Integrated Inventory Model with Decreasing Probability of Imperfect Maintenance in a Supply Chain

Ming Feng Yang, Chien Min Su, Ming Cheng Lo, Hsu Hsin Lee and Wei Chung Tseng

***Abstract*—In today environment, the Just-in-Time (JIT) systems have been widely applied in many enterprises. The JIT system features include small lots size and frequent deliveries, requirements high quantity, low inventory level, low set-up cost and low ordering cost. In the JIT supply chain environment, the companies have to cooperating the vendor and buyer by allying integrated inventory model to reduce the inventory level, and makes the optimal profit with all supply chain members. As for the high quantity requirements, it's necessary to keep whole production system in good condition. And for a good production system, the preventive maintenance (PM) strategy is needed. In the past inventory model research, the PM strategy rarely been considered in integrated inventory model. Hence, in this research we build an integrated inventory model with PM strategy, and develop algorithms to find the optimal solution.**

***Index Terms*—integrated inventory, preventive maintenance, imperfect maintenance, imperfect production, just in time**

I. Introduction

In recent year, enterprises' business models are different from the past due to globalization and informationization. The enterprises have to face many predicaments in internationalization, so there will be the emergence of the concept of supply chain. Compared to the traditional business model, the supply chain system can integrate the upstream and downstream companies between enterprises and use the resources more efficiently to create more profit. Harris (1913) proposed Economic Order Quantity (EPQ) model, after that, many research extend this model to achieve practical environment. This paper will explore the economic production quantity (EPQ) and economic order quantity (EOQ) in manufacturers of integrated supply chain model.

Manuscript received Dec07, 2012
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Inventory management plays an important role in supply chain. The inventory cost of a company may account for 20-30% of the total cost. So there is quite significant effectiveness for reducing the cost of the whole supply chain system by doing well in inventory control. Just-In-Time system, which can effectively reduce stock and the operation time, is mostly taken in today's environment. Ballou (1992) figure out JIT refers to the entire supply pipeline synchronization in response to customer demand and eliminates troubles immediately. Therefore, the time require is important of both production and delivery process, and delivery with small lots. When the unforeseen circumstance occurs can be dealt with immediately. Gunasekaran (1999) pointed out that the JIT system has the following features such as short lead times, low-volume, high quality and short lead time. Closer cooperation between buyers and vendors are extensive to use base on supply chain system. Manufacturers only consider their own profit to make decisions in the past, but such a decision-making will lack of competitive advantage in this environment in long-term. JIT system requires the close cooperation of the buyer and vender to reach the production or materials can be sent to where it needed. The decisions of inventory control system from a single enterprise into the co-operation of all supply chain member, and emphasis on collective profit maximum (Kelle 2002 and Stefan 2004). Therefore, the supply chain can response the customer demand immediately when the supply chain member face to different environment.Goyal (1976) proposed a notion of an integrated inventory model for a single supplier and a single customer that assume the order cycle and order quantity as decision variables. Banerjee (1986) purposed the concept of economic lots size to reduce the inventory cost by extension Goyal's model. Goyal (1988) proved that the combine profit by cooperating would greater than fight alone in the supply chain model. Goyal (1989) proved that the overall profit would greater than individual decision profit in the supply chain model. Lu (1995) considers the integrated inventory model with single vendor and single buyer, and to optimize the total cost. Ha and Kim (1997) proposed a single vendor and single buyer, under a single product integrated strategy that an integrated lot-splitting model of facilitating multiple shipments in small lots is developed and compared with the existing approach in a simple JIT environment. Yang and Pan (2004) proposed integrated model with control leads time and improve the quantity level, assume that the leads time following normal distribution. Yang (2007) proposed a single buyer and a single vendor integrated inventory model ordering policy, application of fuzzy theory to forecast the demand and productivity. Mohan et al. (2008) investigates the optimal replenishment policy for multi-item with the permissible delay in payment and a budget constraint. Yang (2011) consider the time value of the inventory model with single buyer and single vendor, the inventory cost will change follows inventory cycle time. Yang and Lin (2012) propose a single-vendor and multiple buyer integrated inventory model with lead time demand is following normal distribution.

Manufacturers will apply to the EPQ as the basis productions to reduce production cost (Chung and Huang 2003). But this model's assumption is too simple to fit real situation. Our intension is to transform this simplified model to a more realistic one, so we fix the existing model become more realize, so we fix the existing model become more realize. In general, the traditional EPQ model doesn't consider the defective rate of production run. But in the real situation, it will produce defective products in the process, also known as imperfect production (Salameh and Jaber 2000). Therefore, there is a survey during the production line, so non-conforming products will removing out and rework immediately. Owing to the additional cost from reworking, we would like to reduce the rework cost by improving yield rate. Rosenblatt and Lee (1986) assuming the production process to function perfectly at the start of production. Kim and Hong (1999) extend Rosenblatt and Lee (1986) model with the production process shift was arbitrarily distributed. Chen and Lo (2006) purposed an imperfect production system with shortage allows, and product sold with a free minimal repair warranty.

The purpose of Preventive Maintenance (PM) is to maintain the efficacy of production system by regular maintenance and it has been widely used in the production system (Richard Barlow and Larry Hunter 1959).Groenevelt (1992) considered the production of machinery and equipment could be repaired immediately when production system failure occurs, it causes manufacturers a huge profit loss. In generally, the production machinery increase abrasion by time, and the probability of occurrence of defective products will increase. C.T. Lam and R.H. Yeh (1993) illustrates algorithm for decision-making of PM model which five maintenance strategies are considered, failure replacement, age replacement, sequential inspection, periodic inspection and continuous monitoring strategies. Sana (2012) presented an imperfect production system with allowable shortages due to regular preventive maintenance for products sold with free minimal repair warranty In preventive maintenance mode is divided into two types. There are two types of maintenance in the PM model. Fist Tseng (1996) introduced Perfect maintenance, through the perfect maintenance the production system become as good as new. The second is Imperfect maintenance; the failure rate of production system will be the same as before after imperfect maintenance. And when the non-conforming product occurs we have minimal repair immediately

II. Notations and Assumptions

The following notations are used in model:

P average demand per year
D production rate
Q order quantity
T time of inventory cycle
C_h inventory cost rate per unit per year
C_v vender's production cost per unit
C_p purchaser's purchase cost per unit
C_o purchaser's ordering cost
C_s vender's set-up cost
C_m cost of each PM
r breakdown rate of unit
m integer number of lots of items delivered form vender to purchaser
$\bar{p}_j$ probability of j PM are imperfect maintenance
p_j probability of j PM is perfect maintenance which following the (j-1) imperfect PM ;$p_j = \bar{p}_{j-1} - \bar{p}_j$

The assumptions of model are as follow:

(1) Production rate P is finite and P>D.
(2) Inventory is continuously reviewed.
(3) Backorder are not allowed.
(4) There is two type of PM during the production cycle.
- Type-I PM is imperfect maintenance which the system have same failure rate as before with probability $q_j = \bar{p}_j/\bar{p}_{j-1}\ (0 \le q_j \le 1)$;
- Type-II PM is perfect maintenance which the system as good as new after PM with probability $\theta_j = 1 - q_j$

(5) If the failure occurs during production process, the system may goes to out-of -control and the minimal repair can be made immediately, then the system back to in-control, and the repair time is ignored

III. Modeling

In this paper, we have built an integrated inventory model with the PM strategy (Pan and Yang 2004) with the PM strategy, and explore the EOQ, EPQ, inventory cycle and the production system preventive maintenance.

The strategy of PM and minimal repair are based on Liao (2011)'s research. With the two types of maintenance, the repaired units of system are classified into (1) unchanged and (2) renewed. The type I PM is imperfect PM, and the type II PM is known as perfect PM. The probability of the type II PM denotes the number of PM occurring since the first type II PM. This paper assume that the domain of $\bar{p}_j$ is$\{1,2,3,\ldots\}$; $\bar{p}_{j-1} \ge \bar{p}_j (j = 0,1,2,\ldots)$. And let $p_j = \bar{p}_{j-1} - \bar{p}_j = \bar{p}_{j-1}\left(1 - \bar{p}_j/\bar{p}_{j-1}\right)$, with domain$\{1,2,3,\ldots\}$.

A. Vendor's total expected cost

According to the assumptions and notations in chapter 2, the total expected annual cost for vender is given by:

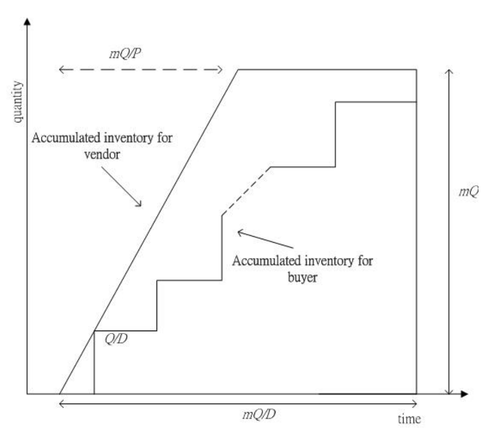

Fig 1.
Inventory model for vendor

ISBN: 978-988-19252-6-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$TEC_v = set - up\ cost + hoding\ cost + PM\ cost + repair\ cost + rework\ cost$$

With C_s is set-up cost and T is inventory cycle time, the expected set-up cost is given byC_s/T. The inventory level of this model is shown in figure 1.Once the vender receives an order, the vender will produce the items immediately until quantity reach tomQ. The item delivered from vender to buyer by each Q unit, and there are *m* lots will delivered in an inventory cycle. For the vendor average inventory can be evaluated as follow:

$$\left\{ \begin{array}{l} \left[mQ\left(\frac{Q}{P}+(m-1)\frac{Q}{D}\right)-\frac{m^2Q^2}{2p}\right]- \\ \left[\frac{Q}{D}\left(1+2+\cdots+(m-1)\right)\right] \end{array} \right\} \Big/ \left(\frac{mQ}{D}\right)$$

$$= \frac{Q}{2} m \left(\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P} \right) \quad (1)$$

Hence, the vendor's expected holding cost per year is:

$$C_h C_v \left(\frac{Q}{2} m \left(\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P} \right) \right) \quad (2)$$

We have

$$T = \frac{mQ}{D}$$

And

$$\frac{TD}{2} C_h C_v \left[\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P} \right] \quad (3)$$

The PM cost is C_{pm} with *T* inventory time, so the expected PM cost is C_{pm}/T. By Liao and Shue 2011 The breakdown cost includes minimal repair cost and rework cost, hence the expected breakdown cost is:

$$C_m \sum_{j=1}^{\infty} \frac{\bar{p}_{j-1} - \bar{p}_j}{\sum_{j=1}^{\infty} \bar{p}_{j-1}} \int_0^{j\left(\frac{D}{P}\right)T} r(t)dt \quad (4)$$

Therefore, the vender's total expected cost can be obtained from the above equation:

$$\frac{C_s}{T} + \frac{TD}{2} C_h C_v \left[\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P} \right] + \frac{C_m}{T} \sum_{j=1}^{\infty} \frac{\bar{p}_{j-1} - \bar{p}_j}{\sum_{j=1}^{\infty} \bar{p}_{j-1}} \int_0^{j\left(\frac{D}{P}\right)T} r(t)dt + \frac{C_{pm}}{T} \quad (5)$$

B. Purchaser's total expected cost

The total expected cost of purchaser as follow:

$$TEC_p = ording\ cost + holding\ cost$$

Ordering cost of order time is C_o and there are *m* lots is an inventory cycle time *T*, therefore, the expected ordering cost of each cycle is $C_o m/T$.:

$$I_p \cong \frac{Q}{2} \quad (6)$$

Hence, the expected holding cost per year is:

$$C_h C_P \frac{Q}{2} \quad (7)$$

We have

$$T = \frac{mQ}{D} \quad (8)$$

And

$$C_h C_P \frac{TD}{2m} \quad (9)$$

Therefore, the buyer's expected cost can be obtained from the above equation as follows:

$$TEC_p = C_o m/T + C_h C_P \frac{TD}{2m} \quad (10)$$

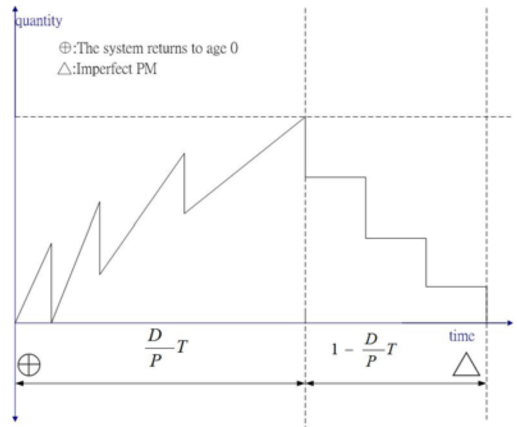

Fig 2.
Production system of PM strategy

C. Joint total expected annual cost of integrated inventory

By adding the TEC_p and TEC_p we can obtain the joint total expected annual cost as follow:

$$\text{JTEC}(T,m) = \frac{C_o m}{T} + C_h C_P \frac{TD}{2m} + \frac{C_s}{T} + \frac{TDm}{2} C_h C_v \left[m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P} \right] + \frac{C_m}{T} \sum_{j=1}^{\infty} \frac{\bar{p}_{j-1} - \bar{p}_j}{\sum_{j=1}^{\infty} \bar{p}_{j-1}} \int_0^{j\left(\frac{D}{P}\right)T} r(t)dt + \frac{C_{pm}}{T} \quad (11)$$

Taking the partial derivatives of *JTEC*(*T*, *m*) to find the optimal inventory runs time T and m, by Shue(2005) and Yang(2010) show that there exist a finite and unique optimal solution which minimize JTEC.

$$\frac{\partial \text{JTEC}(T,m)}{\partial T} = -\frac{C_o m + C_s + C_{pm}}{T^2} - \frac{C_m \sum_{j=1}^{\infty} \frac{\bar{p}_{j-1} - \bar{p}_j}{\sum_{j=1}^{\infty} \bar{p}_{j-1}} \int_0^{j\left(\frac{D}{P}\right)T} r(t)dt}{T^2} + \frac{C_m \sum_{j=1}^{\infty} \frac{\bar{p}_{j-1} - \bar{p}_j}{\sum_{j=1}^{\infty} \bar{p}_{j-1}} j\left(\frac{D}{P}\right) r\left(jT\frac{D}{P}\right)}{T} + \frac{C_h C_P D}{2m} + \frac{C_h C_v D}{2m} \left[m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P} \right] \quad (12)$$

Let $C_m \sum_{j=1}^{\infty} \frac{\bar{p}_{j-1} - \bar{p}_j}{\sum_{j=1}^{\infty} \bar{p}_{j-1}} = Z$

$$\frac{\partial^2 JTEC(T,m)}{\partial^2 T} = \frac{2\left(C_o m + C_s + C_{pm}\right)}{T^3} + \frac{2Z \int_0^{j\left(\frac{D}{P}\right)T} r(t)dt}{T^3} - \frac{2Zj\left(\frac{D}{P}\right) r\left(jT\frac{D}{P}\right)}{T^2} + \frac{Zj\left(\frac{D}{P}\right) r\left(j\frac{D}{P}\right)}{T} \quad (13)$$

ISBN: 978-988-19252-6-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

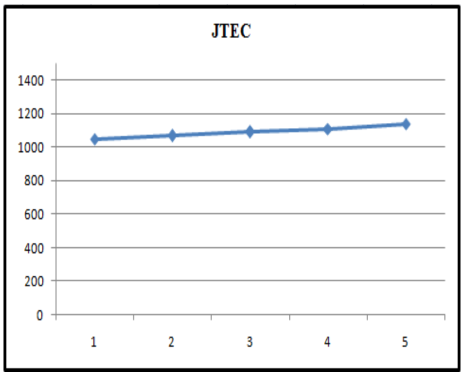

Fig3. Value of j on JTEC

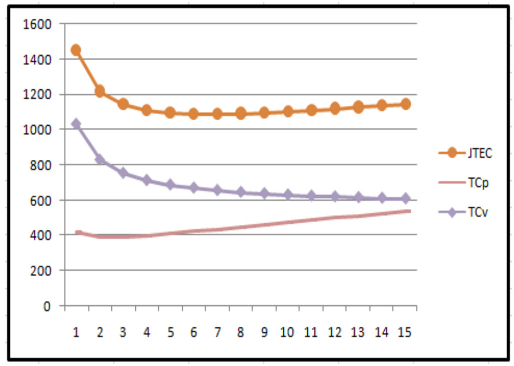

Fig4. Value of m in JTEC

Table 1
Summary of computation results

M	*T*	*JTEC*		
		purchaser	vender	integrated
1	0.186337	413.671	1000.971	1414.642
2	0.276069	388.1659	793.7541	1181.92
3	0.348102	389.5051	716.2762	1105.781
4	0.410062	397.6387	674.5727	1072.211
5	0.465183	408.2665	648.1247	1056.391
6	0.515249	419.9336	629.6804	1049.614
7	0.561386	432.0253	615.9909	1048.016[a]
8	0.604358	444.2467	605.3724	1049.619
9	0.644713	456.4448	596.8606	1053.305
10	0.682858	468.537	589.8617	1058.399

a. The minimum present value of the joint expected total cost.

Table 2
Sensitive analysis of *j*.

j	*m**	*T*	*JTEC*
1	7	0.561386	1048.016
2	7	0.562909	1066.698
3	7	0.555706	1089.769
4	7	0.555706	1140.318

Since (13) >0, hence for fix m, *JTEC(T,m)* is a convex function in T, there exist a unique value of T which minimizes *JTEC(T,m)*, and T can be obtained by solving the equation (12).

D. *Algorithm*

Then, we can use the following procedure to find optimal of T and m.

step 1. Let *m* be equal the minimum feasible value, i.e. 1.
step 2. Calculate *T* using relation (12).
step 3. Calculate *JTEC* by embedding the last calculates T and m in relation (11). If $TC_m < TC_{m-1}$ let $m = m + 1$ ant go to step 2; otherwise go to step 4.
step 4. Find the minimum *JTEC* and the corresponding value of decision variables T and m as the optimal solution.

IV. A NUMERICAL EXAMPLE

To the illustrate the results of the proposed models consider an inventory system with the following characteristics: D = 1000 unit/year, P = 600 unit/year, C_o = \$25/order, $C_s + C_{pm}$ = \$50/set-up, C_p = \$25/unit, C_v = \$20/unit, C_m = \$10/times, C_h = 0.8/unit, $\bar{p}_j = 1/e^j$, *r(t)* following a uniform distribution with a = 0.4 and b = 0.1.

Following the algorithm in lest section, the results of variable value m, T and TC as shown in table 1. The optimal of lots m = 7, and optimal inventory time T =0.561386, and the expected join total cost is 1048.016.The purchaser's total expected cost increase, but the vender's total expected cost is decease when the number of lots m increase.

Based on results in table 1 and Fig 4, we can see the vender's total expected cost declined dramatically before m = 4 then. The purchaser's expected cost is decrease before m = 2, but increase slowly after m = 2. As we can see, the optimal

ISBN: 978-988-19252-6-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

lots size (m=7) is different from the buyer's optimal lots size, the vendor's cost is an absolutely decreasing curve, so the optimal total cost of supply chain should be lower than the optimal total cost by combining each members.

When the PM times of j increase, the optimal numbers of *m*are stay the same, but the joint total expected cost are increase strictly because the value will rise depend on the minimal repair cost and rework cost increase, the result and graph shown that the increase *j* is sensitive to value of *JTEC* in table 2 and Fig. 3.

V. Conclusion and future work

In this paper, we incorporate the preventive maintenance strategy and minimal repair into the production process in the traditional integrated inventory model, although the joint total expected cost is not less than traditional model because we have consider maintenance cost and rework cost, but this model is meet the existing environment, and is not considered in the previous research.

In the future, we will try to expand more considerable variable: leads time demand, time value and permissible delay in payment strategy, and applied the algorithm to compute the optimal value officially. Hence, we will look forward to find a pragmatic sample and actual data to enhance the theory of this model.

ACKNOWLEDGMENTS

The author would like to thank the National Science Council of the Republic of China, Taiwan for financially supporting this research under Contract No. NSC 101-2410-H-019-003.

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