Even-Length Cycles Embedded in Biswapped Networks

Shan Ling and Weidong Chen

Abstract—**Biswapped networks are a family of two-level interconnection networks built by taking multiple copies of a factor network as modules and connecting them in a simple swapped rule, and have been shown to be suitable for building massive parallel computers due to their attractive attributes. In this paper, the bipancyclicity properties of a Biswapped network are investigated based on a given cycle in its factor network. By establishing a close relationship between a cycle in the Biswapped network and two associated closed paths in the factor network, we propose an algorithm for constructing cycles of various even lengths in the Biswapped network. From this algorithm, it is shown that cycles of all even lengths from 8 up to $2l^2$ can be embedded in the Biswapped network if its factor network contains an l-length cycle. This shows that a Biswapped network is 8-bipancyclic if its factor network is Hamiltonian.**

Index Terms—**Biswapped Network, Cycle, Embedding, Interconnection Network**

I. Introduction

Biswapped networks are modified versions of well-known OTIS (Optical Transpose Interconnection System) networks [1]. An OTIS network, also called a swapped network, is built by taking multiple copies of a factor network as modules and connecting these modules in a simple swapped rule. OTIS networks have received considerable attention over the past two decades due to many favorable attributes coming from their simple connectivity rule [2, 3, 4, 5]. However, the very connectivity rule causes a small asymmetry in OTIS networks and many analyses and algorithms for OTIS networks become complicated. Biswapped networks have a similar to but more unified connectivity rule than the corresponding OTIS networks, which removes the inherent asymmetry of OTIS networks, as well as the attendant complications in analyses and applications [1]. Fig.1 depicts an example factor network and the associated swapped network and Biswapped network. It has been shown that Biswapped networks are superior to corresponding OTIS networks with regard to a number of performance criteria [6], including symmetry and fault tolerance, as well as simplicities in analyses and algorithms. These results indicate that the Biswapped network architecture is a competitive scheme for constructing large, scalable, modular, and robust networked systems [7, 8].

Embedding cycles of different lengths in networks is one of the fundamental problems in network virtualization because many applications are based on cycles [9, 10]. For example, cycle embedding can transplant parallel algorithms developed for cycles to a new network without any efficiency loss. Nevertheless, techniques of embedding cycles of various lengths in different networks are distinct because they are generally constructive and topology-specific [11, 12, 13, 14]. Hence, it is hard to extend the techniques from one network topology to another one even in the same family. To the best of our knowledge, previous research in this direction was mainly restricted to the hypercube network and its variants [15, 16, 17, 18]. Recently, it has been proved by a constructive method that if a factor network contains an l-cycle, then cycles of all lengths from 7 up to l^2 can be embedded in the corresponding OTIS network. This indicates that an OTIS network is 7-pancyclic when its factor network is Hamiltonian [19, 20]. This motivates this work. From Fig.1, we easily see that each cycle in the Biswapped network has an even length (no cycles of odd lengths exist). In this paper, we investigate the problem of embedding even-length cycles in Biswapped networks. Using a simple constructive method, we prove that if there exists an l-cycle in the factor network, then cycles of all even lengths from 8 to $2l^2$ are easily embedded in the Biswapped network, which implies that the Biswapped network is 8-bipancyclic if its factor network is Hamiltonian.

The remainder of this paper is organized as follows. Section II introduces some definitions and notations, and Section III gives our algorithm for embedding even-length cycles in Biswapped networks with the bipancyclic properties analyzed. Section VI concludes this paper.

II. Preliminaries

In this paper, terms *graph* and *network* are used interchangeably because networks are modeled as graphs.

A. Bipancyclicity

Let G be a simple undirected graph (graph, for short) with the node set $V(G)$ and the edge set $E(G)$. A path is a sequence $(v_1,v_2, \ldots,v_k)$ of nodes such that, for $1\le i\le k-1$, $(v_i, v_{i+1})\in E(G)$, where the first node v_1 and the last node v_k are called its end-nodes. The length of a path $\boldsymbol{P}$ is the number of edges contained within $\boldsymbol{P}$, denoted by $|\boldsymbol{P}|$. A path is called closed if

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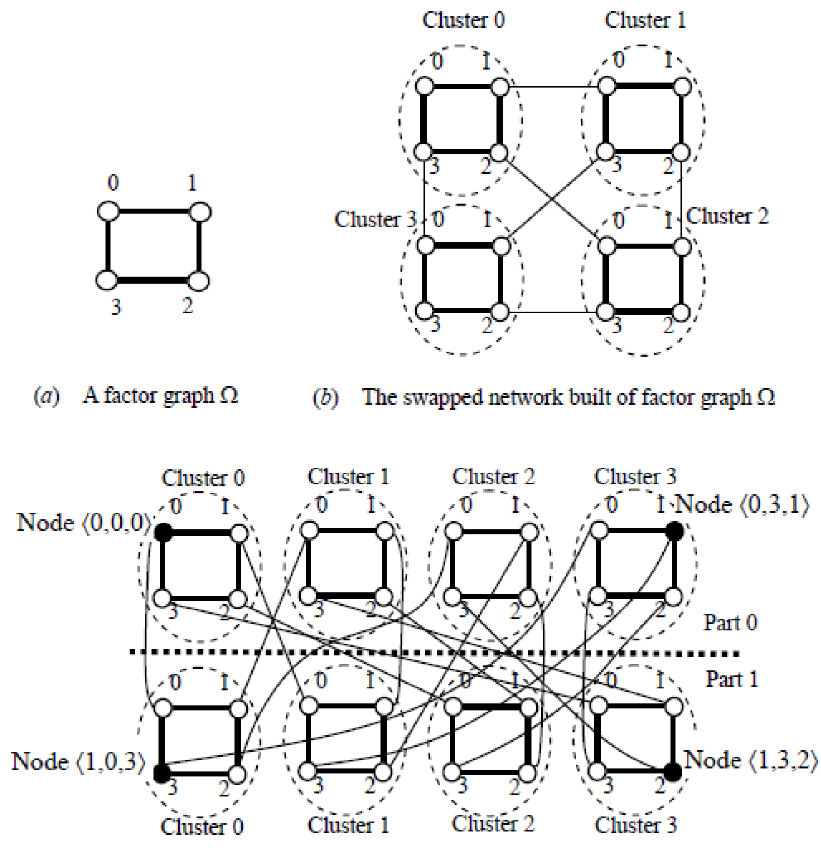

(*a*) A factor graph Ω (*b*) The swapped network built of factor graph Ω

(*c*) The Biswapped network built of factor graph Ω, denoted by *BSN*(Ω)

Fig. 1. An example factor network and the associated swapped network and Biswapped network.

its end-nodes are the same. A cycle is a closed path that repeats no node, except for the end-nodes. An *l*-cycle is a cycle of length *l*, and is called Hamiltonian cycle in the case of $l=|V(G)|$. Graph *G* is called Hamiltonian if it contains a Hamiltonian cycle. Alternatively, we denote by $u\rightarrow v$ the edge $(u,v)\in E(G)$, and by $v_1\rightarrow v_2\rightarrow\ldots\rightarrow v_{k-1}\rightarrow v_k$ the path $(v_1,v_2,\ldots,v_{k-1},v_k)$. Other notation and terminology used in this paper follow [21] unless otherwise stated.

In general, pancyclicity and bipancyclicity of a graph are defined as follows [10].

Definition 1. *A graph G is t-pancyclic if it contains cycles of all lengths from t up to* $|V(G)|$, *where t is an integer such that* $3\leq t\leq|V(G)|$. *A graph G is t-bipancyclic if it contains cycles of all even lengths from t up to* $|V(G)|$, *where t is an even number such that* $4\leq t\leq|V(G)|$.

B. Biswapped Networks

Biswapped networks are defined as follows [1].

Definition 2. *Biswapped Network(BSN): Given a factor graph Ω, the Biswapped network constructed from graph Ω, denoted by BSN(Ω), is a graph with node set* $V(BSN(\Omega))=\{\langle i,g,p\rangle \mid g,p\in V(\Omega),\ i=0,1\}$ *and edge set* $E(BSN(\Omega))$ *that consists of intra-cluster edges* $\{(\langle i,g,p\rangle,\langle i,g,q\rangle) \mid (p,q)\in E(\Omega),\ g\in V(\Omega),\ i=0,1\}$ *and inter-cluster edges* $\{\langle i,g,p\rangle,\langle 1-i,p,g\rangle) \mid g,p\in V(\Omega),\ i=0,1\}$.

Let $|V(\Omega)|=n$ and $V(\Omega)=\{0,1,\ldots,n-1\}$. $BSN(\Omega)$ has $N=2n^2$ nodes and is composed of two parts (called part 0 and part 1), each part of which can be divided into *n* sub-networks (called clusters $0,1,\ldots,n-1$, respectively) with each isomorphic to the factor network Ω. A node with identifier $\langle i,g,p\rangle$ corresponds to node *p* of cluster *g* in part *i*. Node $\langle i,g,p\rangle$ is connected to node $\langle 1-i,p,g\rangle$ via inter-cluster edge $(\langle i,g,p\rangle,\langle 1-i,p,g\rangle)$. Fig. 1 gives an example Biswapped network with a cycle of four nodes as the factor network Ω. Obviously, if every cluster of $BSN(\Omega)$ is viewed as a super-node, then the resulting graph of all the super-nodes along with all the inter-cluster edges will be a complete bipartite graph. Biswapped networks are also called bipartite swapped networks.

C. Notations

In the remainder of this paper, we assume that the factor graph Ω contains a cycle of length *l* ($3\leq l\leq n$), denoted by $\boldsymbol{C}_\Omega^{(l)}$. Without loss of generality, we further assume that the node set of cycle $\boldsymbol{C}_\Omega^{(l)}$ is $\{0,1,\ldots,l-1\}$, and $\boldsymbol{C}_\Omega^{(l)}$ is denoted by $0\rightarrow1\rightarrow2\rightarrow\ldots\rightarrow l-1\rightarrow 0$ clockwise, or by $0\rightarrow l-1\rightarrow l-2\rightarrow\ldots\rightarrow1\rightarrow 0$ counter-clockwise. For convenience, we use the following notations for any two distinct $x,y\in\{0,1,\ldots,l-1\}$:

- $\boldsymbol{P}[x,y]$: The sub-path from *x* to *y* clockwise in $\boldsymbol{C}_\Omega^{(l)}$, i.e., $x\rightarrow x+1\rightarrow x+2\rightarrow\ldots\rightarrow y-1\rightarrow y$.
- $\boldsymbol{P}^{-1}[x,y]$: The sub-path from *x* to *y* counter-clockwise in $\boldsymbol{C}_\Omega^{(l)}$, i.e., $x\rightarrow x-1\rightarrow x-2\rightarrow\ldots\rightarrow y+1\rightarrow y$.
- $x\Rightarrow y$: The sub-path from *x* to *y* in $\boldsymbol{C}_\Omega^{(l)}$, which is either $\boldsymbol{P}[x,y]$ or $\boldsymbol{P}^{-1}[x,y]$.

In the above notations, arithmetic operations "+" and "–" are the corresponding ***mod*** *l* arithmetic operation. We use $|\boldsymbol{P}|$

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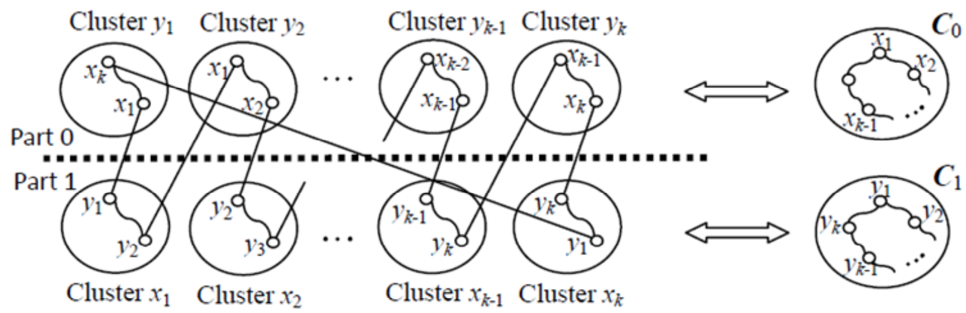

Fig. 2. An illustration of combined closed paths of the factor network Ω derived from a cycle in the Biswapped network $BSN(\Omega)$.

to denote the length of path $\boldsymbol{P}$. For any two distinct $x, y\in\{0, 1, \ldots, l-1\}$, $\boldsymbol{P}[x,y]$ and $\boldsymbol{P}^{-1}[x,y]$ are counter paths of each other, and $|\boldsymbol{P}[x,y]|+|\boldsymbol{P}^{-1}[x,y]| = l$.

III. Constructing Even-Length Cycles

In this section, we first characterize an arbitrary cycle $\boldsymbol{C}_{BSN(\Omega)}$ in $BSN(\Omega)$ based on the given cycle $\boldsymbol{C}_{\Omega}^{(l)}$ in the factor network , and then give an algorithm for constructing cycles of various even lengths in $BSN(\Omega)$.

A. Basic Idea

In order to obtain a method for constructing cycles in Biswapped networks in a general sense, we restrict all intra-cluster edges used in $\boldsymbol{C}_{BSN(\Omega)}$ to those corresponding edges of $\boldsymbol{C}_{\Omega}^{(l)}$ in every cluster since it is isomorphic to the factor network . For convenience, moreover, we allow $\boldsymbol{C}_{BSN(\Omega)}$ to pass through any cluster at most one time. According to the connectivity rule for the Biswapped network architecture, we have either $\boldsymbol{C}_{BSN(\Omega)}$ is completely contained within a cluster, or $\boldsymbol{C}_{BSN(\Omega)}$ goes through even (at least 4) clusters with half clusters in each part. The former case, where $\boldsymbol{C}_{BSN(\Omega)}$ is isomorphic to $\boldsymbol{C}_{\Omega}^{(l)}$, is trivial. Now assume that $\boldsymbol{C}_{BSN(\Omega)}$ passes through one by one cluster y_1 of part 0, cluster x_1 of part 1, cluster y_2 of part 0, cluster x_2 of part 1,..., cluster y_k of part 0, cluster x_k of part 1, where $k\in\{2,3,\ldots,l\}$. An illustration is shown in Fig.2. It is clear that all the sub-paths of $\boldsymbol{C}_{BSN(\Omega)}$ that are contained in the clusters of part 0 form a k-segment combined closed path of factor graph Ω: $x_1\Rightarrow x_2\Rightarrow\ldots\Rightarrow x_{k-1}\Rightarrow x_k\Rightarrow x_1$, denoted by $\boldsymbol{C_0}$, and that all the sub-paths of $\boldsymbol{C}_{BSN(\Omega)}$ that are contained in the clusters of part 1 form a k-segment combined closed path of factor graph Ω: $y_1\Rightarrow y_2\Rightarrow\ldots\Rightarrow y_{k-1}\Rightarrow y_k\Rightarrow y_1$, denoted by $\boldsymbol{C_1}$. Let $\boldsymbol{C_{01}}$ be the set of all inter-cluster edges in $\boldsymbol{C}_{BSN(\Omega)}$ that connect the sub-paths of part 0 with the sub-paths of part 1. Then, the length of cycle $\boldsymbol{C}_{BSN(\Omega)}$ can be calculated by

$$|\boldsymbol{C}_{BSN(\Omega)}| = |\boldsymbol{C_0}| + |\boldsymbol{C_1}| +|\boldsymbol{C_{01}}| \qquad (1)$$

where $|\boldsymbol{C_{01}}|=2k$, and

$$|C_0| = \sum_{i=1}^{k-1} |x_i \Rightarrow x_{i+1}| + |x_k \Rightarrow x_1|$$

$$|C_1| = \sum_{i=1}^{k-1} |y_i \Rightarrow y_{i+1}| + |y_k \Rightarrow y_1|$$

Obviously, we easily obtain a cycle $\boldsymbol{C}_{BSN(\Omega)}$ in $BSN(\Omega)$ based on two k-segment combined closed paths constructed respectively for part 0 and part 1 for any $k\in\{2, 3,\ldots, l\}$. The length of $\boldsymbol{C}_{BSN(\Omega)}$ depends on the value of k and the lengths of all k sub-paths in every combined closed path.

Recall that each sub-path $x \Rightarrow y$ in $\boldsymbol{C}_{\Omega}^{(l)}$ is either $\boldsymbol{P}[x,y]$ or $\boldsymbol{P}^{-1}[x,y]$, which have different lengths (the sum of these two lengths is l). By choosing $\boldsymbol{P}[x,y]$ or $\boldsymbol{P}^{-1}[x,y]$ for each sub-path $x \Rightarrow y$ in every combined closed path, we can get combined closed paths of various different lengths. The following two lemmas give respectively some possible lengths of a 2-segment combined closed path and an l-segment combined closed path provided that the factor network contains an l-length cycle.

Lemma 1. *Given an l-length cycle in factor network Ω, for any $t\in\{2, 4, 6,\ldots,2(l-1)\}$, there exists a 2-segment combined closed path of length t in Ω.*

Proof. For any $i\in\{1, 2,\ldots, l-1\}$, $\boldsymbol{P}[0,i]\cup\boldsymbol{P}^{-1}[i,0]$ is a combined closed path of length $2i$. This completes the proof. □

Lemma 2. *Given an l-length cycle in factor network Ω, for any $t\in\{l, l+2, l+4,\ldots, 3l-4)\}\cup\{2l, 2l+2, 2l+4,\ldots, l^2-l)\}$, there exists an l-segment combined closed path $\boldsymbol{M}[i,j]$ in Ω with $|\boldsymbol{M}[i, j]|=2i+(l-2)j=t$ for $i, j\in\{1, 2,\ldots, l-1\}$.*

Proof. For any $i\in\{1, 2,\ldots, l-1\}$, let $\boldsymbol{M}[i,1]$ be the following path

$0\Rightarrow i\Rightarrow i-1\Rightarrow\ldots\Rightarrow 2\Rightarrow 1\Rightarrow i+1\Rightarrow\ldots\Rightarrow l-2\Rightarrow l-1\Rightarrow 0$

Specifically speaking, the path is

$\boldsymbol{P}[0,i]\cup\boldsymbol{P}^{-1}[i,i-1]\cup...\cup\boldsymbol{P}^{-1}[2,1]\cup\boldsymbol{P}[1,i+1]\cup\ldots\cup\boldsymbol{P}[l-2,l-1]$
$\cup\boldsymbol{P}[l-1,0]$.

Fig. 3 (a) and (b) gives respectively the cycle $\boldsymbol{C}_{\Omega}^{(l)}$ contained in the factor network Ω and the l-segment combined closed path $\boldsymbol{M}[i,1]$. Clearly, $\boldsymbol{M}[i,1]$ is an l-segment combined closed path of length $2i+l-2$, which can take all numbers from set $\{ l, l+2, l+4,\ldots, 3l-4\}$ due to $i\in\{1, 2,\ldots, l-1\}$. Note that all l sub-paths of $\boldsymbol{M}[1,1]$ are of unit length, and for each $i\in\{2, 3,\ldots, l-1\}$, $\boldsymbol{M}[i,1]$ has exactly $l-2$ sub-paths of unit length among all its l sub-paths. Thus, we can replace any j sub-paths of unit length in $\boldsymbol{M}[i,1]$ with their corresponding counter paths of length $l-1$ ($i\in\{1, 2,\ldots, l-1\}$, $j\in\{1, 2,\ldots, l-2\}$). The resulting combined closed path, denoted by $\boldsymbol{M}[i,j]$, has length $2i+(l-2)j$ ($i\in\{1, 2,\ldots, l-1\}$, $j\in\{2, 3,\ldots, l-1\}$), which can take all even numbers from $2l$ to l^2-l. This completes this proof. □

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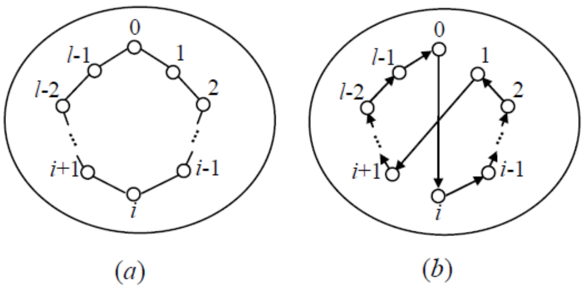

Fig. 3. (*a*) The cycle $C_\Omega^{(l)}$ contained in the factor network Ω. (*b*) The *l*-segment combined closed path $\boldsymbol{M}[i,1]$ in the factor network Ω, where $0\rightarrow i$ and $1\rightarrow i+1$ respectively represent paths $0\Rightarrow i$ and $1\Rightarrow i+1$.

B. Algorithm

Based on the above two lemmas, an algorithm for constructing even-length cycles in a Biswapped network with a factor network containing an *l*-length cycle is described in Alg.1.

Algorithm 1 Constructing Even-Length Cycles in BSNs
Input: A factor network Ω containing a cycle of length *l*, and an even number *t* ($8\leq t\leq 2l^2$).
Output: A cycle $\boldsymbol{C}_{BSN(\Omega)}$ of length *t* in *BSN*(Ω).
Begin
1. **Case** 1 ($8\leq t\leq 2l+4$):
   $k\leftarrow 2$;
   $\boldsymbol{C_0}\leftarrow \boldsymbol{P}[0,1]\cup \boldsymbol{P}^{-1}[1,0]$;
   $\boldsymbol{C_1}\leftarrow \boldsymbol{P}[0,t/2-3]\cup \boldsymbol{P}^{-1}[t/2-3,0]$
2. **Case** 2 ($2l+6\leq t\leq 4l$):
   $k\leftarrow 2$;
   $\boldsymbol{C_0}\leftarrow \boldsymbol{P}[0, l-1]\cup \boldsymbol{P}^{-1}[l-1,0]$;
   $\boldsymbol{C_1}\leftarrow \boldsymbol{P}[0,t/2-l-1]\cup \boldsymbol{P}^{-1}[t/2-l-1,0]$
3. **Case** 3 ($4l+2\leq t\leq 6l-2$):
   $k\leftarrow l$;
   $\boldsymbol{C_0}\leftarrow \boldsymbol{M}[2,1]$;
   $\boldsymbol{C_1}\leftarrow \boldsymbol{M}[t/2-2l,1]$
4. **Case** 4 ($6l\leq t\leq l^2+3l$):
   $k\leftarrow l$;
   $\boldsymbol{C_0}\leftarrow \boldsymbol{M}[2,2]$;
   $\boldsymbol{C_1}\leftarrow \boldsymbol{M}[i,j]$, where $|\boldsymbol{M}[i,j]|= t-4l$ for $i, j\in\{1,2,\ldots,l-1\}$
5. **Case** 5 ($l^2+3l+2\leq t\leq 2l^2$):
   $k\leftarrow l$;
   $\boldsymbol{C_0}\leftarrow \boldsymbol{M}[l-1, l-1]$;
   $\boldsymbol{C_1}\leftarrow \boldsymbol{M}[i,j]$, where $|\boldsymbol{M}[i,j]|= t-l^2-l$ for $i, j\in\{1,2,\ldots,l-1\}$
6. Construct $\boldsymbol{C}_{BSN(\Omega)}$ according to *k*, $\boldsymbol{C_0}$ and $\boldsymbol{C_1}$

End

C. Algorithm Analysis

The following theorem states the properties of Alg.1 with regard to the correctness and the time complexity.

Theorem 1. *Given an l-length cycle in factor network Ω, where $l\geq 3$, for any $t\in\{8, 10, 12,\ldots, 2l^2\}$, Alg.1 yields a cycle of length t in BSN(Ω) in time $O(n^2)$, where n is the number of nodes in Ω.*

Proof. Firstly, according to the above discussion, we have that the path $\boldsymbol{C}_{BSN(\Omega)}$ generated by Alg.1 is a cycle in *BSN*(Ω). Secondly, the length of $\boldsymbol{C}_{BSN(\Omega)}$ can be immediately calculated according to Eq.(1) and Lemmas 1 and 2, which is clearly *t*. The constructions of the combined closed paths $\boldsymbol{C_0}$ and $\boldsymbol{C_1}$ in Steps 1-5 in Alg. 1 is very straightforward, and it is trivial to obtain $\boldsymbol{C}_{BSN(\Omega)}$ in Step 6 based on the generated *k*, $\boldsymbol{C_0}$ and $\boldsymbol{C_1}$. Hence, the algorithm has time complexity $O(n^2)$ due to $l\leq n$. This proof is complete. □

Recall that *BSN*(Ω) has $N=2n^2$ nodes if the factor network Ω has *n* nodes. Alg.1 is very efficient since it yields the required cycle in *BSN*(Ω) in linear time. On the other hand, if the factor network Ω contains a cycle of length *n*, i.e., it is Hamiltonian, *BSN*(Ω) will contain all even-length cycles whose lengths range from 8 up to $2n^2$ by the above theorem. This immediately yields the following result about bipancyclicity of *BSN*(Ω).

Theorem 2. *If factor network Ω is Hamiltonian, then BSN(Ω) is 8-bipancyclic.*

D. Examples

In the following, we give results of Alg. 1 when run on the following input instances: the factor network shown in Fig. 1(*a*) that is Hamiltonian, $l=4$ and $t = 8,10,\ldots,2l^2$.

According to the method proposed in Section II, given a *k* and two *k*-segment combined closed paths $\boldsymbol{C_0}$ and $\boldsymbol{C_1}$ in the factor network, where $\boldsymbol{C_0}= x_1\Rightarrow x_2\Rightarrow\ldots\Rightarrow x_{k-1}\Rightarrow x_k\Rightarrow x_1$ and $\boldsymbol{C_1}= y_1\Rightarrow y_2\Rightarrow\ldots\Rightarrow y_{k-1}\Rightarrow y_k\Rightarrow y_1$, the corresponding cycle in the Biswapped network is as follows.

$$\boldsymbol{C}_{BSN(\Omega)}= \langle 0,y_1,x_k\Rightarrow x_1\rangle\downarrow\langle 1,x_1,y_1\Rightarrow y_2\rangle\uparrow\langle 0,y_2,x_1\Rightarrow x_2\rangle\downarrow$$
$$\langle 1,x_2,y_2\Rightarrow y_3\rangle\uparrow\ldots\langle 0,y_{k-1},x_{k-2}\Rightarrow x_{k-1}\rangle\downarrow\langle 1,x_{k-1},y_{k-1}\Rightarrow y_k\rangle\uparrow$$
$$\langle 0,y_k,x_{k-1}\Rightarrow x_k\rangle\downarrow\langle 1,x_k,y_k\Rightarrow y_1\rangle\uparrow$$

Here we use ↓ (↑) to represent an inter-cluster edge from a node of a cluster in part 0 (part 1) to a node of a cluster in part 1(part 0). For convenience, we only give the *k* and the two *k*-segment combined closed paths $\boldsymbol{C_0}$ and $\boldsymbol{C_1}$ generated by Alg. 1 for every input instance considered here. The results are as follows.

- $t=8$ **(Case 1)**: $k=2$;
  $\boldsymbol{C_0}= 0\rightarrow 1\rightarrow 0$;
  $\boldsymbol{C_1}= 0\rightarrow 1\rightarrow 0$
- $t=10$ **(Case 1)**: $k=2$;
  $\boldsymbol{C_0}= 0\rightarrow 1\rightarrow 0$;
  $\boldsymbol{C_1}= 0\Rightarrow 2(\rightarrow 1\rightarrow 2)\Rightarrow 0\ (\rightarrow 1\rightarrow 0)$
- $t=12$ **(Case 1)**: $k=2$;
  $\boldsymbol{C_0}= 0\rightarrow 1\rightarrow 0$;
  $\boldsymbol{C_1}= 0\Rightarrow 3(\rightarrow 1\rightarrow 2\rightarrow 3)\Rightarrow 0\ (\rightarrow 2\rightarrow 1\rightarrow 0)$
- $t=14$ **(Case 2)**: $k=2$;
  $\boldsymbol{C_0}= 0\Rightarrow 3(\rightarrow 1\rightarrow 2\rightarrow 3)\Rightarrow 0\ (\rightarrow 2\rightarrow 1\rightarrow 0)$;
  $\boldsymbol{C_1}= 0\Rightarrow 2(\rightarrow 1\rightarrow 2)\Rightarrow 0\ (\rightarrow 1\rightarrow 0)$
- $t=16$ **(Case 2)**: $k=2$;
  $\boldsymbol{C_0}= 0\Rightarrow 3(\rightarrow 1\rightarrow 2\rightarrow 3)\Rightarrow 0\ (\rightarrow 2\rightarrow 1\rightarrow 0)$;
  $\boldsymbol{C_1}= 0\Rightarrow 3(\rightarrow 1\rightarrow 2\rightarrow 3)\Rightarrow 0\ (\rightarrow 2\rightarrow 1\rightarrow 0)$
- $t=18$ **(Case 3)**: $k=4$;
  $\boldsymbol{C_0}=\boldsymbol{M}[2,1]= 0\Rightarrow 2(\rightarrow 1\rightarrow 2)\rightarrow 1\Rightarrow 3(\rightarrow 2\rightarrow 3)\rightarrow 0$;
  $\boldsymbol{C_1}=\boldsymbol{M}[1,1]= 0\rightarrow 1\rightarrow 2\rightarrow 3\rightarrow 0$
- $t=20$ **(Case 3)**: $k=4$;
  $\boldsymbol{C_0}=\boldsymbol{M}[2,1]= 0\Rightarrow 2(\rightarrow 1\rightarrow 2)\rightarrow 1\Rightarrow 3(\rightarrow 2\rightarrow 3)\rightarrow 0$;

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$\boldsymbol{C_1}$=$\boldsymbol{M}$[2,1]= $0\Rightarrow2(\rightarrow1\rightarrow2)\rightarrow1\Rightarrow3(\rightarrow2\rightarrow3)\rightarrow0$

- t=22 **(Case 3)**: k=4;
  $\boldsymbol{C_0}$=$\boldsymbol{M}$[2,1]= $0\Rightarrow2(\rightarrow1\rightarrow2)\rightarrow1\Rightarrow3(\rightarrow2\rightarrow3)\rightarrow0$;
  $\boldsymbol{C_1}$=$\boldsymbol{M}$[3,1]= $0\Rightarrow3(\rightarrow1\rightarrow2\rightarrow3)\rightarrow2\rightarrow1\Rightarrow0(\rightarrow2\rightarrow3\rightarrow0)$
- t=24 **(Case 4)**: k=4;
  $\boldsymbol{C_0}$=$\boldsymbol{M}$[2,2]= $0\Rightarrow2(\rightarrow1\rightarrow2)\rightarrow1\Rightarrow3(\rightarrow2\rightarrow3)\Rightarrow0(\rightarrow2\rightarrow1\rightarrow0)$;
  $\boldsymbol{C_1}$=$\boldsymbol{M}$[3,1]= $0\Rightarrow3(\rightarrow1\rightarrow2\rightarrow3)\rightarrow2\rightarrow1\Rightarrow0(\rightarrow2\rightarrow3\rightarrow0)$
- t=26 **(Case 4)**: k=4;
  $\boldsymbol{C_0}$=$\boldsymbol{M}$[2,2]= $0\Rightarrow2(\rightarrow1\rightarrow2)\rightarrow1\Rightarrow3(\rightarrow2\rightarrow3)\Rightarrow0(\rightarrow2\rightarrow1\rightarrow0)$;
  $\boldsymbol{C_1}$=$\boldsymbol{M}$[3,2]= $0\Rightarrow3(\rightarrow1\rightarrow2\rightarrow3)\Rightarrow2(\rightarrow0\rightarrow1\rightarrow2)\rightarrow1\Rightarrow0(\rightarrow2\rightarrow3\rightarrow0)$
- t=28 **(Case 4)**: k=4;
  $\boldsymbol{C_0}$=$\boldsymbol{M}$[2,2]= $0\Rightarrow2(\rightarrow1\rightarrow2)\rightarrow1\Rightarrow3(\rightarrow2\rightarrow3)\Rightarrow0(\rightarrow2\rightarrow1\rightarrow0)$;
  $\boldsymbol{C_1}$=$\boldsymbol{M}$[3,3]= $0\Rightarrow3(\rightarrow1\rightarrow2\rightarrow3)\Rightarrow2(\rightarrow0\rightarrow1\rightarrow2)\Rightarrow1(\rightarrow3\rightarrow0\rightarrow1)\Rightarrow0(\rightarrow2\rightarrow3\rightarrow0)$
- t=30 **(Case 5)**: k=4;
  $\boldsymbol{C_0}$=$\boldsymbol{M}$[3,3]= $0\Rightarrow3(\rightarrow1\rightarrow2\rightarrow3)\Rightarrow2(\rightarrow0\rightarrow1\rightarrow2)\Rightarrow1(\rightarrow3\rightarrow0\rightarrow1)\Rightarrow0(\rightarrow2\rightarrow3\rightarrow0)$;
  $\boldsymbol{C_1}$=$\boldsymbol{M}$[3,2]= $0\Rightarrow3(\rightarrow1\rightarrow2\rightarrow3)\Rightarrow2(\rightarrow0\rightarrow1\rightarrow2)\rightarrow1\Rightarrow0(\rightarrow2\rightarrow3\rightarrow0)$
- t=32 **(Case 5)**: k=4;
  $\boldsymbol{C_0}$=$\boldsymbol{M}$[3,3]= $0\Rightarrow3(\rightarrow1\rightarrow2\rightarrow3)\Rightarrow2(\rightarrow0\rightarrow1\rightarrow2)\Rightarrow1(\rightarrow3\rightarrow0\rightarrow1)\Rightarrow0(\rightarrow2\rightarrow3\rightarrow0)$;
  $\boldsymbol{C_1}$=$\boldsymbol{M}$[3,3]= $0\Rightarrow3(\rightarrow1\rightarrow2\rightarrow3)\Rightarrow2(\rightarrow0\rightarrow1\rightarrow2)\Rightarrow1(\rightarrow3\rightarrow0\rightarrow1)\Rightarrow0(\rightarrow2\rightarrow3\rightarrow0)$

IV. Conclusion

Given an l-cycle in the factor network, a simple algorithm has been proposed for embedding cycles of various even lengths ranging from 8 up to $2l^2$ in the Biswapped network based on the feature of modularity and the interconnection rule in the topology. This result indicates that a Biswapped network is 8-bipancyclic if its factor network is Hamiltonian. The basic idea behind the algorithm is to characterize the structure of a cycle in the Biswapped network, and then establish the relationship between the cycle in the Biswapped network and two associated closed paths in the factor network.

In some Biswapped networks, including the one shown in Fig.1(c), no odd-length cycles exist. This fact may result from the observation that Biswapped networks are closely related with bipartite graphs. An interesting open problem is what odd-length cycles can be embedded in a Biswapped network if its factor network contains an odd-length cycle.

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