

# Service Level Assignment and Container Routing for Liner Shipping Service Networks

M. Hakan AKYÜZ and Chung-Yee LEE

**Abstract**—In this work, a decision tool is developed for a liner shipping company to deploy its fleet considering vessel speeds and to find routes for cargos with transit time constraints simultaneously. This problem is referred as Service level Assignment and Container Routing Problem (SACRP) in the sequel. A Column Generation (CG) algorithm is implemented for the SACRP. Computational experiments are performed on randomly generated test instances. The CG algorithm yields promising solutions for the SACRP.

**Index Terms**—container routing, fleet deployment, liner shipping, sailing speed, column generation.

## I. INTRODUCTION

Liner shipping constitute the backbone of the maritime container transportation. Increasing bunker prices leave no room for liner shipping companies to improve their profit margins. As a result, liner shipping companies are under a high pressure to operate efficiently in order to maintain their competitiveness. A liner shipping company provides weekly (or bimonthly) regular services over a shipping service route. A shipping service route consists of a fixed sequence of port visits to ensure the shipment of cargos among them. Service routes are often circular and ships' journey starts and finishes at the same port after visiting all other ports in their routes. Decisions made by the liner shippers can be grouped in three levels: strategic, tactical and operational level [1]. Figure 1 shows these three levels and the relationship among them.

In this work, we focus on the fleet deployment and speed optimization decisions at the tactical level and cargo routing decision at the operational level. For that purpose, it is assumed that strategic level decisions are already made so that the service routes, fleet size and mix of the liner shipping company are given. At the tactical level, we assume a weekly service frequency is provided. To the best of our knowledge, this is the first attempt to simultaneously decide on fleet deployment and sailing speed optimization problem as well as cargo routing decision.

Efficient transshipment is an important factor to be considered by the liner companies. Indeed, the speed increase in freight distributions is achieved by fast transshipment. Transshipment also enables liner companies to cover a larger port-to-port service area than without transshipment. Moreover,

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M. Hakan AKYÜZ is with the Department of Industrial Engineering, Galatasaray University, Ortaköy, İstanbul, 34357, TÜRKİYE (phone: +90212-2274480(436); fax: 90212-2595557; e-mail: mhakyuz@gsu.edu.tr).

Chung-Yee LEE is with the Department of Industrial Engineering and Logistics Management, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, HONG KONG (e-mail: cylee@ust.hk).

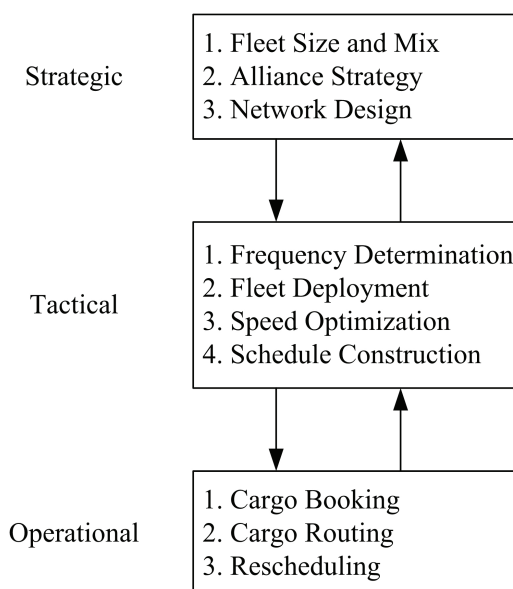


Fig. 1. Three levels of decision making in liner shipping companies. Source: [1]

as mega container vessels of 18000 TEUs started to be used by liner companies in 2013 [2], cargo consolidation at transshipment ports becomes more important for the effectiveness of liner companies. On the other hand, shippers demand shorter transit times which in turn contradicts with the goals of the liner shipping companies since it increases their costs. In particular, faster services significantly increases the bunker costs of the liner shipping companies. Bunker costs constitutes up to 60% of the total ship operating costs [3]. In order to reduce their costs, liner shipping companies use low-steaming policies after a dramatic increase in the oil prices in 2008. The number of ships to meet a given frequency in a service route depends on the sailing speed of vessels and the total distance should be travelled in the route. Reducing the number of ships deployed on a liner's network can reduce operating costs. Consequently, transit times, and thus sailing speed optimization, and cargo routing decisions becomes significantly important for operational efficiency of liner shipping companies.

The objective of this paper is to develop a decision tool for liner shipping companies to jointly deploy its fleet considering vessel speeds, and find routes for cargos with transit time constraints. Unfortunately, modelling fleet deployment and sailing speed optimization simultaneously is not trivial due to nonlinearity which stems from sailing speed optimization. For that purpose, a service level is

defined for a shipping service route as a combination of vessel capacity and its average speed to sail along the route. Indeed, liner shipping companies has a limited number of vessels in their fleet to serve a service route and their average sailing speed is restricted within a minimum and maximum speed level which are all determined by the company policies. Once service levels are determined, they are assigned to service routes so that the containers are routed optimally. Hence, different service quality levels (i.e., sailing speed/transit time) for different customers can be imposed and cost savings can be achieved by using different routes having different transit times for them. Transshipment operations and transit time constraints can be implicitly handled within a path-flow formulation while generating only necessary container routes. Consequently, we propose a path-flow based multi-commodity flow formulation for simultaneous Service level Assignment and Container Routing Problem (SACRP) which considers transit time constraints for specific cargos.

Our main contribution has four-folds. First, we give a mathematical formulation for the SACRP. As far as we know, this is the first work implemented for the SACRP which solves fleet assignment and sailing speed optimization with container routing concurrently considering transshipment operations and transit time. Second, the number of transshipment made by a cargo until its destination port is not limited. Nevertheless, most of the works (if not any) in the literature limits transshipment operations to be less than three. Third, we implement a Column Generation (CG) procedure for the SACRP. Fourth, we perform computational experiments on a set of randomly generated test instances which can be used as benchmark. The rest of this work is organized as follows. In Section 2, we present a brief review of literature. We give the notation used, shipping service network structure and problem definition in Section 3. Section 4 introduces the mathematical formulation of the SACRP. This is followed by Section 5 where we present our solution methodology and our computational results. Conclusions are presented in Section 6.

## II. LITERATURE REVIEW

Álvarez [4] develops a shipping network design model for joint routing and ship deployment over cyclic routes considering multiple vessel types, container transshipment, speed levels of vessels and chartering out vessels at a strategic level. A Tabu Search (TS) algorithm combined with a CG procedure is used as the solution technique. Gelareh and Pisinger [5] focus on a simultaneous fleet deployment and network design problem in a hub-and-spoke network structure. Reinhardt and Pisinger [6] consider joint liner shipping network design and fleet assignment problem. A Mixed Integer Linear Programming (MILP) formulation is proposed and solved by a branch-and-cut algorithm. Liu et al. [7] address ship deployment and cargo routing problem at a tactical level. A joint model and a two-level sequential model formulations are proposed for comparison with an objective to maximize the total revenue considering empty container repositioning. Wang and Meng [8] focus on a liner ship fleet deployment problem taking into account transshipment of containers and devise an efficient origin-based formulation. Meng and Wang [9] address liner ship fleet

deployment problem considering week-dependent demand and maximum permitted transit time for container routing. The problem is modelled as a two level formulation and solved by an exact algorithm. Wang and Meng [10] deal with ship sailing speed optimization in liner shipping. A non-linear MIP model is formulated to minimize the total bunker cost at each voyage leg of ship routes together with total vessel operating cost and container handling charges is also minimized. Brouer et al. [11] address cargo allocation problem with demand rejection and empty container repositioning using devise a path-flow based Multi-commodity Flow Problem (MFP) formulation with inter-balancing constraints for empty container repositioning. The problem is a MILP and it is solved heuristically by a CG algorithm using its LP relaxation. Wang et al. [12] develop a MILP model to generate all possible paths for a single cargo (commodity) to be send from its origin to destination and the suggested model permits to impose total transit time constraints for the cargo to be delivered. For more details, we refer to the works [13], [14] and [1] as excellent surveys of ship routing and scheduling.

## III. NOTATION, SHIPPING NETWORK REPRESENTATION AND PROBLEM DESCRIPTION

A liner company is operating a set, denoted by  $\mathcal{R}$ , of weekly scheduled Service Routes ( $SR$ ) over a set of ports with a vessel of type  $v$  such that each route  $r \in \mathcal{R}$  can be served by a set of vessel types indicated by  $v \in \mathcal{V}^r$  where their union constitutes entire fleet set, denoted by  $\mathcal{V} = \cup_{r \in \mathcal{R}} \mathcal{V}^r$ , of the liner company. An example service network of a liner company consisting of three circular  $SR$ s is illustrated in Figure 2. The first  $SR$  ( $SR1$ ) consists of a voyage sequence visiting ports of Busan, Shanghai, Hong Kong, Singapore, Hong Kong and Busan, respectively. The second  $SR$  ( $SR2$ ) visits Singapore, Jakarta, Port Klang and then returns to Singapore. The third  $SR$  ( $SR3$ ) passes through Sidney, Melbourne, Jakarta, Singapore and Sidney.

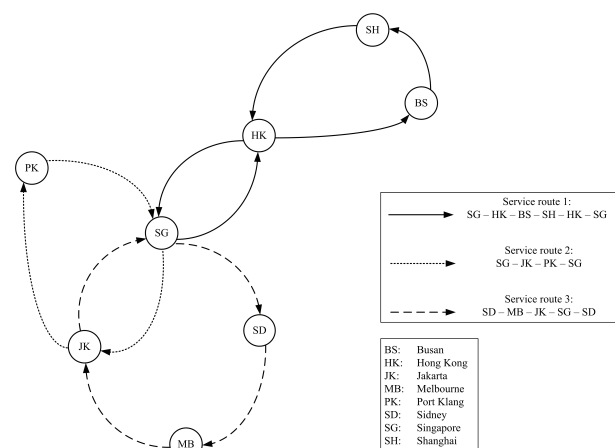


Fig. 2. A sample liner shipping service network.

Each  $SR$   $r$  are associated with one or more service level  $s$  belonging to the service level set, denoted by  $\mathcal{S}^r$ , where the set of all service levels, represented by  $\mathcal{S}$ , is expressed with  $\mathcal{S} = \cup_{r \in \mathcal{R}} \mathcal{S}^r$ . A service level  $s$  is defined with a pair of parameters  $(u_s, f_s)$  for all  $SR$ s: the capacity and the average

speed of ships deployed in the corresponding  $SR$  denoted by  $u_s$  and  $f_s$ , respectively. Note that, we assume types of ships that can be used for a  $SR$  is limited and their speed can be selected from a set of discrete values in the range  $F_v^{min} \leq f_s \leq F_v^{max}$  where  $F_v^{min}$  and  $F_v^{max}$  are minimum and maximum speed in knots for a vessel of type  $v$  deployed in the  $SR$ . Therefore, the number of service levels that can be provided for a  $SR$  is limited by the number of pairs  $(u_s, f_s)$ .

Let  $\mathcal{G} = (\mathcal{N}, \mathcal{A})$  be the graph representing the liner shipping network given in Figure 2 with node set  $\mathcal{N}$  and arc set  $\mathcal{A}$ . Vessels and containers travel along the ship voyage arcs denoted by  $\mathcal{A}^s$  for each service level  $s$ . Each origin-destination pair having demand for a container shipment is named as a commodity  $k$  which belongs to the set denoted by  $\mathcal{K}$ . A commodity  $k$  (a container flow from its origin to destination) can follow a path  $p$  among the set of possible paths defined by  $\mathcal{P}^k$  to reach from its origin port to destination port.

A weekly demand (in TEUs) of a commodity  $k$  is assumed and it is represented with  $d^k$ . Each service level  $s$  is associated with a bunker cost  $c^s$ . Notice that bunker costs change with the speed and type (i.e., the capacity) of the vessel deployed in a  $SR$   $r$ . A path  $p$  of commodity  $k$  is associated with a total cost denoted with  $c^{pk}$  which consists of total loading/unloading and transshipment costs at the nodes of path  $p$  and the transportation cost over arcs used in path  $p \in \mathcal{P}^k$  such that  $c^{pk} = \sum_{(i,j) \in \mathcal{A}} y_{ij}^{pk} c_{ij}^k$  where  $c_{ij}^k$  stands for the cost of loading/unloading, transshipment and transportation of containers on the arc  $(i, j)$  for commodity  $k$ .  $y_{ij}^{pk}$  is the parameter which takes a value of 1 if arc  $(i, j)$  is in the path  $p$  of commodity  $k$ . As can be observed, each service level  $s \in \mathcal{S}^r$  has a different speed level for a given  $SR$   $r$ . Therefore, different vessel speeds imply different travel times between two ports within a service level. Every ship voyage arc is associated with a travel (or waiting) time to move from one node  $i \in \mathcal{N}$  to the other node  $j \in \mathcal{N}$  in the network. We formalize notation used for transit time as follows. Each  $SR$  has a circular structure and its first and last port calls are the same. This holds for all service levels  $s \in \mathcal{S}^r$  of the same  $SR$   $r$ .

Travel time for ship voyage arcs  $(i, j) \in \mathcal{A}^s$  can be calculated by dividing the vessel speed  $f_s$  in knots for service level  $s$  by the nautical distance between port of node  $i$  and port of node  $j$  shown by  $D_{ij}$ . Let  $t_{ij}^s$  denotes the travel time between nodes  $i$  and  $j$  for service level  $s$ , then  $t_{ij}^s = \frac{f_s}{D_{ij}}$  holds. Consider  $t^p$  denotes the total transit time of over a path  $p \in \mathcal{P}$ .  $t^p$  consists of travel time  $t_{ij}^s$  and waiting time in a transshipment port (i.e., transferring time from one ship to another at a port). Let  $T^k$  stands for the desired maximum transit time in days by a customer sending commodity  $k$ . Then, for transit time requirements of customers the following constraints should be satisfied.

$$\max_{p \in \mathcal{P}^k} \{t^p\} \leq 24 \cdot T^k \quad \forall k \in \mathcal{K} \quad (1)$$

which can be also represented as a set of linear constraints as follows

$$t^p \leq 24 \cdot T^k \quad \forall k \in \mathcal{K}; \forall p \in \mathcal{P}^k \quad (2)$$

#### IV. SERVICE LEVEL ASSIGNMENT AND CONTAINER ROUTING PROBLEM

Given weekly demand of customers in TEUs and known weekly scheduled  $SR$ s of a liner shipping company, the SACRP concerns with assigning a service level  $s$  defined by speed and capacity combination of vessels on the service network such that the total cost is minimized subject to transit time constraints of the customers.

The assumptions made to formulate SACRP is summarized in the following. *i)* A weekly demand of cargo is assumed where origin and destination of cargos and transit time requirements of customers are given and deterministic. *ii)* Service routes of the liner shipping company are known a priori and operated with a weekly service frequency. *iii)* Vessels sail with an average speed level through all voyage legs of a  $SR$ . *iv)* Service levels constituted by pairs of speed and capacity of vessels are limited in number and planned by the liner company before. *v)* Number of transshipment operations for cargo is not limited.

Let decision variables  $\lambda^{pk}$  denote the amount of commodity  $k$  sent over path  $p \in \mathcal{P}^k$ . Also denote the binary variables shown with  $\delta^s$  is 1 if and only if service level  $s \in \mathcal{S}$  is selected for operations and zero otherwise. Then, the mathematical formulation of SACRP can be given as follows.

SACRP:

$$\min z = \sum_{k \in \mathcal{K}} \sum_{p \in \mathcal{P}^k} c^{pk} \lambda^{pk} + \sum_{s \in \mathcal{S}} c^s \delta^s \quad (3)$$

$$s.t. \sum_{p \in \mathcal{P}^k} \lambda^{pk} = d^k \quad \forall k \in \mathcal{K} \quad (4)$$

$$\sum_{k \in \mathcal{K}} \sum_{p \in \mathcal{P}^k} y_{ij}^{pk} \lambda^{pk} \leq \delta^s u_{ij} \quad \forall s \in \mathcal{S}; \forall (i, j) \in \mathcal{A}^s \quad (5)$$

$$\sum_{s \in \mathcal{S}^r} \delta^s = 1 \quad \forall r \in \mathcal{R} \quad (6)$$

$$\lambda^{pk} \geq 0 \quad \forall k \in \mathcal{K}; \forall p \in \mathcal{P}^k \quad (7)$$

$$\delta^s \in \{0, 1\} \quad \forall s \in \mathcal{S} \quad (8)$$

The objective function (3) consists of total cost of sending containers and bunker cost of the vessel fleet used. Constraints (4) are demand constraints for cargos. For each commodity  $k$ , the total amount sent through all paths  $p \in \mathcal{P}^k$  of a commodity  $k$  should be equal to a demand of  $d^k$ . Constraints (5) are capacity restrictions over the vessel capacities. The total amount of containers sent through an arc  $(i, j)$  should be less than the arc capacity  $u_{ij}$ . Clearly, each service level  $s$  has a different capacity to be used at each of its arcs  $(i, j) \in \mathcal{A}^s$ . However, its capacity can only be positive as long as the corresponding service level is being used by the liner company. Therefore, binary variable  $\delta^s$  appears on the right hand side of constraints (5) to ensure a positive capacity on the arcs of service level  $s$  when  $s$  is assigned to a  $SR$ . Otherwise, arcs capacities  $u_{ij}$  are equal to zero. Constraints (6) assure that only one service level  $s \in \mathcal{S}^r$  can be assigned for each  $SR$   $r$  in the service network. Nonnegativity restrictions of variables are imposed by constraints (7). Constraints (8) stand for the binary restriction of decision variables  $\delta^s \forall s \in \mathcal{S}$ .

## V. SOLUTION METHODOLOGY AND COMPUTATIONAL EXPERIMENTS

The SACRP is a MILP which becomes difficult to solve when the number of service levels increases. To solve it optimally with a commercial solver, all paths should be generated for each commodity. Our aim is to obtain efficient heuristic solutions for the SACRP. To achieve this, we implement a CG algorithm for the SACRP. In the following, we present the details of the CG algorithm and our results based on the test bed generated for the SACRP.

### A. Using Column Generation

CG algorithm is applied and implemented in detail for the Linear Programming (LP) relaxation of the MFP [15]. Therefore, we only present main steps of the CG algorithm for the SACRP here. Notice that, SACRP reduces to the MFP with side constraints (6). Given the service level assignment variables, CG algorithm generates paths with negative reduced costs for each commodities. The minimum cost (shortest) path is the candidate path to enter the SACRP model. Initially, a path for each commodity is generated and added into the model for feasibility purposes. The dual variables associated with the LP relaxation are used to update arc costs accordingly to generate further paths having negative reduced cost. The path generation procedure continues until all necessary paths are generated and there does not exist any paths with negative reduced cost. Besides, transit time constraints can also be handled with a path based formulation easily. Constraints given in (2) can be implicitly embedded within the CG scheme. For that purpose, the paths, which has longer transit times than the customer demands, are penalized and eliminated from the solution space. Once the CG algorithm terminates, if all service level assignment variables  $\delta_s$  are integer then the solution of the LP relaxation is also optimal for the original SACRP. In case they are fractional, the LP relaxation gives only a lower bound value for the SACRP. To obtain a heuristic upper bound value, the CG algorithm is run again after fixing the fractional service level assignment variables  $\delta^s$  to 0 and 1 considering constraints (6). This is achieved by assigning a value of 1 to a service level assignment variable which has the highest fractional value among the service levels of the corresponding SR  $r$ . The remaining service level assignment variables are set to zero and the reduced problem is solved by the CG algorithm. Consequently, an upper bound value is obtained for the SACRP.

### B. Test Bed

In this section, we introduce our test bed used in the computational experiments. 7 service routes are selected from the routes of OOCL company covering 33 different ports in Europe, Asia and Australia. The capacity of ships are selected as 4000, 5000, 8000, 10000 and 12000 TEUs to comply with the OOCL's ship fleet. We have chosen three speed levels for each ship: 18, 20 and 22 nautical miles per hour. Different speed levels cost different to the liner company. Clearly, ship operating costs are lower in slow speed levels. Fuel costs are determined by using interpolation (i.e., for different ship capacity) values obtained from the work [16]. The number of commodities are chosen from the set

TABLE I. THE AVERAGE PERFORMANCE OF THE CG ALGORITHM ON THE SACRP TEST INSTANCES

Number of Commodities	Capacity Expansion		Capacity Reduction		Current Capacity	
	UB	CPU	UB	CPU	UB	CPU
10	1415626.33	0.03	1342326.54	0.03	1415626.33	0.04
200	26478467.05	0.26	23441626.99	0.30	27135141.75	0.35
400	47887299.53	0.63	43727138.53	0.71	47887299.53	0.90
600	71436063.93	1.18	64545051.09	1.58	71368776.21	1.79

as  $k \in \mathcal{K} = \{10, 200, 400, 600\}$ . Demand amount for each commodity are generated randomly between the interval [1, 250]. For each of these combinations, origin-destination pairs and their corresponding demand amounts 20 different test instances are generated. Loading and unloading cost of containers are selected as 150 dollars per TEU and transshipment cost at a port is set as 200 dollars per TEU. Three scenarios are considered. In the first scenario, liner company tries to increase its capacity in all service routes. In the second scenario, it is assumed that the company maintains its services with its current ship capacities. In the third scenario, current capacities are reduced to obtain savings throughout all service routes. The next section presents the computational results of our CG algorithm.

### C. Computational Results

Now, we report our results obtained with the CG algorithm proposed for the SACRP. Table I gives the average performance of the CG approach on the SACRP instances. Notice that, the values in each cell correspond to the average value obtained over 20 test instances. The first column stands for the number of commodities in the instances. The second and third columns give the total cost "UB" and "CPU" time in seconds respectively when the ship capacities are planned to be increased. Similarly, fourth and fifth columns consider a reduction in ship capacities at the service routes. The sixth and seventh columns present the outcomes of maintaining the current ship deployment for service routes. It is observed that capacity reduction is better than both current capacity usage and capacity expansion decision for all test instances. The CG algorithm yields very short running times within a few seconds.

## VI. CONCLUSION

In this work, we developed a decision tool for liner shipping companies to concurrently deploy its fleet considering vessel speeds, and find routes for cargos with transit time constraints. A path-flow based MFP formulation and a CG algorithm is proposed for the SACRP. The CG algorithm yields heuristic solutions very fast. As a future research direction, an exact solution procedure, that is, a branch and cut algorithm, can be implemented for the SACRP.

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