma_{air} + w_x$$  \hspace{1cm} (4.a)

where $w_x$ is the horizontal wind speed component.

B. Space Indexed Longitudinal Guidance Dynamics

Here we adopt the following notation:

$$\frac{d^4 u}{dx^4} = u^{[4]}$$  \hspace{1cm} (5)

where $u$ is any physical variable depending of the abscissa $x$ of the COG of the aircraft. Then taking into account relations (1.1) and (1.2), we can write the aircraft flight dynamics with respect to space as:

$$z^{[1]} = V_{air} \sin \gamma_{air} + w_z$$  \hspace{1cm} (1.b)

The other state equations are now given by:
It is possible to introduce two auxiliary inputs \( u_1 \) and \( u_2 \) as:

\[
\begin{align*}
u_1 &= \frac{T \cos \alpha - D(z, \dot{V}_w, \alpha)}{m(\dot{V}_w \cos \gamma_{aw} + \dot{w}_i)} \\
u_2 &= \gamma_{aw}
\end{align*}
\]

and

\[
\begin{align*}
u_1 &= \frac{T \cos \alpha - D(z, \dot{V}_w, \alpha)}{m(\dot{V}_w \cos \gamma_{aw} + \dot{w}_i)} \\
u_2 &= \gamma_{aw}
\end{align*}
\]

since \( \alpha = \theta - \gamma_{aw} \) matrix:

\[
\begin{pmatrix}
\dot{\gamma}_{aw} \\
\dot{\theta} \\
\end{pmatrix}
\]

is non-singular.

In [5] it has been shown how to perform on line this numerical inversion using feed-forward neural networks since the guidance dynamics of an aircraft have been shown to be differentially flat. Then we can write:

\[
\begin{pmatrix}
z_2 \\
\dot{t}_2
\end{pmatrix}
\]

with

\[
\begin{pmatrix}
\alpha(V_w, \gamma_{aw}, \dot{w}_i) \\
V_{aw}
\end{pmatrix}
\]

In a constant wind situation with \( w = [\dot{w}_e \ \dot{w}_n] \), we have:

\[
\begin{align*}
z^{[1]} &= \frac{V_{aw} \sin \gamma_{aw} + \dot{w}_w}{V_{aw} \cos \gamma_{aw} + \dot{w}_i} \\
\end{align*}
\]

The acceleration equations are now given by:

\[
\begin{align*}
V_{aw}^{[1]} &= \frac{(T \cos \alpha - D(z, \dot{V}_w, \alpha) - mg \sin \gamma_{aw})}{m(\dot{V}_w \cos \gamma_{aw} + \dot{w}_i)} \\
\gamma_{aw}^{[1]} &= \frac{(T \sin \alpha + L(z, \dot{V}_w, \alpha) - mg \cos \gamma_{aw})}{mV_{aw}(\dot{V}_w \cos \gamma_{aw} + \dot{w}_i)} \\
\end{align*}
\]

and

\[
\begin{align*}
t^{[1]} &= \frac{1}{V_{aw} \cos \gamma_{aw} + \dot{w}_i} \\
\end{align*}
\]

It appears that matrix \( B \) is invertible in normal flying conditions. For example in a no wind condition, matrix \( B \) reduces to:

\[
B = \begin{bmatrix}
0 & V_{aw} \cos \gamma_{aw} \\
-1/(V_{aw}^2 \cos \gamma_{aw}) & -tg \gamma_{aw}/(V_{aw} \cos \gamma_{aw})
\end{bmatrix}
\]

Then, adopting a reference profile \( z_{ref}(x) \) such that there is no conflict with ground proximity and a reference time table \( t_{ref}(x) \) so that minimum and maximum speed constraints are satisfied, stable second order dynamics can be enforced to the altitude and overfly time errors by the spatial controls provided by nonlinear inverse control theory [6]:

\[
\begin{pmatrix}
u_1(x) \\
u_2(x)
\end{pmatrix}
\]

with

\[
\begin{pmatrix}
b_1 & b_2 \\
b_3 & b_4
\end{pmatrix}
\]

III. SPATIAL INDEXED NOMINAL GUIDANCE

In this section we consider the design of a control law to follow accurately a space indexed trajectory composed of altitude and overfly time references. Then the considered control objectives are:

- To follow accurately a space-referenced vertical profile \( z_r(x) \);
- To respect as much as possible a desired time table \( t_s(x) \).

Of course these objectives and reference trajectories must be established in accordance with aircraft performances, air traffic control constraints as well as economic and environmental considerations.

It is possible to write:

\[
\begin{align*}
z^{[2]} &= \frac{V_{aw} \sin \gamma_{aw} - \dot{w}_w \cos \gamma_{aw}}{(V_{aw} \cos \gamma_{aw} + \dot{w}_i)^2} \\
&\quad + \frac{(V_{aw}^2 + V_{aw} \cos \gamma_{aw} + \dot{w}_i \sin \gamma_{aw}) V_{aw}^{[1]} \sin \gamma_{aw}^{[1]}}{(V_{aw} \cos \gamma_{aw} + \dot{w}_i)^2} \\
t^{[2]} &= -\frac{V_{aw} \cos \gamma_{aw}^{[1]} - V_{aw} \sin \gamma_{aw}^{[1]}}{(V_{aw} \cos \gamma_{aw} + \dot{w}_i)^2}
\end{align*}
\]
Here second order damping parameters $\delta$, $\delta_i$ and natural frequencies $\omega_0$ and $\omega_i$, are taken so that the two polynomials $p^2 + 2\delta_0 p + \omega_0^2$ and $p^2 + 2\delta_i p + \omega_i^2$, $i \in [z, t]$ have stable roots. According to Non Linear Inverse control theory, since outputs $z$ and $t$ present relative degrees covering the whole dimension $(1+1=4-2)$ of the considered spatial guidance dynamics there is no internal dynamics which insures the stability of the full nominal spatial guidance dynamics controlled by $\ldots$

Then, the altitude and overfly time tracking errors given by:

$\xi(t) = z(t) - z_s(t)$ \hspace{1cm} (14.a)

$\xi(t) = t(t) - t_s(t)$ \hspace{1cm} (14.b)

obey to the spatial linear decoupled dynamics such as:

$\xi^{(2)}(x) + 2\delta_0 \xi^{(1)}(x) + \omega_0^2 \xi(x) = 0$ \hspace{1cm} (15.a)

$\xi^{(2)}(x) + 2\delta_i \xi^{(1)}(x) + \omega_i^2 \xi(x) = 0$ \hspace{1cm} (15.b)

IV. EFFECTIVE GUIDANCE PERFORMANCE UNDER SPACE INDEXED CONTROL

Here we consider that when applying the proposed control law, various discrepancies appear:

- wind components vary with space and time,
- there is a position error produced by the navigation function.

Then the altitude and overfly time dynamics follow the equations:

$$\begin{bmatrix} \dot{x}^{(2)} \\ \dot{t}^{(2)} \end{bmatrix} = \begin{bmatrix} a_1(V_{aw}, \gamma_{aw}, w_0, \bar{w}) & b_{11} & b_{12} \\ a_1(V_{aw}, \gamma_{aw}, w_0, \bar{w}) & b_{11} & b_{12} \end{bmatrix} \begin{bmatrix} \tilde{u}_1(x) \\ \tilde{u}_2(x) \end{bmatrix}$$ \hspace{1cm} (16)

or

$\chi^{(2)}(x) = g(x) + B(x) \cdot \tilde{u}(x)$ \hspace{1cm} with $\gamma = [z, t]$ \hspace{1cm} (17)

where $\tilde{u}(x)$ is the control applied at true position $x$ while the space indexed estimated position $\hat{x}$ is given by:

$$\hat{x}(x) = x + \Delta x(x)$$ \hspace{1cm} (18)

Here $\Delta x(x)$ is the position estimation error at true but unknown position $x$. It is supposed that measurements of pressures (Pitot), pitch angle $\theta$ and angle of attack $\alpha$, provide online accurate values for the airspeed $V_{aw}$ and the air path angle $\gamma_{aw} = \theta - \alpha$. It is also supposed that an online estimation of the components of the wind speed, $\bar{w}(x)$ and $\bar{w}_0(x)$ are available.

Then vector $\bar{a}$ and matrix $B$ are such as:

$$a_1 = -\frac{(w_s \sin \gamma_{aw} - w_c \cos \gamma_{aw}) g \sin \gamma_{aw}}{(V_{aw} \cos \gamma_{aw} + w_c)^2}$$ \hspace{1cm} (19.a)

$$\begin{align*}
\Delta a_1 &= \frac{(w_s \sin \gamma_{aw} - w_c \cos \gamma_{aw}) (w_0 \cos \gamma_{aw} - w_0^2 \sin \gamma_{aw})}{(V_{aw} \cos \gamma_{aw} + w_c)^2} \\
b_{11} &= \frac{w_s \sin \gamma_{aw} - w_c \cos \gamma_{aw}}{(V_{aw} \cos \gamma_{aw} + w_c)^2} \\
b_{12} &= \frac{-w_s \cos \gamma_{aw}}{(V_{aw} \cos \gamma_{aw} + w_c)^2} \\
b_{21} &= \frac{-w_c \sin \gamma_{aw}}{(V_{aw} \cos \gamma_{aw} + w_c)^2} \\
b_{22} &= \frac{-w_c \cos \gamma_{aw}}{(V_{aw} \cos \gamma_{aw} + w_c)^2}
\end{align*}$$ \hspace{1cm} (19.b)
\[
\left( \frac{\partial \varphi_i(\hat{x},x)}{\partial x} \right)_{\text{inputs}} = z_i^{(1)}(\hat{x}) + 2\delta_i \omega_i z_i^{(1)}(\hat{x}) + \omega_i^2 z_i^{(1)}(\hat{x}) \quad (29.b)
\]

\[
\left( \frac{\partial \varphi_i(\hat{x},x)}{\partial x} \right)_{\text{inputs}} = t_i^{(1)}(\hat{x}) + 2\delta_i \omega_i t_i^{(1)}(\hat{x}) + \omega_i^2 t_i^{(1)}(\hat{x}) \quad (29.c)
\]

Then, outputs \( z \) and \( t \) follow in a first order approximation of \( \varphi \), the dynamics given by:

\[
\zeta^{(2)}(x) = \varphi(x) + \Delta \varphi(x) \quad (30)
\]

or

\[
\zeta^{(2)}(x) + 2\delta_i \omega_i \zeta^{(1)}(x) + \omega_i^2 \zeta(x) = \Delta \varphi_i(x) \quad (31.a)
\]

and

\[
\zeta^{(2)}(x) + 2\delta_i \omega_i \zeta^{(1)}(x) + \omega_i^2 \zeta(x) = \Delta \varphi_i(x) \quad (31.b)
\]

where

\[
\Delta \varphi(x) \approx \Delta \varphi(x) + D_\varphi(x) \cdot (\varphi(x) - \varphi(x)) + \nabla \varphi \cdot \Delta x \quad (32)
\]

This forcing error is equal to zero in a no wind scenario with a zero position estimation error.

V. SIMULATION STUDY

The proposed spatial control approach is illustrated using the Research Civil Aircraft Model (RCAM) which has the characteristics of a wide body transportation aircraft [7] with a maximum allowable landing mass of about 125 tons with a nominal landing speed of 68m/s. Two position estimation errors have been considered: \( \Delta x = 0 \) m and \( \Delta x = 250 \) m, while two wind conditions have been studied: \( w = 0 \) m/s and \( w = 12 \) m/s. 

![Fig. 1 Altitude tracking error, no wind and \( \Delta x = 0 \)](image1)

![Fig. 2 Time tracking error, no wind and \( \Delta x = 0 \)](image2)

![Fig. 3 Altitude tracking error, no wind and \( \Delta x = 250 \) m)](image3)

![Fig. 4 Time tracking error, no wind and \( \Delta x = 250 \) m)](image4)
From the above simulation results it appears that the proposed approach is feasible and provide quite accurate trajectory tracking performance both in altitude and overfly time.

VI. CONCLUSION

In this communication we have considered the use of a spatial reference to provide new capabilities for aircraft operating within a more and more dense traffic. One of the objectives has been to cope more efficiently with overfly and arrival time constraints. The trajectory optimization problem corresponding to reference trajectory generation at the level of flight management, as well as time constrained vertical trajectory tracking at the level of flight guidance have been considered.

The main objective here has been to improve the tracking accuracy performance of the guidance along a 2D+T reference trajectory. This has led to the development of a new representation of longitudinal flight dynamics where the independent variable is ground distance of the aircraft to a reference point. The nonlinear inverse control technique has been applied in this context so that tracking errors follow independent and asymptotically stable spatial dynamics around the desired trajectories. It appears that even in the presence of wind and of a position estimate error up to 250 m, the proposed approach results in good performances as well as in enhanced track predictability.

To get applicability this new approach still should overcome important challenges related mainly with the development of enhanced navigation and wind estimation systems. Then an improved integration of on board flight path functions including the consideration of neighbouring traffic and the guidance functions, will become possible.

REFERENCES


