Parametric Control of Regional Economic Growth based on the One Computable General Equilibrium Model

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Abstract—**Based on computable general equilibrium model (CGE model) "Center-Regions" application efficiency of the proposed method of parametric identification for large mathematical models is illustrated in the paper. Based on the model we conduct analysis of economic growth sources and application efficiency of parametric control theory for conducting state economic policy, directed to economic growth and reduction of disproportion of regional socio-economic development.**

Index Terms—**CGE model, economic growth, parametric control, parametric identification.**

I. Introduction

As it is known, there is a broad consensus among macroeconomists on the application of mathematical models for macroeconomic analysis [1], [2] and solution of problems of economic growth [3], [4]. One of the topical problems of regulation of economic growth is the problem of ensuring sustainable growth of the national economy, taking into account the requirements of smoothing the levels of socio-economic development of certain regions of the country.

Reference [5] describes the CGE model, "Russia: Center - Federal Districts", based on which the scenario approach is considered for assessing the feasibility of smoothing regional disparities in the Russian Federation after solving the calibration problem. However issues of analysis of economic growth sources for each region and problems of finding the optimal rules for state economic policy are not addressed in [5].

A parametric control theory for macroeconomic analysis and evaluating optimal values of economic state policy tools based on the set classes of macroeconomic models are proposed in [6], [7]. The present work contains illustrations of some statements of the parametric control theory on example of a discrete non-autonomous large-scale CGE model "Center-Regions":

- Parametric identification of the model based on statistical data of the Republic of Kazakhstan economy,
- Analysis of sources of regional economic growth based on production functions of estimated model and
- Synthesis of parametric control optimal laws for problems in the sphere of economic growth and reduction in disproportion of regional socio-economic development.

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II. Representation of CGE Models

Considered CGE models, including the CGE model "Center-Regions" are presented in general view with the help of the following system of relations [7].

1) Subsystem of recurrent relations, connecting the values of endogenous variables for the two consecutive years:

$$x_1(t+1) = f_1\big(x_1(t), x_2(t), x_3(t), u(t), a(t)\big). \quad (1)$$

Here $t = 0, 1, \dots, n-1$ – number of a year, discrete time; $x(t) = (x_1(t), x_2(t), x_3(t)) \in R^m$ – vector of endogenous variables of the system;

$$x_i(t) \in X_i(t) \subset R^{m_i},\ i = 1, 2, 3. \quad (2)$$

Here variables

$x_1(t)$ include the values of capital stocks of a regions' economic agents, budgets of economic agents and other;

$x_2(t)$ include demand and supply values of economic agents of regions in different markets and other;

$x_3(t)$ – different types of market prices and budget shares in markets with fixed prices for different economic agents; $m_1 + m_2 + m_3 = m$;

$u(t) \in U(t) \subset R^q$ – vector-function of controllable parameters. Values of the coordinates of this vector correspond to different tools of state economic policy, for example, such as shares of state budget and shares of economic agents' budgets, different tax rates and other;

$a(t) \in A \subset R^s$ – vector-function of uncontrollable parameters (factors). Values of the coordinates of this vector characterize different dependent on time external and internal socio-economic factors: prices for imported and exported goods, population size of the county, parameters of production functions and other;

$X_1(t), X_2(t), X_3(t), U(t)$ – compact sets with non-empty interiors; $X_i = \bigcup_{t=1}^{n} X_i(t)$, $i = 1, 2, 3$; $X = \bigcup_{i=1}^{3} X_i$; $U = \bigcup_{t=0}^{n-1} U(t)$, A – open connected set;

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$f_1: X \times U \times A \to R^{m_1}$ – continuous mapping.

2) A subsystem of algebraic equations characterizing behavior and interaction of agents in different markets during the selected year, these equations allow expression of variables $x_2(t)$ by exogenous parameters and other endogenous variables:

$$x_2(t) = f_2(x_1(t), x_3(t), u(t), a(t)). \quad (3)$$

Here $f_2: X_1 \times X_3 \times U \times A \to R^{m_2}$ – continuous mapping.

3) Subsystem of recurrent relations for iterative calculation of market prices equilibrium values in different markets and budget shares in markets with state prices for different economic agents:

$$x_3(t)[Q+1] = f_3(x_2(t)[Q], x_3(t)[Q], L, u(t), a(t)). \quad (4)$$

Here $Q = 0, 1, \dots$ – number of iteration; L – set of positive numbers (adjusted coefficients of iteration, when their values decrease economic system reaches equilibrium faster, but the risk that prices go to negative domain increases; $f_3: X_2 \times X_3 \times (0, +\infty)^{m_3} \times U \times A \to R^{m_2}$ – continuous mapping (contracting at fixed t; $x_1(t) \in X_1(t); u(t) \in U(t)$; $a(t) \in A$ and some fixed L. In this case f_3 mapping has the unique fixed point, where the iteration process (3), (4) converges.

CGE model (1), (3), (4) at fixed values of the functions $u(t)$ and $a(t)$ at each point of time t defines values of endogenous variables $x(t)$, corresponding to equilibrium of demand and supply prices in markets of goods and services of agents within the framework of the following algorithm.

1) On the first step we assume that $t = 0$ and initial values of the variables $x_1(0)$ are set.

2) On the second step for the current t we set initial values for variables $x_3(0)[0]$ in different markets and for different agents; with the help of (3) we compute values $x_2(t)[0] = f_2(x_1(t), x_3(t)[0], u(t), a(t))$, (initial demand and supply values of agents in markets of goods and services).

3) On the third step for the current t the process of iteration (4) is run. Meanwhile for each value of Q current demand and supply values are found with the help of (3): $x_2(t)[Q] = f_2(x_1(t), x_3(t)[Q], u(t), a(t))$, through refinement of market prices and budget shares of economic agents.

A condition for iteration process to stop is the equality of supply and demand in different markets (accurate within 0.01%). As a result we obtain equilibrium values of market prices for each market and budget shares in markets with state prices for different economic agents. Q index is omitted for such equilibrium values of endogenous variables.

4) On the next step values of variables $x_1(t+1)$ are found with the help of obtained equilibrium solution for time t applying differential equations (1). A value of t increases by 1. Jump to step 2.

The number of steps 2, 3, 4 iterations is defined according to problems of parametric identification, forecasting and control for the time intervals selected in advance.

The considered CGE model can be presented as continuous mapping $f: X \times U \times A \to R^m$, giving transformation of values of the system's endogenous variables for a null year to the corresponding values of the consecutive year according to the algorithm stated above. Here the compacts $X(t) = X_1(t) \times X_2(t) \times X_3(t)$, giving a compact X in the space of endogenous variables are determined by set of possible values of x_1 variable and corresponding equilibrium values of variables x_2 and x_3, estimated by the ratios (3) and (4).

We assume that for selected point $x_1(0) \in \text{Int}(X_1)$ and corresponding point $x(0) = (x_1(0), x_2(0), x_3(0))$, computed with the help of (3) and (4), an inclusion $x(t) = f^t(x(0)) \in \text{Int}(X(t))$ is true at some fixed $u(t) \in \text{Int}(U(t))$, $a(t) \in A$ for $t = 0, \dots, n$. (n – fixed non-negative integer number). This mapping f defines a discrete dynamic system in the set X, on the trajectory of which the following initial condition is imposed:

$$\{f^t,\ t = 0,1, \dots\}, x|_{t=0} = x_0. \quad (5)$$

Based on this expression below we consider a particular CGE model "Center-Regions".

III. Brief Description and Parametric Identification of the CGE Model "Center-Regions"

The considered model on statistical data for the Republic of Kazakhstan and its 16 regions is presented by the following 66 economic agents (sectors):

- 16 legal sectors of economy of all regions;
- 16 shadow sectors of economy of all regions;
- 16 aggregate consumers of all regions;
- 16 regional authorities;
- Government, represented by central government and also by non-budget funds. This sector also includes noncommercial organizations serving households (political parties, trade unions, public associations etc.);
- Banking sector, involving Central bank and commercial banks.

Here the first 32 economic sectors are producing agents.

The considered model is presented within the framework of general expressions of ratios (1), (3), (4) respectively by $m_1 = 240$, $m_2 = 4554$, $m_3 = 160$ expressions, with the help of which values of its 4954 endogenous variables are calculated. This model also contains 391,122 estimated exogenous parameters.

The problem of parametric identification of the researched macroeconomic model is to find estimates of unknown values of its parameters at which a minimum value of the objective function is reached. This objective function characterizes deviations of values of the model's output variables from corresponding observed values (known statistical data for the time interval $t = t_1, t_1 + 1, \dots, t_2$). This problem is to find minimum value of the function of several variables (parameters) at some closed set in the domain D of the Euclidian space with constraints of type (2), imposed on values of endogenous variables. Standard methods of finding the function's minimums are often inefficient due to existence of multiple local minimums of an objective function in case of high dimensionality of the

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region of possible arguments' values. Below we present an algorithm, that considers peculiarities of the parametric identification problem of macroeconomic models and that allows to avoid the problem of "local extremums".

The domain of type $D = \prod_{i=1}^{(q+s)(t_2-t_1+1)+m_1}[a^i, b^i]$, where $[a^i, b^i]$ – possible values interval of the parameter p^i; $i = 1, \dots, (q+s)(t_2 - t_1 + 1) + m_1$, is considered as a domain $D \subset \prod_{t=t_1}^{t_2}[U(t) \times A(t)] \times X_1(t_1)$ for estimating possible values of exogenous parameters (values of exogenous functions $u(t)$, $a(t)$ and initial conditions of dynamic equations (1)). Meanwhile, parameter values, for which we have observed values, are searched at intervals $[a^i, b^i]$ with centers at corresponding observed values (in case if there is one such value) or at some intervals, covering observed values (in case if there are several such values). Other intervals $[a^i, b^i]$ for parameter search have been selected with the help of indirect estimations of their possible values. To find minimal values of continuous multivariable function $K: D \to R$ with additional constraints imposed on endogenous variables of type (2) at computational experiments the Nelder-Mead algorithm [8] of directed search has been applied. Using this algorithm for the starting point $p_1 \in D$ can be interpreted as converging to the point (of local minimum) $p_0 = \underset{D,(2)}{\operatorname{argmin}} K$ of sequence $\{p_1, p_2, p_3, \dots\}$, where $K(p_{j+1}) \leq K(p_j)$, $p_j \in D$, $j = 1,2, \dots$ In description of the following algorithm we assume that p_0 can be found sufficiently accurately.

To solve the problem of parametric identification of the considered CGE model based on the assumption of apparent discrepancy (in general) of minimum points of two different functions two criterions of the following type are proposed:

$$K_A(p) = \sqrt{\frac{1}{n_\alpha(t_2-t_1+1)} \sum_{t=t_1}^{t_2} \sum_{i=1}^{n_A} \alpha_i \left(\frac{y^i(t) - y^{i*}(t)}{y^{i*}(t)}\right)^2},$$

$$K_B(p) = \sqrt{\frac{1}{n_\beta(t_2-t_1+1)} \sum_{t=t_1}^{t_2} \sum_{i=1}^{n_B} \beta_i \left(\frac{y^i(t) - y^{i*}(t)}{y^{i*}(t)}\right)^2}. \quad (6)$$

Here $\{t_1, \dots, t_2\}$ – identification time interval; $y^i(t)$, $y^{i*}(t)$ – estimated and observed values of output variables of the model respectively, $K_A(p)$ – auxiliary criterion, $K_B(p)$ – main criterion; $n_B > n_A$; $\alpha_i > 0$ and $\beta_i > 0$ – some weight coefficients, their values are determined during the process of solving the parametric identification problem for the dynamic system; $\sum_{i=1}^{n_A} \alpha_i = n_\alpha$, $\sum_{i=1}^{n_B} \beta_i = n_\beta$.

Algorithm of solving the problem of parametric identification of the model is selected with the help of following steps.

1) Problems A and B are solved simultaneously for a vector of initial values of parameters $p_1 \in D$. As a result points p_{A0} and p_{B0} of minimums criteria K_A and K_B are found respectively.

2) If $K_B(p_{B0}) < \varepsilon$ is true for some sufficiently small value ε, then the problem of parametric identification of the model (1), (3), (4) is solved.

3) Otherwise problem A is solved applying point p_{B0} as initial point p_1, problem B is solved applying point p_{A0} as p_1. Jump to step 2.

Quite large number of iterations of steps 1, 2, 3 provides an opportunity for searched values of parameters to exit from neighborhood points of nonglobal minimums of one criterion with the help of another criterion, thus solve the problem of parametric identification.

As a result of joint solution of problems A и B according to the specified algorithm applying statistical data on evolution of the Republic of Kazakhstan economy, the Nelder-Mead algorithm [8] we have obtained the following values $K_A = 0.034$ and $K_B = 0.047$. Relative magnitude of deviations of parameter calculated values used in the main criterion from corresponding observed values is less than 4.7%.

Further calculation of the estimated model on the parametric identification interval and outside the period of parametric identification (forecasted estimation) with the help of extrapolated values $u(t)$, $a(t)$ is called a basic calculation.

Results of calculation and of retrospective basic calculation of the model for 2011, partially presented in Table I, demonstrate estimated, observed values and deviations of estimated values of main output variables of the model from corresponding observed values. Here the time interval 2000–2010 corresponds to the period of parametric identification of the model; 2011 – is a period of retroforecasting; $Y(t)$ – total gross output of a legal sector ($\times 10^{12}$ tenge, in prices of 2000; tenge – national currency of Kazakhstan); $Y_g(t)$ – GDP of a state ($\times 10^{12}$ tenge, in prices of 2000); a sign "*" corresponds to observed values, a sign "Δ" corresponds to deviations (in percentage) of estimated values from corresponding observed values.

TABLE I
OBSERVED, CALCULATED VALUES OF OUTPUT VARIABLES OF THE MODEL AND CORRESPONDING DEVIATIONS

Indicator	Year					
	2000	2001	2002	2003	2004	2005
$Y^*(t)$	5.30	6.26	6.33	6.87	7.84	8.44
$Y(t)$	5.16	6.42	6.44	6.98	7.81	8.23
$\Delta Y(t)$	-2.90	-1.00	-3.00	0.10	0.60	2.40
$Y_g^*(t)$	2.31	2.62	2.88	3.18	3.52	3.86
$Y_g(t)$	2.25	2.58	2.93	3.21	3.47	3.81
$\Delta Y_g(t)$	0.60	2.30	0.10	-1.50	0.50	2.30

TABLE I CONTINUED

Indicator	Year					
	2006	2007	2008	2009	2010	2011
$Y^*(t)$	9.04	9.87	9.92	1.04	1.09	1.14
$Y(t)$	8.79	9.65	9.85	1.05	1.13	1.15
$\Delta Y(t)$	1.50	-2.80	0.40	0.30	-0.10	0.20
$Y_g^*(t)$	4.72	5.14	5.30	5.36	5.50	5.65
$Y_g(t)$	4.70	5.19	5.17	5.52	5.36	5.58
$\Delta Y_g(t)$	-2.80	1.90	2.50	-1.90	-2.80	-2.20

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IV. Analysis of Regional Economic Growth Sources on the Basis of the CGE Model "Center-Regions"

In this section we make analysis of economic growth sources of legal sectors of the Republic of Kazakhstan regions on the basis of the CGE model "Center-Regions", which exogenous functions and parameters have been evaluated as a result of solving the parametric identification problem of the model based on the statistical data of the socio-economic development of the Republic of Kazakhstan for 2000–2010.

The researched model uses the following expressions of multiplicative production functions of legal sectors of 16 regions:

$$Y_i(t+1) = A_i^{\mathrm{r}}(t) \times \left[\sum_{j=1}^{16}\left(D_i^{zj1}(t) + D_i^{zj2}(t)\right)\right]^{A_i^{\mathrm{z}}} \times \exp\left[A_i^{\mathrm{zIm}} \times D_i^{\mathrm{zIm}}(t)\right] \times \left[\frac{K_i(t)+K_i(t+1)}{2}\right]^{A_i^{\mathrm{k}}} \times \exp\left[A_i^{\mathrm{l}} \times D_i^{\mathrm{l}}(t)\right]. \quad (7)$$

Here t – time in years; Y_i – real output of a legal sector in region i (i – number of a region, i = 1, …, 16, see Table II); D_i^{zj1} – real demand of a i region's legal sector for intermediate goods, produced by a legal sector of a region j; D_i^{zj2} – real demand of a i region's legal sector for intermediate goods, produced by a shadow sector of a region j; D_i^{zIm} – real demand of a i region's legal sector for imported intermediate goods; D_i^{l} – demand of a i region's legal sector for labor; K_i – real capital funds of a i region's legal sector; A_i^{r}, A_i^{z}, A_i^{zIm}, A_i^{k}, A_i^{l} – known exogenous functions.

Let us evaluate influence of growth rate of this function's arguments on growth rates of output $Y_i(t+1)$ of legal sector in a region in the assumption of constant exogenous functions A_i^{z}, A_i^{zIm}, A_i^{k}, A_i^{l}. Such assumption is used at extrapolation of these functions for the period of forecasting: 2012–2015.

Having taking the logarithms of both sides (7), then having found the total increment of the function and having dropped the high-order infinitesimals we obtain the following estimate for growth rate $y_i = \frac{\Delta Y_i}{Y_i}$ of real output of a legal sector in a region i depending on growth rates of endogenous arguments ($D_i^{\mathrm{z}} = \sum_{j=1}^{16}\left(D_i^{zj1} + D_i^{zj2}\right)$, D_i^{zIm}, $K_i^{\mathrm{m}}(t) = \frac{K_i(t)+K_i(t+1)}{2}$, D_i^{l}) of production functions and an exogenous coefficient of technical progress (A_i^{r}).

$$y_i = \frac{\Delta A_i^{\mathrm{r}}}{A_i^{\mathrm{r}}} + A_i^{\mathrm{z}}\frac{\Delta D_i^{\mathrm{z}}}{D_i^{\mathrm{z}}} + \left(A_i^{\mathrm{zIm}} D_i^{\mathrm{zIm}}\right)\frac{\Delta D_i^{\mathrm{zIm}}}{D_i^{\mathrm{zIm}}} + A_i^{\mathrm{k}}\frac{\Delta K_i^{\mathrm{m}}}{K_i^{\mathrm{m}}} + \left(A_i^{\mathrm{l}} D_i^{\mathrm{l}}\right)\frac{\Delta D_i^{\mathrm{l}}}{D_i^{\mathrm{l}}}. \quad (8)$$

Here D_i^{z} – total demand of a i region's legal sector for intermediate goods, produced by legal as well as shadow sectors of all regions; K_i^{m} – annual average real capital stock of the legal sector in a region i.

Let $a_i = \frac{\Delta A_i^{\mathrm{r}}}{A_i^{\mathrm{r}}}$ denote the rate of technical progress of a i region's legal sector; $z_i = \frac{\Delta D_i^{\mathrm{z}}}{D_i^{\mathrm{z}}}$ – intermediate goods consumption rate by a legal (or shadow) sector in a region j; $z_i^{\mathrm{Im}} = \frac{\Delta D_i^{\mathrm{zIm}}}{D_i^{\mathrm{zIm}}}$ – imported intermediate goods consumption rate by a legal sector in a region i, $k_i = \frac{\Delta K_i^{\mathrm{m}}}{K_i^{\mathrm{m}}}$ – capital accumulation rate in a i region's legal sector; $l_i = \frac{\Delta D_i^{\mathrm{l}}}{D_i^{\mathrm{l}}}$ – labor costs growth rate in a region i, where the sign "Δ" indicates increment of a variable in one year; time in (8) is omitted for briefness.

Coefficients at the right-hand side of (8) at the rates indicated above characterize degree of influence of the considered factors on economic growth and allows to compare their influence with influence of technical progress growth rate, at which the coefficient is equal to 1. Having denoted these coefficients in terms of $\alpha_i = A_i^{\mathrm{z}}$, $\beta_i = A_i^{\mathrm{zIm}} D_i^{\mathrm{zIm}}$, $\gamma_i = A_i^{\mathrm{k}}$, $\delta_i = A_i^{\mathrm{l}} D_i^{\mathrm{l}}$, we get brief version of (8):

$$y_i = a_i + \alpha_i z_i + \beta_i z_i^{\mathrm{Im}} + \gamma_i k_i + \delta_i l_i. \quad (9)$$

Below we present the values of the coefficients that determine the contributions of sources of economic growth in legal sector in each region on the basis of the researched model for 2011. (See Table II). The coefficients in Table II show by how many percent (approximately) rate of output growth in legal sector of a region will increase if growth factor (growth rates for corresponding intermediate goods, investment goods, labor) increases by 1% compared to the base case.

TABLE II
COEFFICIENTS CHARACTERIZING THE EFFECTS OF FACTORS OF ECONOMIC GROWTH

i	Region	Value of coefficient			
		α_i	β_i	γ_i	δ_i
1	Akmola	0.764	0.584	0.524	0.951
2	Aktobe	2.088	1.630	2.947	1.528
3	Almaty	0.170	2.812	0.938	0.299
4	Atyrau	2.449	1.372	2.930	2.755
5	West Kazakhstan	0.903	2.911	2.835	0.302
6	Zhambyl	1.442	0.681	0.315	2.174
7	East Kazakhstan	1.087	0.512	2.471	0.334
8	Karaganda	1.644	1.035	2.394	2.446
9	Kostanay	1.672	2.584	2.324	0.616
10	Kyzylorda	0.251	1.489	1.258	1.173
11	Mangystau	2.516	0.260	0.559	2.508
12	Pavlodar	1.606	1.290	2.291	2.098
13	North Kazakhstan	2.527	1.378	2.554	1.192
14	South Kazakhstan	1.964	1.742	2.383	0.539
15	Astana city	2.097	2.802	0.382	1.962
16	Almaty city	1.851	1.954	1.980	2.978

Analysis of the coefficients α_i, β_i, γ_i, δ_i in Table II shows that if we drop the rate of technical progress, which influence on legal sectors growth rate of all regions in this model is the same, then out of four rest rates of economic growth factors, the greatest impact on the rate of real output in regions 1, 6, 8, 16 has a rate of labor costs; in regions 2, 4, 7, 12, 13, 14 – capital accumulation growth rate; in regions 3, 5, 9, 10, 15 – consumption rate of imported intermediate goods; and for the rest region 11 – consumption rate of imported intermediate goods produced by all regions.

The results of the analysis enable to select the following

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budget shares of legal sectors in 16 regions as tools for solving regional economic growth problem: $O_{ij}^{1}(t)$ – budget share of a legal sector in a region i, assigned to pay for goods and services, purchased from legal sector in a region j; $O_{ij}^{2}(t)$ – budget share of a legal sector in a region i, assigned to pay for goods and services purchased from a shadow sector in a region j; $O_{i}^{\mathrm{Im}}(t)$ – budget share of a legal sector in a region i, assigned to purchase imported intermediate goods and services; $O_{i}^{\mathrm{l}}(t)$ – budget share of a legal sector in a region i, assigned to labor costs; $O_{i}^{\mathrm{n}}(t)$ – budget share of a legal sector in a region i, assigned to purchase investment goods. This approach is implemented in the following section.

V. Finding Optimal Values of Economic Instruments based on the CGE Model "Center-Regions"

A method of selecting optimal values of economic tools for considered problems of regional economic growth within the framework of parametric regulation theory is associated with the following model, presented by

- optimality criterion K_r, characterizing average growth rate of GRP (gross regional product) as well as relative deviations of per capita GRP in regions from per capita GRP in region №4 (Atyrau region – region, that has the highest value of the stated indicator among all regions of a country in 2000–2011) in 2012–2015:

$$K_r = \frac{1}{4}\sum_{t=2012}^{2015} tYg(t) - \frac{1}{4\sum_{i=1,i\neq 4}^{16}\varepsilon_i}\times \sum_{i=1,i\neq 4}^{16}\left(\varepsilon_i \sum_{t=2012}^{2015}\left|\frac{Yg_i^{\mathrm{l}}(t)-Yg_4^{\mathrm{l}}(t)}{Yg_4^{\mathrm{l}}(t)}\right|\right). \quad (10)$$

Here: $tYg(t)$ – annual GDP rate of a country; $Yg_i^{\mathrm{l}}(t)$ – per capita GRP in a region i; ε_i – weight coefficient, its value is equal to $\varepsilon_i = 1$ for underdeveloped regions where per capita GRP is lower than national average (regions 1, 3, 5, 6, 9, 10, 13, 14) and $\varepsilon_i = 0.1$ for developed regions, where per capita GRP is higher than national average (regions 2, 7, 8, 11, 12, 15, 16).

- constraints on the state that include constraints imposed on values of consumer price levels in all regions:

$$P_{ic}(t) \le \overline{P_{ic}}(t),\ i = 1,\dots,16, t = 2012,\dots,2015, \quad (11)$$

also constraints on per capita GRP in regions:

$$Yg_i^{l}(t) \ge \overline{Yg_i^{l}}(t),\ i = 1,\dots,16, t = 2012,\dots,2015. \quad (12)$$

Here: P_{ic} – consumer price level in region i with parametric regulation; $Yg_i^{\mathrm{l}}(t)$ – per capita GRP in region i with parametric regulation; a sign "$\overline{\ \ }$" indicates basic values of the corresponding indicator (without parametric regulation).

- explicit constraints on control ($O_{ij}^{1}(t)$, $O_{ij}^{2}(t)$, $O_{i}^{\mathrm{Im}}(t)$, $O_{i}^{\mathrm{l}}(t)$, $O_{i}^{\mathrm{n}}(t)$; $i,j = 1,\dots,16$; $t = 2012,\dots,2015$) of Problem 1 (see below):

$$O_{ij}^{1}(t) \ge 0; O_{ij}^{2}(t) \ge 0; O_{i}^{\mathrm{Im}}(t) \ge 0; O_{i}^{\mathrm{l}}(t) \ge 0; O_{i}^{\mathrm{n}}(t) \ge 0; \sum_{j=1}^{16}\left(O_{ij}^{1}(t) + O_{ij}^{2}(t)\right) + O_{i}^{\mathrm{Im}}(t) + O_{i}^{\mathrm{l}}(t) + O_{i}^{\mathrm{n}}(t) \le 1;\ 0.5 \le O_{ij}^{1}(t)/\overline{O_{ij}^{1}} \le 2; 0.5 \le O_{ij}^{2}(t)/\overline{O_{ij}^{2}} \le 2;\ 0.5 \le O_{i}^{\mathrm{Im}}(t)/\overline{O_{i}^{\mathrm{Im}}} \le 2; 0.5 \le O_{i}^{\mathrm{l}}(t)/\overline{O_{i}^{\mathrm{l}}} \le 2; 0.5 \le O_{i}^{\mathrm{n}}(t)/\overline{O_{i}^{\mathrm{n}}} \le 2. \quad (13)$$

Here $\overline{O_{ij}^{1}}$, $\overline{O_{ij}^{2}}$, $\overline{O_{i}^{\mathrm{Im}}}$, $\overline{O_{i}^{\mathrm{l}}}$, $\overline{O_{i}^{\mathrm{n}}}$ – fixed basic values of the stated shares, obtained as a result of solving the problem of parametric identification of the model applying the data for 2000–2010.

- explicit constraints on control ($O_{i}^{\mathrm{k}}(t)$; $i = 1,\dots,16$; $t = 2012,\dots,2015$) of Problem 2 (see below):

$$O_{i}^{\mathrm{k}}(t) \ge 0; \sum_{i=1}^{16}\sum_{t=2012}^{2015} O_{i}^{\mathrm{k}}(t) \le 9.2\times 10^{12}. \quad (14)$$

Here $O_{i}^{\mathrm{k}}(t)$ – additional investments for subsidizing the legal sector of the region i in year t; 9.2×10^{12} – constraints imposed on the total volume of the stated additional investments in tenge for the period of 2012–2015.

With the help of relations (10)–(13) and (10)–(12), (14) we formulate correspondingly the following problems of regional economic growth.

Problem 1. *Based on the CGE model "Center-Regions" find such values of budget shares* ($O_{ij}^{1}(t)$, $O_{ij}^{2}(t)$, $O_{i}^{\mathrm{Im}}(t)$, $O_{i}^{\mathrm{l}}(t)$, $O_{i}^{\mathrm{n}}(t)$; $i,j = 1,\dots,16$; $t = 2012,\dots,2015$) *of legal sectors of regions' economy, satisfying the condition* (13), *so that corresponding to it solution of the CGE model "Center-Regions" satisfies the conditions* (11)–(12) *and provides maximum of the criterion* (10).

Problem 2. *Based on the CGE model "Center-Regions" find values of additional investments* ($O_{i}^{\mathrm{k}}(t)$; $i = 1,\dots,16$; $t = 2012,\dots,2015$), *assigned for subsidizing legal sectors of economy in regions, satisfying the condition* (14), *so that corresponding to it solution of the CGE model "Center-Regions" satisfies the conditions* (11)–(12) *and provides maximum of the criterion* (10).

As a result of solving the Problem 1 by a numerical method, applying the Nelder-Mead algorithm a value of the criterion (10) turned out to be $K_r = -0.7235$, the value of the criterion increased by 4.7% as compared with the base case, whereas average growth rate of GDP is equal to 6.60% in 2012–2015, which is 2.25 percentage points higher than the corresponding value for the base case.

As a result of solving the Problem 1 we obtained alignment of regional socio-economic development, which is characterized by reduction of a ratio of maximum to minimum per capita GRP among all regions by 16.5% in 2015 as compared with the case without control, and by 17.3% in 2015 as compared with 2011. We also achieved per capita GRP increase by 18.5% – 24.8% in 2015 for regions, where this indicator is lower than national average.

For regions where per capita GRP is higher than national average, growth of this indicator is within 3.5% – 5.7%. Per capita GRP growth rate in the country is 10.01% compared to the case without control. Average quadratic deviation of per capita GRP among all regions from maximum per capita GRP in 2015 decreased by 13.8% compared to the case without control. Fig. 1 presents the result of solving the

Problem 1 – graphics of per capita GRP for Akmola region (in tenge in prices of 2000) without control and with parametric control. Per capita GRP growth is 21.2% by 2015 compared to the base case. If we compare a value reached in 2015 with corresponding value for 2011, then we observe a growth by 69% (the highest growth among all regions compared with data for 2011).

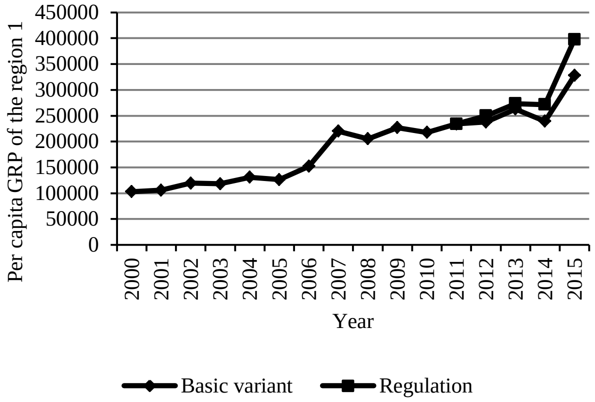

Fig. 1. Per capita GRP of the region 1 (Akmola region).

As a result of solving the Problem 2 by computational method applying the Nelder-Mead algorithm the value of the criterion (10) turned out to be $K_r = -0.7162$, the value of the criterion increased by 5.7% as compared to the base case, meanwhile GDP average growth rate in 2012–2015 is equal to 6.98%, which is 2.63 percentage points higher than of the corresponding value of the base case.

As a result of solving the Problem 2 we have obtained alignment of regional economic development, which is characterized by reduction of ratio of maximum to minimum per capita GRP among all regions by 17.07% in 2015 compared to the case without control, and by 17.28% in 2015 compared to 2011. Also the growth of per capita GRP in 2015 by 20.4% – 28.4% has been achieved for regions, where this indicator is below than national average.

Growth of per capita GRP has been within 3.4% – 5.9% in 2015 in regions where this indicator has been higher than national average. Per capita GRP growth rate of a country is 10.5% compared to the base case without regulation. Average quadratic deviation of per capita GRP among all regions from maximum per capita GRP in 2015 decreased by 12.72% compared to the case without regulation. Fig. 2 presents results of solving the Problem 2 – graphics of per capita GRP (in tenge in prices for 2000) for South Kazakhstan region without control and with parametric control. Per capita GRP in this region is 28.4% by 2015 compared to the base case (the highest growth among all regions). The achieved in 2015 value of this indicator exceeds 1.5 times its value for 2011.

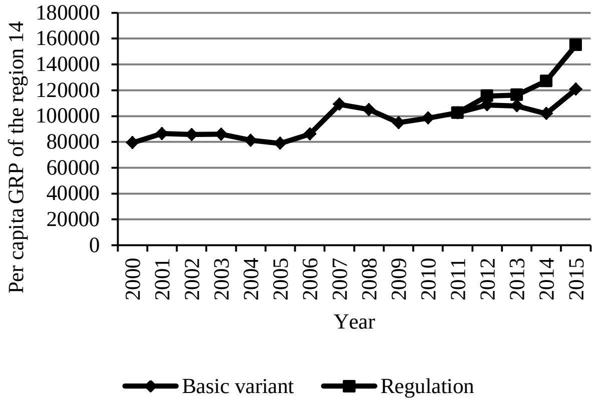

Fig. 2. Per capita GRP of the region 14 (South Kazakhstan region).

VI. Conclusion

The paper shows efficiency of the proposed method of parametric identification of large-scale CGE models.

The results of computational experiments on evaluation and analysis of sources of regional economic growth on the basis of CGE model "Center-Regions" are presented.

Efficiency of parametric control theory application at solving the problems of regional economic growth on the basis of CGE model "Center-Regions" is shown.

The obtained results can be applied during the development and implementation of an effective state policy in the sphere of economic growth and reduction of regional socio-economic growth disproportion.

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