Optimization of Multiple Responses in the Taguchi Method Using Desirability Function and Fuzzy Regression

Abbas Al-Refaie, Ibrahim Rawabdeh, Issam Jalham, Nour Bata, and Reema Abu-Alhaj

***Abstract*—This research proposes an approach for optimizing fuzzy multiple responses using desirability function and fuzzy regression. Each response repetition is transformed into signal to noise ratio then modeled using statistical multiple regression. A trapezoidal fuzzy regression model is formulated for each response utilizing the statistical regression coefficients. The most desirable response values and the deviation function are determined for each response. Finally, four optimization models are formulated for the trapezoidal membership fuzzy numbers to obtain optimal factor level at each number. A a case study is employed for illustration. In conclusion, the proposed approach successfully dealt with inherent variability and fuzziness in multiple responses. This shall be valuable to process and product engineers for optimizing fuzzy multiple responses in manufacturing applications on the Taguchi method.**

***Index Terms*—Desirability function, Fuzzy regression, Optimization, Taguchi method**

I. Introduction

THE Taguchi [1] method utilizes a fractional factorial designs, or the so-called orthogonal arrays (OAs), to reduce the number of experiments under permissive reliability. This method has been widely accepted for obtaining robust designs in many business applications. Nevertheless, most of the published researches on the applications of the Taguchi method have been conducted to optimize a single quality response of a process or product [2, 3]. Recently, several optimization approaches have been proposed for the optimization of multiple responses [4-8].

Statistical regression has many applications [9-10], in many manufacturing processes the behavior of processes is usually vague and the observed data is irregular, hence the statistical regression models have an unnaturally wide possibility range. A fuzzy regression approach in modeling manufacturing processes, which has a high degree of fuzziness, possesses the distinct advantage of being able to generate models using only a small number of experimental data sets. The fuzzy regression analysis uses fuzzy numbers expressed as intervals with membership values as the regression coefficients. Fuzzy linear regression approaches have been successfully applied in many business applications [11-13].

In fact, many manufacturing processes tend to be very complex in behavior and have inherent system fuzziness; such as, fluctuation of process pressure and temperature due to environmental effects. This research, therefore, aims at optimizing multiple responses in the manufacturing application on the Taguchi method using fuzzy regression analysis.

II. The Proposed Optimization Approach

The proposed approach for optimizing multiple responses in the Taguchi method is outlined in the following steps:

***Step* 1**: Typically, the quality response, y, is divided into three main types; involving the smaller-the-better (STB), the nominal-the-best (NTB), and the larger-the-better (LTB) response types. The Taguchi's OA conducts n experiments to investigate f factors concurrently. Let q denotes the number of responses of main concern that are measured in each experiment. To include all fuzziness in the observations values for each response, calculate the S/N ratio, η_{ijr}, at experiment i for each repetition of response j, using an appropriate equation from the following formulas:

$$\eta_{ijr} = \begin{cases} -10\log_{10}(y_{ijr}^{2}) & \text{for STB,} \\ 10\log_{10}(s_{ijr}^{2}/\bar{y}_{ijr}^{2}) & \text{for NTB,} \qquad \forall i,\ \forall j,\ \forall r \\ -10\log_{10}(1/y_{ijr}^{2}) & \text{for LTB,} \end{cases} \tag{1}$$

where $\bar{y}_i$ and s_i are the estimated average and standard deviation of y_{ir} replicates at the ith experiment. Determine the optimal factor setting for rth repetition of response j using S/N ratio. Let $\bar{\eta}_{lfr}$ denotes the average of η_i values at level l of factor f for the rth repetition. Calculate the $\bar{\eta}_{lfr}$ values for all factor levels. Identify the combination of optimal factor levels for each repetition as the levels that maximize the $\bar{\eta}_{lfr}$ for this factor.

Manuscript received June, 2012; Accepted Nov., 2012. This work was supported by the University of Jordan, Amman.

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***Step* 2**: Let η_{jr} denotes the S/N ratio for *r*th repetition of response *j*. Obtain the multiple linear regressions η_{jr} for all factor level combinations using the values of η_{ijr} . That is,

$$\eta_{jr} = \beta_{0r} + \sum_{f=1}^{v} \beta_{fr} x_f + \sum_{f=1}^{v} \beta_{ffr} x_f^2 + \sum_{g<f} \sum \beta_{fgr} x_f x_g + \varepsilon, \quad r = 1, 2, \ldots, k \tag{2}$$

where x_f , x_f^2, and $x_f x_g$ are the independent factor variables, and the coefficients, β_f , β_{fg} , β_{ff} are crisp values, and ε is random error observed in the response value. Then, determine the best-fit models for describing the functional relationship between the S/N ratio for response *j* and process factors. The fuzzy regression expressed as

$$\tilde{\eta}_j = \tilde{\beta}_0 + \sum_{f=1}^{v} \tilde{\beta}_f x_f + \sum_{f=1}^{v} \tilde{\beta}_{ff} x_f^2 + \sum_{g<f} \sum \tilde{\beta}_{fg} x_f x_g + \varepsilon \quad \forall j, \forall f \tag{3}$$

A trapezoidal fuzzy number $\tilde{B}$ as shown in Fig. 1 can be defined as (*l*,*b*,*c*,*u*), where *l*, *b*, *c*, and *u* are trapezoidal limits.

The $\mu_{\tilde{B}}(x)$ is then defined as:

$$\mu_{\tilde{B}}(x) = \begin{cases} \dfrac{x-l}{b-l}, & l \le x \le b \\ 1, & b \le x \le c \\ \dfrac{x-u}{c-u}, & c \le x \le u \\ 0, & x \le l, x \ge u \end{cases} \tag{4}$$

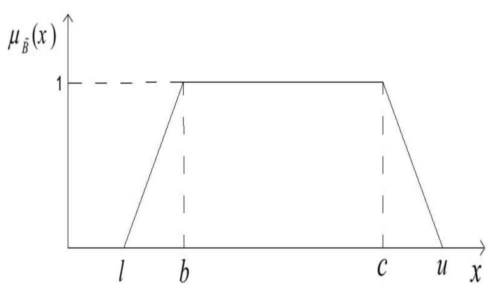

Fig. 1. Trapezoidal membership function.

Let $\tilde{\beta} = (\beta^l, \beta^b, \beta^c, \beta^u)$ are trapezoidal fuzzy coefficients. Obtain $\beta^l, \beta^b, \beta^c$ and β^u as follows:

$$\tilde{\beta} = \begin{cases} \beta^{\mathrm{m}} = \mathrm{Average}(\beta_1, \ldots, \beta_k) \\ \beta^{\mathrm{l}} = \beta^{\mathrm{m}} - \mathrm{s} \\ \beta^{\mathrm{b}} = \beta^{\mathrm{m}} - \lambda \mathrm{s} \\ \beta^{\mathrm{c}} = \beta^{\mathrm{m}} + \lambda \mathrm{s} \\ \beta^{\mathrm{u}} = \beta^{\mathrm{m}} + \mathrm{s} \end{cases} \tag{5}$$

where *s* is standard deviation of $(\beta_1, \ldots, \beta_k)$, and λ is a positive constant $0 \le \lambda \le 1$, chosen by an expert depending on experience about the repetitiveness of the proposed data. For instance a large λ means that the expert has a poor opinion about their repetitiveness.

***Step* 3**: Let $\tilde{\eta}_j(\tilde{x}_q)$ be *j*th response value by substituting the optimal fuzzy factor levels of *q*th response. Calculate the $\tilde{\eta}_j(\tilde{x}_q)$ or all $\tilde{x}_q$ values. Provide the "most desirable" response values for all response types using

$$\tilde{d}_j(\tilde{\eta}_j(X)) = \begin{cases} 0, & \tilde{\eta}_j \le \eta_{\min} \\ \dfrac{\tilde{\eta}_j(X) - \eta_{\min}}{\eta_{\max} - \eta_{\min}}, & \eta_{\min} \le \eta_j(X) \le \eta_{\max} \\ 1, & \tilde{\eta}_j \ge \eta_{\max} \end{cases} \tag{6}$$

Let $\tilde{U}_j$ and $\tilde{L}_j$ denote the upper and lower limit for the desirability functions, respectively. Calculate $\tilde{U}_j$ and $\tilde{L}_j$ as follows

$$\tilde{U}_j = \tilde{d}_j(\tilde{\eta}_j(\tilde{X}_j^*)), \quad j = 1, \ldots, q \tag{7}$$

$$\tilde{L}_j = Min\left\{\tilde{d}_j(\tilde{\eta}_j(\tilde{X}_1^*)), \ldots, \tilde{d}_j(\tilde{\eta}_j(\tilde{X}_q^*))\right\}, \quad j = 1, \ldots, q \tag{8}$$

***Step* 4**: Let $D_j(x)$ denotes the deviation function to be minimized then calculate the $D_j(x)$ using (9).

$$D_j(x) = \frac{\eta_j^u(x) - \eta_j^c(x)}{(1-\lambda)}, \quad j = 1, \ldots, q \tag{9}$$

Then, calculate the followings

$$\tilde{P}_j = \tilde{D}_j(\tilde{x}_j^*) \tag{10}$$

$$\tilde{Q}_j = Max\left\{\tilde{D}_j(\tilde{x}_1), \ldots, \tilde{D}_j(\tilde{x}_q)\right\}, \quad j = 1, \ldots, q \tag{11}$$

***Step* 5**: Formulate the final model as follows

$$\begin{aligned} &Max\left\{\tilde{d}_1(x), \ldots, \tilde{d}_j(x)\right\}, \quad j = 1, \ldots, q \\ &Min\left\{D_1(x), \ldots, D_j(x)\right\}, \quad j = 1, \ldots, q \\ &s.t \\ &x \in [\text{Factor Levels}] \end{aligned} \tag{12}$$

Equation (12) is converted to single objective by introducing two functions $\tilde{S}_j(x)$ and $\tilde{T}_j(x)$. Let

$$\tilde{S}_j(x) = (S_j^l(x), S_j^b(x), S_j^c(x), S_j^u(x)) \tag{13}$$

and

$$\tilde{T}_j(x) = (T_j^l(x), T_j^b(x), T_j^c(x), T_j^u(x)) \tag{14}$$

ISBN: 978-988-19252-6-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

where $\tilde{S}_j(x)$ and $\tilde{T}_j(x)$ indicate the degrees of satisfaction from desirability and robustness, respectively. Then, estimate $\tilde{S}_j(x)$ and $\tilde{T}_j(x)$ as follows:

$$\tilde{S}_j(x)=\begin{cases}0, & \tilde{d}_j(x)\le\tilde{L}_j\\ \dfrac{\tilde{d}_j(x)-\tilde{L}_j}{\tilde{U}_j-\tilde{L}_j}, & \tilde{L}_j\le\tilde{d}_j(x)\le\tilde{U}_j\\ 1, & \tilde{d}_j(x)\ge\tilde{U}_j\end{cases} \tag{15}$$

and

$$\tilde{T}_j(x)=\begin{cases}1, & \tilde{D}_j(x)\le\tilde{P}_j\\ \dfrac{\tilde{Q}_j-\tilde{D}_j(x)}{\tilde{Q}_j-\tilde{P}_j}, & \tilde{P}_j\le\tilde{D}_j(x)\le\tilde{Q}_j\\ 0, & \tilde{D}_j(x)\ge\tilde{Q}_j\end{cases} \tag{16}$$

Consequently, the objective is to maximize $\tilde{S}_j(x)$ and $\tilde{T}_j(x)$. That is,

$$\begin{aligned}&Max \quad \tilde{S}_j(x) \qquad j=1,\dots,q\\ &Max \quad \tilde{T}_j(x) \qquad j=1,\dots,q\\ &s.t\\ &x\in[\text{Factor Levels}]\end{aligned} \tag{17}$$

Zimmerman Max-Min operator will be applied to convert the two-objective model to a single objective, which maximizes the minimum degree of satisfaction [14]. Let

$$Min \quad \tilde{S}_j(X)=\tilde{S} \tag{18}$$

and

$$Min \quad \tilde{T}_j(X)=\tilde{T} \tag{19}$$

Then, the final model is formulated as:

$$\begin{aligned}&Max \quad \tilde{S}(X)\\ &Max \quad \tilde{T}(X)\end{aligned} \tag{20}$$

Subject to:

$$\tilde{S}\le\frac{\tilde{d}_j(X)-\tilde{L}_j}{\tilde{U}_j-\tilde{L}_j}, \text{then } \tilde{d}_j(X)-\tilde{S}(\tilde{U}_j-\tilde{L}_j)\ge\tilde{L}_j,$$
$$j=1,\dots,q$$
$$\tilde{T}\le\frac{\tilde{Q}_j-\tilde{D}_j(X)}{\tilde{Q}_j-\tilde{P}_j}, \text{then } \tilde{D}_j(X)+\tilde{T}(\tilde{Q}_j-\tilde{P}_j)\le\tilde{Q}_j,$$
$$j=1,\dots,q$$
$$x\in[\text{Factor Levels}]$$

Finally, let w_1 and w_2 indicate the important weights for desirability and robustness expressed by user based on cost, quality loss, and warranty. The final model with only one objective is transformed to

$$\begin{aligned}&Max \quad w_1\tilde{S}+w_2\tilde{T}\\ &s.t\\ &\tilde{d}_j(X)-\tilde{S}(\tilde{U}_j-\tilde{L}_j)\ge\tilde{L}_j \quad, j=1,\dots,q\\ &\tilde{D}_j(x)+\tilde{T}(\tilde{Q}_j-\tilde{P}_j)\le\tilde{Q}_j \quad, j=1,\dots,q\\ &w_1+w_2=1\\ &0\le\tilde{S}\le1\\ &0\le\tilde{T}\le1\\ &X\in[\text{Factor Levels}]\end{aligned} \tag{21}$$

Solve the models *l*, *b*, *c*, and *u* to determine the values of factor fuzzy levels.

III. Illustrative Case Study

Chen *et. al* [15] investigated sputtering process of GZO films using the grey-Taguchi method. Five process factors were studied including: R.F. power, x_1, sputtering pressure, x_2, deposition time, x_3, substrate temperature, x_4, and post-annealing temperature, x_5. The deposition rate (DR, y_1, LTB), electrical resistively (ER, y_2, STB), and optical transmittance (OT, y_3, LTB) were the main responses. Let η_{i1r}, η_{i2r}, and η_{i3r} denote the S/N ratio for DR, ER, and OT responses at experiment i (i =1,…, 18) with r (r =1, 2) repetitions, respectively. The η_{i1r}, η_{i2r}, and η_{i3r} are calculated for each repetition using the appropriate formula in (1). The obtained η_{i1r}, η_{i2r}, and η_{i3r} are then summarized for both repetitions in Table I. The $\bar{\eta}_{lfr}$ values in each response repetition are calculated for all factor levels for the DR, ER, and OT responses. Table II displays the $\bar{\eta}_{lfr}$ values for DR, the combination of optimal factor levels for the two repetitions are identified as $x1_{(3)}x2_{(2)}x3_{(2)}x4_{(3)}x5_{(2)}$. In a similar manner, the combination of optimal factor levels for the first and second repetitions of ER are found respectively as $x1_{(3)}x2_{(2)}x3_{(3)}x4_{(3)}x5_{(3)}$ and $x1_{(3)}x2_{(2)}x3_{(2)}x4_{(3)}x5_{(3)}$. Finally, the combination of optimal factor levels is $x1_{(1)}x2_{(1)}x3_{(1)}x4_{(3)}x5_{(3)}$ for both OT repetitions. It is noticed that there is a conflict among the combinations that optimize the three responses concurrently. Moreover, there are two distinct combinations of optimal factor levels for ER repetitions. This shows the fuzziness effect on ER response. The multiple linear regression equations for η_{11} and η_{12} are respectively written as:

$$\begin{aligned}\eta_{11}&=10.29+0.08x_1+0.39x_2+1.2\times10^{-3}x_3+2.64\times10^{-3}x_4\\&\quad+8.75\times10^{-4}x_5\\ \eta_{12}&=10.42+0.08x_1+0.38x_2+7.2\times10^{-4}x_3+4.6\times10^{-4}x_4\\&\quad+7.18\times10^{-4}x_5\end{aligned}$$

Whereas, the multiple linear regression equations for η_{21} and η_{22} are respectively represented by:

ISBN: 978-988-19252-6-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$\eta_{21} = -27.499 + 0.1063x_1 + 0.016x_2 + 0.0105x_3 + 0.0165x_4 + 0.0113x_5 \text{; } \textit{R-Sq (adj)} = 92.4\%$$

$$\eta_{22} = -27.947 + 0.105x_1 + 0.394x_2 + 0.0109x_3 + 0.0217x_4 + 0.0112x_5 \text{; } \textit{R-Sq (adj)} = 93.1\%$$

Finally, the multiple linear regression equations for η_{31} and η_{32} are respectively expressed as:

$$\eta_{31} = 39.202 - 2.34\times10^{-3}x_1 - 0.048x_2 - 4.75\times10^{-3}x_3 + 1.11\times10^{-3}x_4 + 5.78\times10^{-4}x_5 \text{; } \textit{R-Sq (adj)} = 84.9\%$$

$$\eta_{32} = 39.211 - 2.39\times10^{-3}x_1 - 0.058x_2 - 4.64\times10^{-3}x_3 + 1.02\times10^{-3}x_4 + 5.53\times10^{-4}x_5 \text{; } \textit{R-Sq (adj)} = 84\%$$

Using (5), the $\tilde{\beta} = (\beta^l, \beta^b, \beta^c, \beta^u)$ values are obtained. The trapezoidal fuzzy regression $\tilde{\eta}_1$, $\tilde{\eta}_2$ and $\tilde{\eta}_3$ are then formulated for the three responses as follows:

$\tilde{\eta}_1 =$ (10.28, 10.31, 10.40, 10.44) + (0.08796, 0.0797, 0.0800, 0.08022) x_1 + (0.3709, 0.3772, 0.3897, 0.396) x_2 + (0.0006, 0.0008, 0.0011, 0.0013) x_3 + (0.00001, 0.0008, 0.0023, 0.0031) x_4 + (0.0007, 0.00074, 0.00085, 0.0009) x_5

$\tilde{\eta}_2 =$ (-28.04, -27.88, -27.57, -27.41) + (0.1047, 0.1052, 0.1061, 0.1065) x_1 + (-0.0622, 0.0713, 0.3383, 0.4718) x_2 + (0.0104, 0.0105, 0.0108, 0.0109) x_3 + (0.0154, 0.0172, 0.0209, 0.0227) x_4 + (0.0112, 0.01121, 0.0113, 0.01132) x_5

and

$\tilde{\eta}_3 =$ (39.1999, 39.2030, 39.2093, 39.2124) + (-0.00239, -0.00238, -0.00235, -0.00233) x_1 + (-0.0603, -0.0567, -0.0496,-0.0461) x_2 + (-0.0048, -0.0047, -0.00465,-0.0046) x_3 + (0.001, 0.00102, 0.00107, 0.00109) x_4 + (0.00055, 0.00056, 0.000575, 0.00058) x_5

From Tables III to V, the values of the combinations of optimal fuzzy factor levels for DR are given calculated as $\tilde{x}_1$ = (200, 200, 200, 200), $\tilde{x}_2$ = (0.67, 0.67, 0.67, 0.67), $\tilde{x}_3$ = (60, 60, 60, 60), $\tilde{x}_4$ = (100, 100, 100, 100), and $\tilde{x}_5$ = (100, 100, 100, 100). The corresponding values of $\tilde{\eta}_1$ are (26.5432, 26.7056, 27.0392, 27.2276). Similarly, for ER the values of the combinations of optimal fuzzy factor levels are $\tilde{x}_1$ = (200, 200, 200, 200), $\tilde{x}_2$ = (0.67, 0.67, 0.67, 0.67), $\tilde{x}_3$ = (53.79, 64.39, 85.61, 96.21), $\tilde{x}_4$ = (100, 100, 100, 100), and $\tilde{x}_5$ = (200, 200, 200, 200). The corresponding values of $\tilde{\eta}_2$ are (-2.8021, -2.1555, -0.8434,-0.2114). Finally, the values of the combinations of optimal fuzzy factor levels for OT are $\tilde{x}_1^*$ = (50, 50, 50, 50), $\tilde{x}_2$ = (0.13, 0.13, 0.13, 0.13), $\tilde{x}_3$ = (30, 30, 30, 30), $\tilde{x}_4$ = (100, 100, 100, 100), $\tilde{x}_5$ = (200, 200, 200, 200). The corresponding values of $\tilde{\eta}_3^*$ are (39.1386, 39.1496, 39.1679, 39.1769).

Assuming that the acceptable ranges of $\eta_{\min}$ and $\eta_{\max}$ for DR values are equal to (12, 12, 12) and (30, 30, 30), respectively. Similarly, the values of $\eta_{\min}$ and $\eta_{\max}$ for ER are (30, 30, 30) and (-24, -24, -24), respectively. Finally, the values of $\eta_{\min}$ and $\eta_{\max}$ for OT are determined as (38, 38, 38) and (40, 40, 40), respectively. The desirability function for each response is calculated using (6) and (7) then the following values are determined:

$$\tilde{U}_1 = (0.8080, 0.8170, 0.8355, 0.8460),$$
$$\tilde{L}_1 = (0.1364, 0.1443, 0.1600, 0.1684)$$

$$\tilde{U}_2 = (0.8832, 0.9102, 0.9649, 0.9912),$$
$$\tilde{L}_2 = (0.2200, 0.2360, 0.2691, 0.2849)$$

$$\tilde{U}_3 = (0.5693, 0.5748, 0.5840, 0.5885),$$
$$\tilde{L}_3 = (0.2490, 0.2650, 0.3002, 0.3167)$$

Next, the fuzzy deviation functions for DR, ER, and OT are expressed respectively as:

$$\tilde{D}_1 = 0.0864 + 0.00044\tilde{x}_1 + 0.0126\tilde{x}_2 + 0.0004\tilde{x}_3 + 0.0016\tilde{x}_4 + 0.0001\tilde{x}_5$$
$$\tilde{D}_2 = 0.3168 + 0.0008\tilde{x}_1 + 0.267\tilde{x}_2 + 0.0002\tilde{x}_3 + 0.0036\tilde{x}_4 + 0.00004\tilde{x}_5$$
$$\tilde{D}_3 = 0.0062 + 0.00004\tilde{x}_1 + 0.007\tilde{x}_2 + 0.0001\tilde{x}_3 + 0.00004\tilde{x}_4 + 0.00001\tilde{x}_5$$

Using (10) and (11), the $\tilde{P}_j$ and $\tilde{Q}_j$ values are calculated and found respectively equal to

$$\tilde{P}_1 = (0.3768, 0.3768, 0.3768, 0.3768),$$
$$\tilde{Q}_1 = (0.3844, 0.3886, 0.3971, 0.4013)$$

$$\tilde{P}_2 = (1.0344, 1.0366, 1.0408, 1.0429),$$
$$\tilde{Q}_2 = (1.0344, 1.0366, 1.0408, 1.0429)$$

$$\tilde{P}_3 = (0.0181, 0.0181, 0.0181, 0.0181),$$
$$\tilde{Q}_3 = (0.0303, 0.0313, 0.0335, 0.0345)$$

By applying Zimmerman Max-Min operator, the final model is categorized to four models *l*, *b*, *c*, and *u*. Table III displays the obtained results. For illustration, the models *b* and *c* are formulated as follows. The values of DR, ER, and OT (dB) at the optimal fuzzy factors levels given in Table VI are calculated and found equal to (11.84, 12.26, 12.75, 21.94), (1.39, 2.91, 3.19, 4.32), and (86.48, 86.55, 87.67, 87.80), respectively. It is found that the trapezoidal membership function increases the flexibility of the fuzzy models by providing ranges of optimal solution. There are

ISBN: 978-988-19252-6-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

wide ranges between the lower and maximal DR and ER values, whereas a tight range is noted for OT. Excluding the fuzziness will result in misleading response values if solved by the traditional regression technique. Thus, by considering each response repetition separately, the inherent variability in the collected data is minimized.

Model *b*

$$
\begin{aligned}
&Max \quad 0.5\times S^b + 0.5\times T^b \\
&s.t \\
&-0.09384 + 0.00443x_1^b + 0.021x_2^b + 4.44\times10^{-5}x_3^b + 4.44\times10^{-5}x_4^b + 4.11\times10^{-5}x_5^b - 0.6727S^b \ge 0.1443 \\
&-0.16173 + 0.00438x_1^b - 0.00297x_2^b + 4.375\times10^{-4}x_3^b + 7.17\times10^{-4}x_4^b + 4.67\times10^{-4}x_5^b - 0.6742S^b \ge 0.236 \\
&0.6015 - 0.00119x_1^b - 0.0284x_2^b - 0.00235x_3^b + 0.00051x_4^b + 2.8\times10^{-4}x_5^b - 0.3098S^b \ge 0.2650 \\
&0.0863 + 0.00044x_1^b + 0.126x_2^b + 0.0004x_3^b + 0.0016x_4^b + 0.0001x_5^b + 0.0118T^b \le 0.3886 \\
&0.3168 + 0.0008x_1^b + 0.267x_2^b + 0.0002x_3^b + 0.0036x_4^b + 0.00004x_5^b + 0T^b \le 1.0366 \\
&0.0062 + 0.00004x_1^b + 0.007x_2^b + 0.0001x_3^b + 0.00004x_4^b + 0.00001x_5^b + 0.0132T^b \le 0.0313 \\
&w_1 + w_2 = 1 \\
&0 \le S^b, T^b \le 1 \\
&X = \left\{x_1^b, x_2^b, x_3^b, x_4^b, x_5^b\right\} \in \text{[Factor Levels]}
\end{aligned}
$$

Model *c*

$$
\begin{aligned}
&Max \quad 0.5\times S^c + 0.5\times T^c \\
&s.t \\
&-0.0867 + 0.0045x_1^c + 0.022x_2^c + 7.22\times10^{-5}x_3^c + 1.72\times10^{-4}x_4^c + 5\times10^{-5}x_5^c - 0.6776S^c \ge 0.1684 \\
&-0.1419 + 0.0044x_1^c + 0.0197x_2^c + 4.5\times10^{-4}x_3^c + 9.46\times10^{-4}x_4^c + 4.71\times10^{-4}x_5^c - 0.7063S^c \ge 0.2849 \\
&0.6062 - 0.00117x_1^c - 0.0231x_2^c - 0.0023x_3^c + 5.45\times10^{-4}x_4^c + 2.9\times10^{-4}x_5^c - 0.2718S^c \ge 0.3167 \\
&0.0864 + 0.00044x_1^c + 0.126x_2^c + 0.0004x_3^c + 0.0016x_4^c + 0.0001x_5^c + 0.0245T^c \le 0.4013 \\
&0.3168 + 0.0008x_1^c + 0.267x_2^c + 0.0002x_3^c + 0.0036x_4^c + 0.00004x_5^c + 0T^c \le 1.0429 \\
&0.0062 + 0.00004x_1^c + 0.0071x_2^c + 0.0001x_3^c + 0.00004x_4^c + 0.00001x_5^c + 0.0164T^c \le 0.0345 \\
&w_1 + w_2 = 1 \\
&0 \le S^c, T^c \le 1 \\
&X = \left\{x_1^c, x_2^c, x_3^c, x_4^c, x_5^c\right\} \in \text{[Factor Levels]}
\end{aligned}
$$

TABLE I
The S/N ratios for the repetitions of sputtering experiment

Exp. *i*	DR		ER		OT	
	η_{i11}	η_{i12}	η_{i21}	η_{i22}	η_{i31}	η_{i32}
1	13.064	13.442	-23.464	-23.694	38.929	38.929
2	14.964	14.964	-19.825	-19.735	38.860	38.860
3	13.979	13.804	-17.953	-17.842	38.900	38.900
4	19.645	19.370	-14.648	-14.964	39.007	39.017
5	20.906	21.062	-13.255	-12.669	38.800	38.790
6	20.000	20.000	-16.258	-16.391	38.558	38.558
7	25.977	26.107	-4.0824	-4.609	38.750	38.750
8	26.689	26.690	-5.5751	-6.021	38.308	38.319
9	26.403	26.403	-5.1055	-4.082	38.649	38.619
10	13.625	13.255	-17.266	-16.902	38.850	38.850
11	13.979	13.804	-16.777	-17.025	38.998	38.998
12	13.804	13.625	-17.842	-17.730	38.830	38.830
13	19.735	19.735	-15.707	-15.417	38.790	38.7900
14	20.906	21.289	-15.563	-15.269	38.455	38.455
15	20.588	20.668	-14.807	-15.117	38.929	38.919
16	25.801	25.753	0.000	-0.828	38.392	38.392
17	26.888	26.848	-1.584	-2.279	38.660	38.600
18	26.235	26.235	-2.923	-2.279	38.475	38.455

TABLE II
THE OPTIMAL FACTOR LEVELS FOR THE TWO REPETITIONS OF DR

Factor	**Replicate** r =1			**Replicate** r =2		
(f)	Level 1	Level 2	Level 3	Level 1	Level 2	Level 3
x_1	16.002	20.297	26.332	13.816	20.354	26.340
x_2	19.641	20.722	20.168	19.611	20.776	20.123
x_3	20.095	20.270	20.167	20.089	20.288	20.132
x_4	20.027	20.248	20.256	20.146	20.179	20.185
x_5	20.030	20.296	20.205	20.057	20.253	20.200

TABLE III
THE OPTIMAL FACTOR LEVELS FOR THE SPUTTERING PROCESS

	Model l	Model b	Model c	Model u
x_1^*	139.84	140.73	142.52	200
x_2^*	0.13	0.13	0.13	0.13
x_3^*	30	30	30	30
x_4^*	25	25	25	25
x_5^*	0	200	200	200

IV. CONCLUSIONS

This research proposed an optimization approach using desirability function and fuzzy regression to deal with fuzzy multiple responses. A case study from previous literature are employed for illustration. It is found that the proposed approach (1) successfully deals with inherent variability and fuzziness in multiple responses by considering trapezoidal membership for each response, (2) provides ranges for optimal solution in contrast with traditional optimization techniques, (3) deals with response repetition rather than average value of response repetitions and hence provides reliable results, and (3) considers fuzzy process factor levels rather than crisp settings, hence allows flexibility in changing factor levels that may be affected during operation by uncontrollable factors. In conclusion, the proposed approach shall provide great assistance to process engineers in minimizing deviation and obtain desirable response values in manufacturing applications on the Taguchi method.

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