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Interactive Decision Making for Multiobjective Fuzzy Random Linear Programming Problems Using Expectations and Coefficients of Variation

Hitoshi Yano, Kota Matsui and Mikiya Furuhashi

***Abstract*—In this paper, an interactive decision making method for multiobjective fuzzy random linear programming problems using expectations and coefficients of variation is proposed. In the proposed method, it is assumed that the decision maker intends to not only maximize the expected degrees of possibilities that the original objective functions attain the corresponding fuzzy goals, but also minimize coefficients of variation for such possibilities, and such fuzzy goals are quantified by eliciting the corresponding membership functions. Using the fuzzy decision, such two kinds of membership functions are integrated. In the integrated membership space, a satisfactory solution is obtained from among an EV-Pareto optimal solution set through the interaction with the decision maker.**

***Index Terms*—multiobjective programming, fuzzy random variables, expectations, coefficients of variation, fuzzy decision, interactive method.**

I. Introduction

In the real world decision making situations, we often have to make a decision under uncertainty. In order to deal with decision problems involving uncertainty, stochastic programming approaches [1], [2], [3], [6] and fuzzy programming approaches [12], [14], [20] have been developed. Recently, mathematical programming problems with fuzzy random variables [11] have been proposed [13], [15], [16] whose concept includes both probabilistic uncertainty and fuzzy one simultaneously. Extensions to multiobjective fuzzy random linear programming problems (MOFRLP) have been done and interactive methods to obtain the satisfactory solution for the decision maker have been proposed [7], [9], [15]. In their methods, it is required in advance for the decision maker to specify permissible possibility levels in a probability maximization model or permissible probability levels in a fractile optimization model. However, it seems to be very difficult for the decision maker to specify such permissible levels appropriately. From such a point of view, a fuzzy approach to MOFRLP, in which the decision maker specifies the membership functions for the fuzzy goals of both the original objective functions and the corresponding permissible levels has been proposed [17]. In the proposed method, it is assumed that the decision maker adopts the fuzzy decision [5], [14] to integrate the membership functions. As a natural extension of such a method, interactive fuzzy decision making methods for MOFRLP to obtain the satisfactory solution from among an extended Pareto optimal solution set have been proposed [18], [19]

In this paper, it is assumed that the decision maker intends to not only maximize the expected degrees of possibilities [5] that the original objective functions involving fuzzy random variable coefficients attain the corresponding fuzzy goals, but also minimize coefficients of variation for such possibilities in MOFRLP [8], [10]. In order to deal with such decision making situations in MOFRLP, we introduce an EV-Pareto optimal solution concept, in which both the expected degrees of possibilities and the corresponding coefficients of variation for such possibilities are integrated through the fuzzy decision [5], [14]. To obtain an EV-Pareto optimal solution, minmax problem is formulated. An interactive algorithm is proposed to obtain the satisfactory solution from among an EV-Pareto optimal solution set by solving the minmax problem on the basis of convex programming technique. In order to illustrate the proposed method, a three-objective fuzzy random linear programming problem is formulated, and the interactive processes under the hypothetical decision maker are demonstrated.

II. Multiobjective fuzzy random linear programming problems

In this section, we focus on multiobjective programming problems involving fuzzy random variable coefficients in objective functions called multiobjective fuzzy random linear programming problem (MOFRLP).
[MOFRLP]

$$\min_{\boldsymbol{x}\in X} \tilde{\bar{C}}\boldsymbol{x} = (\tilde{\bar{\boldsymbol{c}}}_1\boldsymbol{x}, \cdots, \tilde{\bar{\boldsymbol{c}}}_k\boldsymbol{x}) \tag{1}$$

where $\boldsymbol{x} = (x_1, \cdots, x_n)^T$ is an n dimensional decision variable column vector. X is a linear constraint set with respect to $\boldsymbol{x}$. $\tilde{\bar{\boldsymbol{c}}}_i = (\tilde{\bar{c}}_{i1}, \cdots, \tilde{\bar{c}}_{in}), i = 1, \cdots, k$ are coefficient vectors of objective function $\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}$, whose elements are fuzzy random variables (The symbols "-" and "~" mean randomness and fuzziness respectively).

In this paper, we assume that under the occurrence of each scenario $\ell_i \in \{1, \cdots, L_i\}$, $\tilde{c}_{ij\ell_i}$ is a realization of a fuzzy random variable $\tilde{\bar{c}}_{ij}$ which is a fuzzy number whose membership function is defined as follows [15].

$$\mu_{\tilde{c}_{ij\ell_i}}(t) = \begin{cases} \max\left\{1 - \frac{d_{ij\ell_i} - t}{\alpha_{ij}}, 0\right\}, & t \le d_{ij\ell_i} \\ \max\left\{1 - \frac{t - d_{ij\ell_i}}{\beta_{ij}}, 0\right\}, & t > d_{ij\ell_i} \end{cases} \tag{2}$$

where the parameters $\alpha_{ij} > 0, \beta_{ij} > 0$ are constants and $d_{ij\ell_i}$ varies depending on which a scenario ℓ_i occurs.

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Moreover, we assume that a scenario ℓ_i occurs with a probability $p_{i\ell_i}$, where $\sum_{\ell_i=1}^{L_i} p_{i\ell_i} = 1$ for $i = 1, \cdots, k$.

By Zadeh's extension principle, the realization $\tilde{\boldsymbol{c}}_{i\ell_i}\boldsymbol{x}$ becomes a fuzzy number which characterized by the following membership function.

$$\mu_{\tilde{\boldsymbol{c}}_{i\ell_i}\boldsymbol{x}}(y) = \begin{cases} \max\left\{1 - \frac{\boldsymbol{d}_{i\ell_i}\boldsymbol{x} - y}{\boldsymbol{\alpha}_i\boldsymbol{x}}, 0\right\}, & y \leq \boldsymbol{d}_{i\ell_i}\boldsymbol{x} \\ \max\left\{1 - \frac{y - \boldsymbol{d}_{i\ell_i}\boldsymbol{x}}{\boldsymbol{\beta}_i\boldsymbol{x}}, 0\right\}, & y > \boldsymbol{d}_{i\ell_i}\boldsymbol{x} \end{cases} \tag{3}$$

where $\boldsymbol{d}_{i\ell_i} = (d_{i1\ell_i}, \cdots, d_{in\ell_i})$, $\boldsymbol{\alpha}_i = (\alpha_{i1}, \cdots, \alpha_{in}) \geq \mathbf{0}$, $\boldsymbol{\beta}_i = (\beta_{i1}, \cdots, \beta_{in}) \geq \mathbf{0}$.

Considering the imprecise nature of the decision maker's judgment, it is natural to assume that the decision maker has a fuzzy goal for each objective function in MOFRLP. In this paper, it is assumed that such a fuzzy goal $\tilde{G}_i$ can be quantified by eliciting the corresponding membership function defined as follows.

$$\mu_{\tilde{G}_i}(y_i) = \begin{cases} 1 & y_i < z_i^1 \\ \dfrac{y_i - z_i^0}{z_i^1 - z_i^0} & z_i^1 \leq y_i \leq z_i^0 \\ 0 & y_i > z_i^0 \end{cases} \tag{4}$$

where z_i^0 represents the minimum value of an unacceptable level of the objective function, and z_i^1 represents the maximum value of a sufficiently satisfactory level of the objective function. By using a concept of possibility measure [5], the degree of possibility that the objective function value $\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}$ satisfies the fuzzy goal $\tilde{G}_i$ is expressed as follows [9].

$$\Pi_{\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}}(\tilde{G}_i) \stackrel{\text{def}}{=} \sup_y \min\{\mu_{\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}}(y), \mu_{\tilde{G}_i}(y)\} \tag{5}$$

It should be noted here that if a scenario ℓ_i occurs with probability $p_{i\ell_i}$ then the value of possibility measure can be represent as

$$\Pi_{\tilde{\boldsymbol{c}}_{i\ell_i}\boldsymbol{x}}(\tilde{G}_i) \stackrel{\text{def}}{=} \sup_y \min\{\mu_{\tilde{\boldsymbol{c}}_{i\ell_i}\boldsymbol{x}}(y), \mu_{\tilde{G}_i}(y)\}. \tag{6}$$

Using the above possibility measure, MOFRLP can be transformed into the following multiobjective stochastic programming problem (MOSP).

[MOSP]

$$\max_{\boldsymbol{x}\in X}(\Pi_{\tilde{\bar{\boldsymbol{c}}}_1\boldsymbol{x}}(\tilde{G}_1), \cdots, \Pi_{\tilde{\bar{\boldsymbol{c}}}_k\boldsymbol{x}}(\tilde{G}_k)) \tag{7}$$

III. An Expectation model and A variance model for MOFRLP

Katagiri et al.[8], [10] formulated MOFRLP as the multiobjective programming problems through expectation model (E-model) and variance model (V-model) respectively. At First, we explain E-model for MOFRLP formulated as follows.

[MOP-E1]

$$\max_{\boldsymbol{x}\in X}(E[\Pi_{\tilde{\bar{\boldsymbol{c}}}_1\boldsymbol{x}}(\tilde{G}_1)], \cdots, E[\Pi_{\tilde{\bar{\boldsymbol{c}}}_k\boldsymbol{x}}(\tilde{G}_k)]) \tag{8}$$

where $E[\cdot]$ denotes the expectation operator. In order to deal with MOP-E1, we introduce an E-Pareto optimal solution concept.

Definition 1: $\boldsymbol{x}^* \in X$ is said to be an E-Pareto optimal solution to MOP-E1, if and only if there does not exist another $\boldsymbol{x} \in X$ such that $E[\Pi_{\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}}(\tilde{G}_i)] \geq E[\Pi_{\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}^*}(\tilde{G}_i)]$, $i = 1, \cdots, k$ with strict inequality holding for at least one i.

It should be noted here that (6) can be represented as follows [15].

$$\Pi_{\tilde{\boldsymbol{c}}_{i\ell_i}\boldsymbol{x}}(\tilde{G}_i) = \frac{\sum_{j=1}^n(\alpha_{ij} - d_{ij\ell_i})x_j + z_i^0}{\sum_{j=1}^n \alpha_{ij}x_j - z_i^1 + z_i^0} \tag{9}$$

Since the probability that a scenario ℓ_i occurs is $p_{i\ell_i}$, $E[\Pi_{\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}}(\tilde{G}_i)]$ can be computed as follows.

$$\begin{aligned} & E[\Pi_{\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}}(\tilde{G}_i)] \\ = & \sum_{\ell_i=1}^{L_i} p_{i\ell_i}\Pi_{\tilde{\boldsymbol{c}}_{i\ell_i}\boldsymbol{x}}(\tilde{G}_i) \\ = & \frac{\sum_{j=1}^n(\alpha_{ij} - \sum_{\ell_i=1}^{L_i} p_{i\ell_i}d_{ij\ell_i})x_j + z_i^0}{\sum_{j=1}^n \alpha_{ij}x_j - z_i^1 + z_i^0} \\ \stackrel{\text{def}}{=} & Z_i^E(\boldsymbol{x}) \end{aligned} \tag{10}$$

Then, MOP-E1 can be transformed into MOP-E2.

[MOP-E2]

$$\max_{\boldsymbol{x}\in X}(Z_1^E(\boldsymbol{x}), \cdots, Z_k^E(\boldsymbol{x})) \tag{11}$$

Next, consider V-model for MOFRLP. The multiobjective programming problem based on V-model can be formulated as follows.

[MOP-V1]

$$\min_{\boldsymbol{x}\in X}(V[\Pi_{\tilde{\bar{\boldsymbol{c}}}_1\boldsymbol{x}}(\tilde{G}_1)], \cdots, V[\Pi_{\tilde{\bar{\boldsymbol{c}}}_k\boldsymbol{x}}(\tilde{G}_k)]) \tag{12}$$

subject to

$$E[\Pi_{\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}}(\tilde{G}_i)] \geq \xi_i, \quad i = 1, \cdots, k \tag{13}$$

where $V[\cdot]$ denotes the variance operator, and ξ_i represents a permissible expectation level for $E[\Pi_{\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}}(\tilde{G}_i)]$. Now, we denote the feasible set of MOP-V1 as

$$X(\boldsymbol{\xi}) \stackrel{\text{def}}{=} \{\boldsymbol{x} \in X | E[\Pi_{\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}}(\tilde{G}_i)] \geq \xi_i, i = 1, \cdots, k\}. \tag{14}$$

Similar to E-model, in order to deal with MOP-V1, a V-Pareto optimal solution concept is defined.

Definition 2: $\boldsymbol{x}^* \in X(\boldsymbol{\xi})$ is said to be a V-Pareto optimal solution to MOP-V1, if and only if there does not exist another $\boldsymbol{x} \in X(\boldsymbol{\xi})$ such that $V[\Pi_{\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}}(\tilde{G}_i)] \leq V[\Pi_{\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}^*}(\tilde{G}_i)]$, $i = 1, \cdots, k$ with strict inequality holding for at least one i.

It should be noted here that $V[\Pi_{\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}}(\tilde{G}_i)]$ can be represented as follows [15].

$$\begin{aligned} & V[\Pi_{\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}}(\tilde{G}_i)] \\ = & \frac{1}{(\sum_{j=1}^n \alpha_{ij}x_j - z_i^1 + z_i^0)^2} V\left[\sum_{j=1}^n \bar{d}_{ij}x_j\right] \\ = & \frac{1}{(\sum_{j=1}^n \alpha_{ij}x_j - z_i^1 + z_i^0)^2}\boldsymbol{x}^T\boldsymbol{V}_i\boldsymbol{x} \\ \stackrel{\text{def}}{=} & Z_i^V(\boldsymbol{x}) \end{aligned} \tag{15}$$

where $\boldsymbol{V}_i$ is the variance-covariance matrix of $\bar{\boldsymbol{d}}_i$ expressed by

$$\boldsymbol{V}_i = \begin{pmatrix} v_{11}^i & v_{12}^i & \dots & v_{1n}^i \\ v_{21}^i & v_{22}^i & \dots & v_{2n}^i \\ \vdots & \vdots & \ddots & \vdots \\ v_{n1}^i & v_{n2}^i & \dots & v_{nn}^i \end{pmatrix}, i = 1, \cdots, k, \tag{16}$$

and

$$\begin{aligned} v^i_{jj} &= V[\bar{d}_{jj}] \\ &= \sum_{\ell_i=1}^{L_i} p_{i\ell_i} d^2_{ij\ell_i} - \left(\sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i} \right)^2, \\ & \quad j = 1, \cdots, n, \qquad (17) \\ v^i_{jr} &= \mathrm{Cov}[\bar{d}_{ij}, \bar{d}_{ir}] \\ &= E[\bar{d}_{ij} \cdot \bar{d}_{ir}] - E[\bar{d}_{ij}]E[\bar{d}_{ir}] \\ &= \sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i} d_{ir\ell_i} - \sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i} \sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ir\ell_i}, \\ & \quad j, r = 1, \cdots, n, j \neq r \qquad (18) \end{aligned}$$

Furthermore, the inequalities (13) can be expressed by the following forms.

$$\sum_{j=1}^{n} \left(\sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i} - (1-\xi_i)\alpha_{ij} \right) x_j \leq z^0_i - \xi_i(z^0_i - z^1_i), i = 1, \cdots, k \qquad (19)$$

Then, MOP-V1 can be transformed into MOP-V2.

[MOP-V2]

$$\min_{\boldsymbol{x} \in X} (Z^V_1(\boldsymbol{x}), \cdots, Z^V_k(\boldsymbol{x})) \qquad (20)$$

subject to

$$\sum_{j=1}^{n} \left(\sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i} - (1-\xi_i)\alpha_{ij} \right) x_j \leq z^0_i - \xi_i(z^0_i - z^1_i), \quad i = 1, \cdots, k.$$

From the fact that $\sum_{j=1}^{n} \alpha_{ij} x_j - z^1_i + z^0_i > 0$, $\boldsymbol{x}^T \boldsymbol{V}_i \boldsymbol{x} > 0$, due to the positive-semidefinite property of V_i, MOP-V2 can be equivalently transformed to MOP-V3

[MOP-V3]

$$\min_{\boldsymbol{x} \in X} (Z^{SD}_1(\boldsymbol{x}), \cdots, Z^{SD}_k(\boldsymbol{x})) \qquad (21)$$

subject to

$$\sum_{j=1}^{n} \left(\sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i} - (1-\xi_i)\alpha_{ij} \right) x_j \leq z^0_i - \xi_i(z^0_i - z^1_i), \quad i = 1, \cdots, k,$$

where

$$Z^{SD}_i(\boldsymbol{x}) \stackrel{\text{def}}{=} \frac{\sqrt{\boldsymbol{x}^T \boldsymbol{V}_i \boldsymbol{x}}}{\sum_{j=1}^{n} \alpha_{ij} x_j - z^1_i + z^0_i}.$$

It should be noted here that $Z^E_i(\boldsymbol{x})$ and $Z^{SD}_i(\boldsymbol{x})$ are the statical values for the same random function $\Pi_{\tilde{\bar{\boldsymbol{c}}}_i \boldsymbol{x}}(\tilde{G}_i)$. When solving MOFRLP, it is natural for the decision maker to consider both $Z^E_i(\boldsymbol{x})$ and $Z^{SD}_i(\boldsymbol{x})$ for each objective function $\Pi_{\tilde{\bar{\boldsymbol{c}}}_i \boldsymbol{x}}(\tilde{G}_i)$ of MOSP simultaneously, rather than considering either of them. Moreover, it seems be difficult for the decision maker to express his/her preference for the standard deviations $Z^{SD}_i(\boldsymbol{x})$, $i = 1, \cdots, k$. From such a point of view, in the following sections, we propose the hybrid model for MOFRLP, in which E-model and V-model are incorporated simultaneously, and define an EV-Pareto optimality concept. In order to derive a satisfactory solution of the decision maker from among an EV-Pareto optimal solution set, the interactive algorithm is developed.

IV. EV-Model for MOFRLP

In this section, we consider the following hybrid model for MOFRLP, where both E-model and V-model are considered simultaneously.

[MOP-EV1]

$$\begin{aligned} \max_{\boldsymbol{x} \in X} \quad & \big(Z^E_1(\boldsymbol{x}), \cdots, Z^E_k(\boldsymbol{x}), \\ & -Z^{SD}_1(\boldsymbol{x}), \cdots, -Z^{SD}_k(\boldsymbol{x}) \big) \qquad (22) \end{aligned}$$

In MOP-EV1, $Z^E_i(\boldsymbol{x})$ and $Z^{SD}_i(\boldsymbol{x})$ means the expected value and the standard deviation of the objective function $\Pi_{\tilde{\bar{\boldsymbol{c}}}_i \boldsymbol{x}}(\tilde{G}_i)$ in MOSP. It should be noted here that $Z^E_i(\boldsymbol{x})$ can be interpreted as an expected value of the satisfactory degree for $\Pi_{\tilde{\bar{\boldsymbol{c}}}_i \boldsymbol{x}}(\tilde{G}_i)$, but $Z^{SD}_i(\boldsymbol{x})$ does not mean the satisfactory degree itself. Here, instead of $Z^{SD}_i(\boldsymbol{x})$, let us consider the coefficient of variation defined as follows.

$$\begin{aligned} Z^{CV}_i(\boldsymbol{x}) &\stackrel{\text{def}}{=} \frac{Z^{SD}_i(\boldsymbol{x})}{Z^E_i(\boldsymbol{x})} \\ &= \frac{\sqrt{\boldsymbol{x}^T \boldsymbol{V}_i \boldsymbol{x}}}{\sum_{j=1}^{n} (\alpha_{ij} - \sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i}) x_j + z^0_i} \qquad (23) \end{aligned}$$

By using the coefficient of variation $Z^{CV}_i(\boldsymbol{x})$, we can transform MOP-EV1 into MOP-EV2.

[MOP-EV2]

$$\begin{aligned} \max_{\boldsymbol{x} \in X} \quad & \big(Z^E_1(\boldsymbol{x}), \cdots, Z^E_k(\boldsymbol{x}), \\ & -Z^{CV}_1(\boldsymbol{x}), \cdots, -Z^{CV}_k(\boldsymbol{x}) \big) \qquad (24) \end{aligned}$$

In MOP-EV2, we assume that the decision maker has fuzzy goals for $Z^{CV}_i(\boldsymbol{x}), i = 1, \cdots, k$, and the corresponding membership functions are defined as $\mu^{CV}_i(Z^{CV}_i(\boldsymbol{x})), i = 1, \cdots, k$. In order to elicit the membership function $\mu^{CV}_i(Z^{CV}_i(\boldsymbol{x}))$ appropriately, we can compute a range of $\mu^{CV}_i(Z^{CV}_i(\boldsymbol{x}))$ as follows.

$$CV_{i\min} \stackrel{\text{def}}{=} \min_{\boldsymbol{x} \in X} \frac{\sqrt{\boldsymbol{x}^T \boldsymbol{V}_i \boldsymbol{x}}}{\sum_{j=1}^{n} (\alpha_{ij} - \sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i}) x_j + z^0_i} \qquad (25)$$

$$CV_{i\max} \stackrel{\text{def}}{=} \max_{\boldsymbol{x} \in X} \frac{\sqrt{\boldsymbol{x}^T \boldsymbol{V}_i \boldsymbol{x}}}{\sum_{j=1}^{n} (\alpha_{ij} - \sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i}) x_j + z^0_i} \qquad (26)$$

The problem (25) for $CV_{i\min}$ is easily solved by applying a Dinkelbach-type algorithm [4], or a hybrid method of the bisection method and convex programming technique. Unfortunately, the problem (26) for $CV_{i\max}$ becomes a non-convex optimization problem. On the interval $[CV_{i\min}, CV_{i\max}]$, the decision maker sets his/her membership function $\mu^{CV}_i(Z^{CV}_i(\boldsymbol{x}))$, which is strictly decreasing and continuous.

From the point of view that both $Z^E_i(\boldsymbol{x})$ and $\mu^{CV}_i(Z^{CV}_i(\boldsymbol{x}))$ means the satisfactory degree for $\Pi_{\tilde{\bar{\boldsymbol{c}}}_i \boldsymbol{x}}(\tilde{G}_i)$, we introduce the integrated membership function in which the both satisfactory levels $Z^E_i(\boldsymbol{x})$ and $\mu^{CV}_i(Z^{CV}_i(\boldsymbol{x}))$ are incorporated simultaneously through the fuzzy decision [5], [14].

$$\mu_{D_i}(\boldsymbol{x}) \stackrel{\text{def}}{=} \min\{Z^E_i(\boldsymbol{x}), \mu^{CV}_i(Z^{CV}_i(\boldsymbol{x}))\} \qquad (27)$$

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Then, MOP-EV2 can be transformed into the following multiobjective programming problem.

[MOP-EV3]

$$\max_{\boldsymbol{x}\in X} \quad (\mu_{D_1}(\boldsymbol{x}),\cdots,\mu_{D_k}(\boldsymbol{x})) \tag{28}$$

$\mu_{D_i}(\boldsymbol{x})$ can be interpreted as an overall satisfactory degree for the fuzzy goal $\tilde{G}_i$. For MOP-EV3, we introduce an EV-Pareto optimal solution concept defined as follows.

Definition 3: $\boldsymbol{x}^* \in X$ is an EV-Pareto optimal solution to MOP-EV3, if and only if there does not exist another $\boldsymbol{x} \in X$ such that $\mu_{D_i}(\boldsymbol{x}) \geq \mu_{D_i}(\boldsymbol{x}^*)$, $i=1,\cdots,k$ with strict inequality holding for at least one i.

In order to generate a candidate of a satisfactory solution from among an EV-Pareto optimal solution set, the decision maker is asked to specify the reference membership values [14]. For the reference membership values $\hat{\boldsymbol{\mu}} = (\hat{\mu}_1,\cdots,\hat{\mu}_k)$, the corresponding EV-Pareto optimal solution is obtained by solving the following minmax problem.

[MINMAX($\hat{\boldsymbol{\mu}}$)]

$$\min_{\boldsymbol{x}\in X,\lambda\in\Lambda} \lambda \tag{29}$$

subject to

$$\hat{\mu}_i - Z_i^E(\boldsymbol{x}) \leq \lambda, i=1,\cdots,k \tag{30}$$

$$\hat{\mu}_i - \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x})) \leq \lambda, i=1,\cdots,k \tag{31}$$

where

$$\Lambda \stackrel{\text{def}}{=} \left[\max_{i=1,\cdots,k}\hat{\mu}_i - 1, \max_{i=1,\cdots,k}\hat{\mu}_i\right] = [\lambda_{\min},\lambda_{\max}] \tag{32}$$

From the definition of $Z_i^E(\boldsymbol{x})$ and $\mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}))$, the constraints (30) and (31) can be equivalently transformed into the following forms respectively.

$$\begin{aligned}&\sum_{j=1}^{n}(\alpha_{ij} - \sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i})x_j + z_i^0 \\ &\geq \left(\sum_{j=1}^{n}\alpha_{ij}x_j - z_i^1 + z_i^0\right)\cdot(\hat{\mu}_i - \lambda), i=1,\cdots,k\end{aligned} \tag{33}$$

$$\begin{aligned}&\left(\sum_{j=1}^{n}\left(\alpha_{ij} - \sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i}\right)x_j + z_i^0\right) \\ &\cdot {\mu_i^{CV}}^{-1}(\hat{\mu}_i - \lambda) \geq \sqrt{\boldsymbol{x}^T\boldsymbol{V}_i\boldsymbol{x}}, i=1,\cdots,k\end{aligned} \tag{34}$$

The relationship between the optimal solution $(\boldsymbol{x}^*,\lambda^*)$ of MINMAX($\hat{\boldsymbol{\mu}}$) and EV-Pareto optimal solutions of MOP-EV3 can be characterized by the following theorems.

Theorem 1: If $\boldsymbol{x}^* \in X$, $\lambda^* \in \Lambda$ is an unique optimal solution of MINMAX($\hat{\boldsymbol{\mu}}$) then $\boldsymbol{x}^*$ is an EV-Pareto optimal solution of MOP-EV3.

(Proof)

Let us assume that $\boldsymbol{x}^* \in X$ is not an EV-Pareto optimal solution of MOP-EV3. Then, there exists $\boldsymbol{x} \in X$ such that $\mu_{D_i}(\boldsymbol{x}) \geq \mu_{D_i}(\boldsymbol{x}^*), i=1,\cdots,k,$ with strict inequality holding for at least one i. This implies that

$$\begin{aligned}& && \mu_{D_i}(\boldsymbol{x}) \geq \mu_{D_i}(\boldsymbol{x}^*) \\ &\Leftrightarrow && \hat{\mu}_i - \min\{Z_i^E(\boldsymbol{x}), \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}))\} \\ & &\leq\ & \hat{\mu}_i - \min\{Z_i^E(\boldsymbol{x}^*), \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}^*))\} \\ &\Leftrightarrow && \max\{\hat{\mu}_i - Z_i^E(\boldsymbol{x}), \hat{\mu}_i - \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}))\} \\ & &\leq\ & \max\{\hat{\mu}_i - Z_i^E(\boldsymbol{x}^*), \hat{\mu}_i - \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}^*))\} \\ & &\leq\ & \lambda^*, i=1,\cdots,k.\end{aligned}$$

This contradicts the assumption that $\boldsymbol{x}^* \in X$, $\lambda^* \in \Lambda$ is an unique optimal solution of MINMAX($\hat{\boldsymbol{\mu}}$). □

Theorem 2: If $\boldsymbol{x}^* \in X$ is an EV-Pareto optimal solution of MOP-EV3, then there exists a reference membership values $\hat{\boldsymbol{\mu}} = (\hat{\mu}_1,\cdots,\hat{\mu}_k)$ such that $\boldsymbol{x}^* \in X$, $\lambda^* = \hat{\mu}_i - \mu_{D_i}(\boldsymbol{x}^*), i=1,\cdots,k$ is an optimal solution of MINMAX($\hat{\boldsymbol{\mu}}$)

(Proof)

Let us assume that $\boldsymbol{x}^* \in X$, $\lambda^* = \hat{\mu}_i - \mu_{D_i}(\boldsymbol{x}^*) = \max\{\hat{\mu}_i - Z_i^E(\boldsymbol{x}^*), \hat{\mu}_i - \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}^*))\}$, $i=1,\cdots,k$, is not an optimal solution of MINMAX($\hat{\boldsymbol{\mu}}$). Then, there exists $\boldsymbol{x} \in X$ and $\lambda < \lambda^*$ such that

$$\begin{aligned}& && \begin{cases}\hat{\mu}_i - Z_i^E(\boldsymbol{x}) \leq \lambda < \lambda^* \\ \hat{\mu}_i - \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x})) \leq \lambda < \lambda^*\end{cases} \\ &\Leftrightarrow && \hat{\mu}_i - \mu_{D_i}(\boldsymbol{x}) \leq \lambda < \lambda^* \\ &\Leftrightarrow && \hat{\mu}_i - \mu_{D_i}(\boldsymbol{x}) < \hat{\mu}_i - \mu_{D_i}(\boldsymbol{x}^*) \\ &\Leftrightarrow && \mu_{D_i}(\boldsymbol{x}) > \mu_{D_i}(\boldsymbol{x}^*)\end{aligned}$$

for all $i=1,\cdots,k$. This contradicts the fact that $\boldsymbol{x}^* \in X$ is an EV-Pareto optimal solution of MOP-EV3. □

Unfortunately, since MINMAX($\hat{\boldsymbol{\mu}}$) is a nonlinear programming problem, it seems to be difficult to solve it directly. To overcome such difficulties, we consider the following function with respect to the constraints (33) and (34).

$$\begin{aligned}g_i(\boldsymbol{x},\lambda) &\stackrel{\text{def}}{=} \sqrt{\boldsymbol{x}^T\boldsymbol{V}_i\boldsymbol{x}} - {\mu_i^{CV}}^{-1}(\hat{\mu}_i - \lambda) \\ &\quad\cdot\left(\sum_{j=1}^{n}\left(\alpha_{ij} - \sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i}\right)x_j + z_i^0\right), \\ &\quad i=1,\cdots,k\end{aligned} \tag{35}$$

$$\begin{aligned}h_i(\boldsymbol{x},\lambda) &\stackrel{\text{def}}{=} \left(\sum_{j=1}^{n}\alpha_{ij}x_j - z_i^1 + z_i^0\right)\cdot(\hat{\mu}_i - \lambda) \\ &\quad -\sum_{j=1}^{n}(\alpha_{ij} - \sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i})x_j - z_i^0, \\ &\quad i=1,\cdots,k\end{aligned} \tag{36}$$

It should be noted here that $g_i(\boldsymbol{x},\lambda)$, $h_i(\boldsymbol{x},\lambda)$, $i=1,\cdots,k$ are convex with respect to $\boldsymbol{x} \in X$ for any fixed $\lambda \in \Lambda$. Let us define the following feasible set $X(\lambda)$ for some fixed $\lambda \in \Lambda$.

$$X(\lambda) \stackrel{\text{def}}{=} \{\boldsymbol{x} \in X \mid g_i(\boldsymbol{x},\lambda) \leq 0, h_i(\boldsymbol{x},\lambda) \leq 0, i=1,\cdots,k\} \tag{37}$$

Then, it is clear that $X(\lambda)$ is a convex set and satisfies the following property.

Property 1: If $\lambda_1,\lambda_2 \in \Lambda, \lambda_1 \leq \lambda_2$, then it holds that $X(\lambda_1) \subset X(\lambda_2)$.

In the following, it is assumed that $X(\lambda_{\min}) = \phi$, $X(\lambda_{\max}) \neq \phi$. From Property 1, we can obtain the optimal solution $(\boldsymbol{x}^*,\lambda^*)$ of MINMAX($\hat{\boldsymbol{\mu}}$) using the following simple algorithm which is based on the bisection method and the convex programming technique.

[Algorithm 1]

Step 1: Set $\lambda_0 \leftarrow \lambda_{\min}$, $\lambda_1 \leftarrow \lambda_{\max}$, $\lambda \leftarrow (\lambda_0 + \lambda_1)/2$.

Step 2: Solve the convex programming problem for the fixed $\lambda \in \Lambda$,

$$\min_{\boldsymbol{x} \in X} h_j(\boldsymbol{x}, \lambda)$$

subject to

$$\begin{aligned} g_i(\boldsymbol{x}, \lambda) &\leq 0, i = 1, \cdots, k, \\ h_i(\boldsymbol{x}, \lambda) &\leq 0, i = 1, \cdots, k, \end{aligned}$$

where the index j is one of $\{1, 2, \cdots, k\}$, and denote the optimal solution as $\boldsymbol{x}(\lambda)$.

Step 3: If $|\lambda_0 - \lambda_1| < \delta$, go to Step 4, where δ is a sufficiently small positive number. If $g_i(\boldsymbol{x}(\lambda), \lambda) \leq 0$ and $h_i(\boldsymbol{x}(\lambda), \lambda) \leq 0$, for any $i = 1, \cdots, k$, set $\lambda_1 \leftarrow \lambda$, $\lambda \leftarrow (\lambda_0 + \lambda_1)/2$. Otherwise, set $\lambda_0 \leftarrow \lambda$, $\lambda \leftarrow (\lambda_0 + \lambda_1)/2$. And return to Step 2.

Step 4: Adopt $\boldsymbol{x}^* \leftarrow \boldsymbol{x}(\lambda)$, $\lambda^* \leftarrow \lambda$ as an optimal solution of MINMAX($\hat{\boldsymbol{\mu}}$).

V. An interactive algorithm

In Theorem 1, if the optimal solution $(\boldsymbol{x}^*, \lambda^*)$ of MINMAX($\hat{\boldsymbol{\mu}}$) is not unique, the EV-Pareto optimality can not be guaranteed. In order to guarantee the EV-Pareto optimality for $(\boldsymbol{x}^*, \lambda^*)$, we formulate the EV-Pareto optimality test problem. Before formulating such a test problem, without loss of generality, we assume that the following inequalities hold at the optimal solution $\boldsymbol{x}^* \in X, \lambda^* \in \Lambda$.

$$Z_i^E(\boldsymbol{x}^*) \leq \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}^*)), i \in I_1 \quad (38)$$

$$Z_i^E(\boldsymbol{x}^*) > \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}^*)), i \in I_2 \quad (39)$$

$$I_1 \cup I_2 = \{1, \cdots, k\}, I_1 \cap I_2 \neq \phi \quad (40)$$

Under the above conditions, we formulate the following EV-Pareto optimality test problem.

[EV-Pareto optimality test problem]

$$\max_{\boldsymbol{x} \in X, \epsilon_i \geq 0, i=1,\cdots,k} \sum_{i=1}^{k} \epsilon_i$$

subject to

$$\begin{aligned} & Z_i^E(\boldsymbol{x}) \geq Z_i^E(\boldsymbol{x}^*) + \epsilon_i, i \in I_1 \\ & \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x})) \geq Z_i^E(\boldsymbol{x}^*) + \epsilon_i, i \in I_1 \\ & Z_i^E(\boldsymbol{x}) \geq \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}^*)) + \epsilon_i, i \in I_2 \\ & \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x})) \geq \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}^*)) + \epsilon_i, \in I_2 \end{aligned}$$

The following theorem shows the relationships between the optimal solution of EV-Pareto optimality test problem and the EV-Pareto optimal solution for MOP-EV3.

Theorem 3: Let $\check{\boldsymbol{x}} \in X$, $\check{\epsilon}_i \geq 0$, $i = 1, \cdots, k$ be an optimal solution of the EV-Pareto optimality test problem for $(\boldsymbol{x}^*, \lambda^*)$. If $\sum_{i=1}^{k} \check{\epsilon}_i = 0$, then $\boldsymbol{x}^* \in X$ is an EV-Pareto optimal solution.

(Proof)

Assume that $\check{\epsilon}_i = 0$, $i = 1, \cdots, k$. If $\boldsymbol{x}^* \in X$ is not an EV-Pareto optimal solution, there exists some $\boldsymbol{x} \in X$ such that $\mu_{D_i}(\boldsymbol{x}) \geq \mu_{D_i}(\boldsymbol{x}^*)$,$i = 1, \cdots, k$, with strict inequality holding for at least one i. From the inequalities (38) and (39), this is equivalent to the following relations.

$$\begin{aligned} & \min\{Z_i^E(\boldsymbol{x}), \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}))\} \\ \geq & \min\{Z_i^E(\boldsymbol{x}^*), \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}^*))\} \\ = & \begin{cases} Z_i^E(\boldsymbol{x}^*), & i \in I_1 \\ \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}^*)), & i \in I_2 \end{cases} \end{aligned}$$

As a result, the following inequalities holds.

$$\begin{cases} Z_i^E(\boldsymbol{x}) \geq Z_i^E(\boldsymbol{x}^*), & i \in I_1 \\ \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x})) \geq Z_i^E(\boldsymbol{x}^*), & i \in I_1 \\ Z_i^E(\boldsymbol{x}) \geq \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}^*)), & i \in I_2 \\ \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x})) \geq \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}^*)), & i \in I_2 \end{cases} \quad (41)$$

with strict inequality holding for at least one $i \in I_1 \cup I_2$. Hence, there must exist at least one i such that $\check{\epsilon}_i > 0$. This contradicts the assumption that $\check{\epsilon}_i = 0$, $i = 1, \cdots, k$. □

Now, following the above discussions, we can construct the interactive algorithm in order to derive a satisfactory solution from among an EV-Pareto optimal solution set.

[An interactive algorithm]

Step 1: The decision maker sets the membership function $\mu_{\tilde{G}_i}(y)$, $i = 1, \cdots, k$ for the fuzzy goals of the objective functions in MOFRLP.

Step 2: Considering the interval $CV_{i\min}, CV_{i\max}$, the decision maker sets the membership function $\mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}))$, $i = 1, \cdots, k$.

Step 3: Set the initial reference membership values as $\hat{\mu}_i = 1$, $i = 1, \cdots, k$

Step 4: Solve MINMAX($\hat{\boldsymbol{\mu}}$) by applying Algorithm 1, and obtain the optimal solution $\boldsymbol{x}^* \in X, \lambda^* \in \Lambda$. In order to guarantee EV-Pareto optimality, solve the EV-Pareto optimality test problem for $\boldsymbol{x}^* \in X$.

Step 5: If the decision maker is satisfied with the current value of the EV-Pareto optimal solution $\boldsymbol{x}^* \in X$, then stop. Otherwise, the decision maker updates his/her reference membership values $\hat{\mu}_i$, $i = 1, \cdots, k$ and return to Step 4.

VI. Numerical example

In order to demonstrate the proposed method and the interactive processes, we consider the following three-objective

TABLE I
PARAMETERS OF OBJECTIVE FUNCTIONS

	$\ell_1 = 1$	$\ell_1 = 2$	$\ell_1 = 3$	α_{1j}, β_{1j}
$d_{11\ell_1}$	-2.5	-2.0	-1.5	0.4
$d_{12\ell_1}$	-3.5	-3.0	-2.5	0.5
$d_{13\ell_1}$	-2.25	-2.0	-1.75	0.4
p_1	0.25	0.4	0.35	
	$\ell_2 = 1$	$\ell_2 = 2$	$\ell_2 = 3$	α_{2j}, β_{2j}
$d_{21\ell_2}$	-2.5	-2.0	-1.5	0.3
$d_{22\ell_2}$	-0.75	-0.5	-0.25	0.4
$d_{23\ell_2}$	-2.5	-2.25	-2.0	0.3
p_2	0.3	0.5	0.2	
	$\ell_3 = 1$	$\ell_3 = 2$	$\ell_3 = 3$	α_{3j}, β_{3j}
$d_{31\ell_3}$	3.0	3.25	3.5	0.4
$d_{32\ell_3}$	2.5	2.75	3.0	0.5
$d_{33\ell_3}$	4.5	4.75	5.0	0.4
p_3	0.2	0.45	0.35	

TABLE II
PARAMETERS OF MEMBERSHIP FUNCTIONS

	z_i^0	z_i^1	q_i^0	q_i^1
$i=1$	-7.5	-23.8181	0.45	0.25
$i=2$	-0.9375	-16.25	0.25	0.08
$i=3$	33.5	9.375	0.3	0.03

linear programming problem with fuzzy random variable coefficients.

[MOFRLP]

$$\min_{\boldsymbol{x}\in X} z_1(\boldsymbol{x}) = \tilde{\bar{c}}_{11}x_1 + \tilde{\bar{c}}_{12}x_2 + \tilde{\bar{c}}_{13}x_3$$
$$\min_{\boldsymbol{x}\in X} z_2(\boldsymbol{x}) = \tilde{\bar{c}}_{21}x_1 + \tilde{\bar{c}}_{22}x_2 + \tilde{\bar{c}}_{23}x_3$$
$$\min_{\boldsymbol{x}\in X} z_3(\boldsymbol{x}) = \tilde{\bar{c}}_{31}x_1 + \tilde{\bar{c}}_{32}x_2 + \tilde{\bar{c}}_{33}x_3$$

where $X = \{(x_1, x_2, x_3) \in \mathrm{R}^3_+ \mid 3x_1+2x_2+x_3 \leq 18, 2x_1+x_2+2x_3 \leq 13, 3x_1+4x_2+3x_3 \geq 15, x_1+3x_2+2x_3 \leq 17\}$ and it is assumed that a realization $\tilde{c}_{ij\ell_i}$ of a fuzzy random variable $\tilde{\bar{c}}_{ij}$ is an triangular-type fuzzy number whose membership function is defined as (2) where the parameters $d_{ij\ell_i}$, α_{ij}, β_{ij} are given in Table I. According to (17) and (18), the variance-covariance matrices $V_i, i = 1, 2, 3$ are computed as follows.

$$V_1 = \begin{pmatrix} 0.1475 & 0.1475 & 0.07375 \\ 0.1475 & 0.1475 & 0.07375 \\ 0.07375 & 0.07375 & 0.036875 \end{pmatrix}$$

$$V_2 = \begin{pmatrix} 0.1225 & 0.06125 & 0.06125 \\ 0.06125 & 0.030625 & 0.030625 \\ 0.06125 & 0.030625 & 0.030625 \end{pmatrix}$$

$$V_3 = \begin{pmatrix} 0.032969 & 0.032969 & 0.032969 \\ 0.032969 & 0.032969 & 0.032969 \\ 0.032969 & 0.032969 & 0.032969 \end{pmatrix}$$

Let us assume that the hypothetical decision maker sets the membership functions $\mu_{\tilde{G}_i}(\cdot)$, $\mu_i^{CV}(\cdot)$, $i = 1, 2, 3$ as follows.

$$\begin{aligned} \mu_{\tilde{G}_i}(y) &= \frac{y - z_i^0}{z_i^1 - z_i^0}, \quad z_i^1 \leq y \leq z_i^0, i = 1, 2, 3 \\ \mu_i^{CV}(s) &= \frac{s - q_i^0}{q_i^1 - q_i^0}, \quad q_i^1 \leq s \leq q_i^0, i = 1, 2, 3 \end{aligned}$$

where the parameters z_i^0, z_i^1, q_i^0, q_i^1 are given in Table II. The interactive processes under the hypothetical decision maker are summarized in Table III.

TABLE III
INTERACTIVE PROCESSES

Iteration	1	2	3
$\hat{\mu}_1$	1	0.85	0.85
$\hat{\mu}_2$	1	1	1
$\hat{\mu}_3$	1	0.85	0.75
x_1^*	0.5683	1.0762	0.8326
x_2^*	3.1269	2.1054	2.1693
x_3^*	2.3510	2.8859	3.3730
$Z_1^E(\boldsymbol{x}^*)$	0.5358	0.4825	0.5214
$Z_2^E(\boldsymbol{x}^*)$	0.5358	0.6325	0.6714
$Z_3^E(\boldsymbol{x}^*)$	0.5358	0.4825	0.4214
$\mu_1^{CV}(Z_1^{CV}(\boldsymbol{x}^*))$	0.5358	0.5255	0.6209
$\mu_2^{CV}(Z_2^{CV}(\boldsymbol{x}^*))$	0.6270	0.6325	0.6714
$\mu_3^{CV}(Z_3^{CV}(\boldsymbol{x}^*))$	0.8009	0.7630	0.6732

VII. CONCLUSION

In this paper, under the assumption that the decision maker intends to not only maximize the expected degrees of possibilities that the original objective functions attain the corresponding fuzzy goals, but also minimize coefficients of variation for such possibilities, an interactive decision making method for MOFRLP is proposed. In the proposed method, a satisfactory solution is obtained from among an EV-Pareto optimal solution set through the interaction with the decision maker.

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