The Formation of Cavities in Castings and Their Impact on the Conditions of Heat Dissipation

Robert Dyja, Norbert Sczygiol, and Zbigniew Domański

***Abstract*—The presented article explores the impact that the formation of shrinkage cavities has on the heat transfer conditions in a mold. The conditions of heat dissipation have a significant impact on the rate of cooling of different parts of the casting, which in turn affects the microstructure formed in the casting.**

The results presented in this article should allow the reader to assess how significant changes in the solidification time and cooling curves in different parts of the casting are.

***Index Terms*—casting, computer simulation, shrinkage cavities, solidification processing.**

I. INTRODUCTION

SHRINKAGE is a phenomenon concerning the reduction in the size of a casting during its transition from a liquid to a solid state. Shrinkage occurs during all stages of solidification. The volume in both the liquid and solid phases change under the influence of temperature. The difference in density of the liquid and solid phases, which causes a significant difference in the volume of these phases, should be taken into account [1], [2].

The phenomenon of metal shrinkage has a significant impact on the quality of castings. In general, the quality of the casting is affected by such factors as the selection of an appropriate molding and casting technology as well as the proper design and construction of the supply and gating system.

The phenomenon of casting shrinkage cannot be avoided. It is however possible to minimize the occurrence of its negative effects on the casting. A common approach is to design the mold in such a way that solidification proceeds in accordance with the assumed direction. As a rule, it is desirable that the last parts to solidify are removed later. These parts, which are cut off, are the gating system components and feeders. Thanks to this, the regions of shrinkage cavities and the occurrence of microporosity are removed from the cast.

The use of risers results from need for areas that will contain a sufficient amount of metal to compensate for the loss caused by the shrinkage. Moreover, in order to effectively supply the casting, a number of factors should be considered, for example the feeders must solidify after the supplied part of the casting, there must be a permeable flow channel of molten metal from the feeders to that part, and since the flow is due to gravity, the feeders must be placed above the part [3]. Taking all these factors into account is not an easy task and is the subject of current research [4], [5].

II. DESCRIPTION OF METHOD

A. Heat conduction problem

The presented algorithm is based on the results of previous analysis of the solidification model. The required results are the time depended temperature change and the solid phase fraction distribution. Numerical simulations of solidification are based on a heat transfer equation with heat source term [6]:

$$\nabla \cdot (\lambda \nabla T) + \dot{q} = c\rho \frac{\partial T}{\partial t} \tag{1}$$

where λ is heat conductivity, T is temperature, c is specific heat, ρ is density, t is time and q is the heat source term, which in the case of solidification is related to the phase change phenomena.

The solution to the heat transfer equation with a source term uses the enthalpy in the form of apparent heat formulation [7]:

$$\nabla \cdot (\lambda \nabla T) = c^* \frac{\partial T}{\partial t} \tag{2}$$

where c^* is the effective heat capacity, which can be obtained with the use of specific formulas [8].

As metal alloys solidify in a temperature range, one can distinguish temperature T_L, above which there is only a liquid phase in the casting and the temperature T_S, below which there is only a solid phase in the casting. The so-called mushy zone lies between these two temperatures.

A characteristic feature of enthalpic formulations is that they allow us to deal with the area of casting as a single area. There is no need to track the solidification front and distinguish areas for different groups of material properties.

Assuming that the material properties of the mushy zone are dependent on the proportion of the solid and liquid phase fraction in the area, one can write the relationship for thermal capacity as:

$$c\rho_f(T) = f_s(T)c\rho_s + (1 - f_s(T))c\rho_l \tag{3}$$

where f_s is the solid phase fraction, $c\rho_f$ is the heat capacity of the mushy zone, $c\rho_s$ is the heat capacity of the solid phase, and $c\rho_l$ is the heat capacity of the liquid phase. Similarly, the thermal conductivity is calculated according to the formula:

$$\lambda_f(T) = f_s(T)\lambda_s + (1 - f_s(T))\lambda_l \tag{4}$$

where the subscripts have the same meaning as in (3).

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In a similar way the change in the material properties due to the formation of a shrinkage cavity is taken into account. Assuming that the degree of filling element f_r is given, the heat capacity and heat transfer coefficient are calculated as follows:

$$c\rho(T) = f_r(T)c\rho_r + (1 - f_r(T))c\rho_a \tag{5}$$

$$\lambda(T) = f_r(T)\lambda_r + (1 - f_r(T))\lambda_a \tag{6}$$

where the subscript $_a$ is the air, and the subscript $_m$ means any phase of metal.

Calculation of the material properties using these formulas requires a knowledge of the degree of filling element f_r. This value is obtained by carrying out a simulation of the formation of the shrinkage cavities, which is described in detail in the next section.

To take into account the mutual influence of heat conduction and the formation of shrinkage cavities, the steps of these simulations are made alternately, i.e. one step of the heat conduction simulation is performed to obtain the solid phase fraction in the elements. Then one time step of cavities formation simulation, which takes as input the growth of the solid phase in that time step, and returns as a result the degree of filling of elements (that is the amount of solid and liquid fraction, which takes into account the shrinkage).

The solution to equation (1) is obtained numerically with the use of the Finite Element Method [9]. After spatial discretization of equation (1) the following can be obtained:

$$\mathbf{M}\dot{\mathbf{T}} + \mathbf{K}(T)\mathbf{T} = \mathbf{b}(t) \tag{7}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{K}$ is the conductivity matrix, $\mathbf{T}$ is the temperature vector, and $\mathbf{b}$ denotes the vector of right-hand side of equation, whose elements are calculated from the boundary conditions. The elements of these matrices and the vector are calculated from formulas (given for an individual finite element):

$$\begin{aligned}
\mathbf{M}^e &= \int\limits_{\Omega^e} \mathbf{N}^T\mathbf{N}\mathrm{d}\Omega \\
\mathbf{K}^e &= \int\limits_{\Omega^e} \lambda(T)\nabla^T\mathbf{N}\cdot\nabla\mathbf{N}\mathrm{d}\Omega \\
\mathbf{b}^e &= \int\limits_{\Omega^e} \mathbf{N}_\Gamma^T\mathbf{N}_\Gamma\mathrm{d}\Gamma\mathbf{q}^T(T)
\end{aligned}$$

where $\mathbf{N}$ is a vector of shape function of the finite element in space Ω, $\mathbf{N}_\Gamma$ denotes a vector of the shape function of boundary Γ, and $\mathbf{q}$ denotes the flux node vector.

In presented paper, time discretization was done with the use of a two-step time discretization scheme, represented by the Dupont II scheme [10]. The amount of solid phase growth in the solidifying casting was calculated according to the indirect model [11]. The indirect model assumes full solute diffusion in the liquid phase and finite solute diffusion in the solid phase. A specific feature of the indirect solid phase growth model is the variable temperature at the end of solidification, whose values range from eutectic temperature to solidus temperature. The amount of solid phase in this model is calculated from the formula:

$$f_s = \frac{1}{1-2k\alpha}\left(1 - \left(\frac{T_L - T}{T_M - T}\right)^{\frac{1-2k\alpha}{k-1}}\right) \tag{8}$$

where k is the partition coefficient, T_L is the liquidus temperature, T_M is the pure metal melting temperature, α is the Brody-Flemmings coefficient.

A full mathematical description of the solidifying phenomena requires the boundary conditions to be defined. In presented work, two types of boundary conditions have been used.

One of them is Newton's boundary condition, which expresses exchanging heat with the environment through a given boundary and the second is a contact boundary condition, which expresses heat exchange through a given boundary between the mold and the casting. In the case of the contact boundary condition it is assumed that heat exchange goes through an additional separating layer.

B. Shrinkage cavities

The method used in this paper is based on the method presented in [12]. In that method it is assumed that the results of the simulation of the temperature distribution during the casting solidification are obtained using the finite element method. This algorithm also relies on the assumption that the casting area was divided into elements corresponding to triangular finite elements.

The presented algorithm takes into account the behavior of liquid metal consistent with physical laws. Therefore, it is assumed that: the movement of liquid takes place under the effect of gravity, fully solidified canals prevent the motion of liquid, and that there must be enough liquid metal to compensate for any loss caused by solidification. It requires that every element is described by: solidification time, area, the current amount of liquid metal and a list of adjacent elements. These elements must be sorted in ascending order by the solidification time.

The operation of the proposed method requires the following steps to be carried out for each of the elements:

1) Determination the liquid metal loss A_C due to solidification, according to the formula (9).
2) Identifying elements that are not fully solidified and are connected to the current element.
3) Subtracting from A_C the amount of liquid contained in the current element.
4) Further subtraction from A_C of the amount of liquid contained in the elements that are connected to the current element [12]. The first chosen elements are those that have the highest location. A condition of feeding is that the elements that supply liquid metal have a higher geometrical centre than the current element.
5) If a subtraction from A_C amount of the liquid occurs, then the degree of filling of the current element and the element that provides liquid metal is updated. The degree of filling is a ratio between the amount of liquid metal in the element and the area of element.

The algorithm ends when the value of A_C drops to zero, or when there are no elements that could feed the current element. The loss of liquid due to solidification is calculated by the following formula:

Fig. 1. Searching for elements representing the neighborhood of the current finite element

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Require: $e$ is the current element
    $t$ is a list of all elements in the casting
Ensure: $p$ is a list of the neighbouring elements of $e$
 1: Set $Q$ as queue and initialize it with $e$
 2: while $Q$ is not empty do
 3:     Initialize $e\_lok$ with first element of queue $Q$ with values taken from $t$
 4:     If there is liquid metal in $e\_lok$ and $e\_lok$ is not in list $p$, add $e\_lok$ to the list $p$
 5:     Remove $e\_lok$ from queue $Q$
 6:     for all $n$ from the list of neighbors do
 7:         if neighbor of $n$ is not solidified then
 8:             Add neighbor of $n$ to the queue $Q$ and list $p$
 9:         end if
10:     end for
11: end while
12: return $p$
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$$A_C = A_E \cdot (1 + \beta) \tag{9}$$

where A_C is the amount of liquid needed to supply the solidified finite element of given area A_E, and β is a given in advance value of volumetric shrinkage.

An essential role of the algorithm is to find all the elements that have not yet solidified and represent uniform group of connected elements. The connection condition was introduced in order to satisfy the assumption that the flow of the liquid phase through the solidified part of the casting is impossible. A method of determining the elements that can potentially offset the loss of liquid metal is presented on Fig. 1. Similar algorithm is used in computer graphics [13].

To obtain results for the impact of the formation of shrinkage cavities on heat exchange conditions, the simulations were performed in three stages:

1) heat transfer simulation,
2) cavities formation simulation,
3) heat transfer and cavities formation simulation done alternately step by step.

During the first stage, the material properties were obtained with the use of formulas (3) and (4). During the second stage, the simulation of cavities formation was performed exactly as described in this section. However, during the third stage, the material properties for heat capacity and the heat transfer coefficient are obtained with the use of formulas (5) and (6). During the third stage of simulation slight modifications to procedure described in this section are necessary. Steps 1) – 5) are made for all elements for which solid fraction coefficient has increased. As it is impossible to prepare solidification time map, the order of the elements depends on their height (elements located lower are taken first). Searching for elements representing the neighborhood of the current finite element is done with the use of algorithm presented in Fig. 1, without any modifications. However, the feeding occurs in different way, because results from second stage are also considered. First, the metal is subtracted from the elements that contain liquid metal and during the second stage are marked as empty, than if there are no more such elements, the metal is subtracted from elements marked during second stage as empty, but that contain no liquid and only solidified metal, and finally, if necessary, the metal is subtracted from other neighbourhood elements that contain liquid metal. This complicated procedure is to prevent the growth of the solid phase in areas of casting that do not offer strong enough support.

TABLE I
PHYSICAL PROPERTIES OF CAST MATERIAL (AL-2%CU), MOLD, AND AIR

Quantity	Unit symbol	Value
Thermal conductivity of solid phase	$\frac{W}{m \cdot K}$	262
Thermal conductivity of liquid phase	$\frac{W}{m \cdot K}$	104
Density of solid phase	$\frac{kg}{m^3}$	2824
Density of liquid phase	$\frac{kg}{m^3}$	2498
Specific heat of solid phase	$\frac{J}{kg \cdot K}$	1077
Specific heat of liquid phase	$\frac{J}{kg \cdot K}$	1275
Solidus temperature	K	853
Liquidus temperature	K	926
Eutectic temperature	K	821
Melting temperature of pure metal	K	933
Latent heat of solidification	$\frac{J}{kg \cdot K}$	390000
Thermal conductivity of mold	$\frac{W}{m \cdot K}$	40
Density of mold	$\frac{kg}{m^3}$	7500
Specific heat of mold	$\frac{J}{kg \cdot K}$	620
Thermal conductivity of air	$\frac{W}{m \cdot K}$	0.025
Density of air	$\frac{kg}{m^3}$	1.168
Specific heat of air	$\frac{J}{kg \cdot K}$	1005

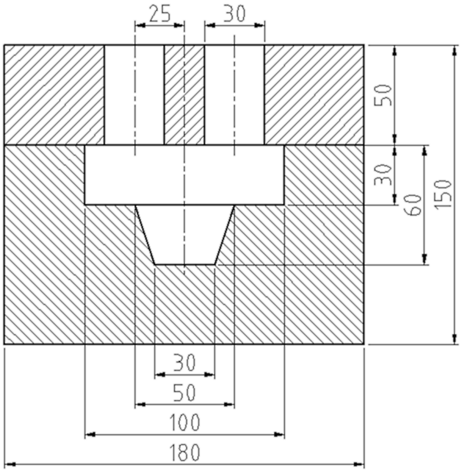

Fig. 2. The shape of the casting used in the simulation

III. NUMERICAL EXPERIMENT

The presented method was tested on the casting, presented in Fig. 2. The test casting is a simple shape but includes also the feeding system.

The calculations for obtaining temperature, solidification time, and solid phase fraction were made with finite element method software. NuscaS software, developed by the Institute of Computer and Information Sciences [14], was used to obtain the results in this paper.

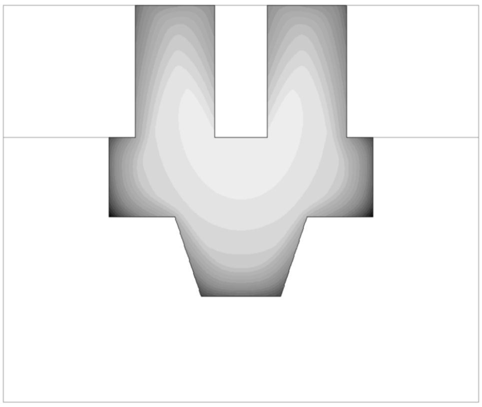

Fig. 3. Distribution of solidification time. Minimal solidification time is 259.9 s, and is marked in black. Maximal solidification time is 292.7 s, and is marked in light gray.

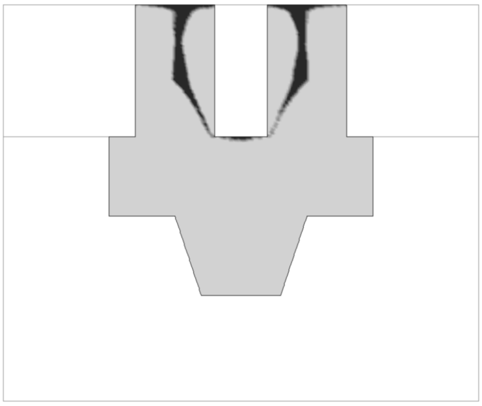

Fig. 4. Filling of the mold after end of solidification. Black colour marks cavities

The area of task was divided into finite elements of triangular shape. 10343 finite elements and 5611 nodes were used during the computations, of which 7560 elements and 3971 nodes occur in the casting.

The casting material is aluminum alloy with the addition of copper and the mold is assumed to be made from steel. The material properties are listed in Table I, which also shows the adopted values of air properties. Al-2%Cu alloy is characterized by a relatively broad range of solidification temperature. The initial temperature of the pouring was equal to 950 K and this value was the value of initial condition temperature for region of cast in performed calculations. The initial temperature of the mold was set to 600 K. The time of the simulation was calculated from the moment, when the mold was wholly filled with liquid metal. The software used did not take into account the phenomenon related to pouring or the motion of the liquid phase during solidification, which is the result of convective forces.

The complete set of task parameters requires the values of the boundary conditions coefficients to be defined. Newton's boundary condition is assumed on the upper and side surfaces of the mold assuming heat exchange with the environment with a coefficient equal to 200 $\frac{W}{m^2 \cdot K}$. On the lower surface of the mold, due to difficulty with the heat exchange, the heat exchange coefficient was reduced to 50 $\frac{W}{m^2 \cdot K}$. On upper surface of the open risers, the coefficient also was also reduced to 50 $\frac{W}{m^2 \cdot K}$, but in this case it was due to assumed use of insulation powder on these surfaces. The ambient temperature has a value of 300 K in all occurrences of this type of boundary condition.

The heat exchange between the casting and the mold is obtained from type IV boundary condition, which assumes the heat exchange through the insulation layer of the conductivity coefficient of separating layer 1000 $\frac{W}{m^2 \cdot K}$.

Simulations ended after a period of 300.0 s. This was sufficient time for the solidification of the whole casting, because the maximum solidification time was 292.7 s. As can be seen from Fig. 3, the corners of the cast solidified first and the last place to solidifying is located in the middle of the cast.

As was stated in the previous section, the algorithm for tracking the location of shrinkage cavities in the casting uses the concept of elements, to name the small portion of the casting area. In this case, the division of the casting into smaller fragments (elements) is equivalent to the division of the casting area into finite elements and both divisions are characterized by the same number of elements.

However, in the shrinkage occurring simulation, an important additional parameter, which must be specified, is the value of the shrinkage of the casting material. In case of aluminum alloy with an admixture of 2% copper, 6.3% volume shrinkage is assumed [15].

The cross sectional area of casting is equal to 0.0072 m^2. Taking into account the shrinking of the material caused by the difference in density between the liquid phase and solid phase, the solidified sectional area of the casting should be of 0.0068 m^2 in area.

The results of the shrinkage simulation are presented in the form of a distribution of value of the element filling at the end of the simulation (Fig. 4). Shortages of the elements filling in the solidification process appeared at the top of the risers, which means that the relevant part of the casting, according to the results of the presented algorithm, should be without any of the defects associated with the formation of shrinkage cavities.

The topic of this work was to discuss the influence of cavity formation on heat exchange conditions. While no differences between cavities distribution after stage 2) and 3) of simulation were observed, some differences were observed in temperature distributions. These differences are presented in the form of cooling curves obtained from stage 1) and stage 3) of the simulation. Fig. 5 presents two cooling curves taken from a point located in the middle of the riser. As can be seen there is a slight difference between these two curves. However, because the difference is observable, it can be stated that the formation of cavities have some impact on the heat dissipation conditions.

IV. CONCLUSION

The article discusses the problems associated with the formation of the shrinkage cavities. As this phenomenon is unavoidable it is reasonable to account for it in simulations. The proposed method offers one possible solution: including shrinkage formation in the heat flow simulation. As it can be seen, on the basis of the results, it is possible to perform simulations, which give reliable results. Obviously, both

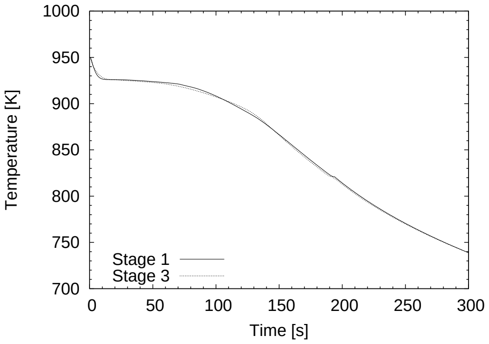

Fig. 5. Cooling curves obtained for a point inside the riser

types of simulation are already linked as they influence each other' results.

One of the advantages of the presented solution is its relative simplicity. The presented method uses relatively simple mathematical tools to achieve its objectives.

The simulations are also quick and simple to prepare, the only additional parameter, beyond those needed for the heat transfer simulation, which the user must supply is the value of material contraction, which is a material property and cannot be easily calculated. Besides, most foundry engineers are used to taking this coefficient into account and, what is more, this coefficient is given for many materials. During modifications for performing simultaneous simulations of heat transfer and cavities formation no additional actions from the user, such as: dividing the casting into areas, indicating feeding areas, etc. which maintains the relative simplicity of the proposed method. Nevertheless, it can be seen that the shape, that has been chosen as an example, is not of the simple shape, but close approximation of real cast.

Referring to the presented results it seems reasonable to conclude that the presented simulations give interesting results and the further development of the proposed algorithms can be purposeful. For example, as a further improvement, it would be interesting to perform simulations in three-dimensional areas.

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