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Characteristic of Spectral Exponent of Epileptic ECoG Data Corresponding to Levels in Wavelet-Based Fractal Analysis

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***Abstract*—In this study, the wavelet-based fractal analysis is applied to analyze epileptic ECoG data obtained during non-seizure period and epileptic seizure events. The spectral exponents of the epileptic ECoG data obtained using the wavelet-based fractal analysis from various intervals of levels are examined. The computational results show that the estimated spectral exponents of the epileptic ECoG epochs vary according to the levels m used in the estimation of slope of $\log_2 \mathrm{var}(d_{m,n})$-$m$ graphs. Also, it is shown that the spectral exponents of epileptic ECoG data obtained during epileptic seizure events are different from those of epileptic ECoG data obtained during non-seizure period. The most difference between the spectral exponents of epileptic ECoG data obtained during non-seizure period and epileptic seizure events is observed in the 125.0–15.625 frequency band.**

***Index Terms*—wavelet analysis, fractals, epilepsy, seizure, electrocorticogram,**

I. Introduction

Epilepsy is a common brain disorder in which clusters of neurons signal abnormally [1]. More than 50 million individuals worldwide, about 1% of the world's population are affected by epilepsy [2]. In epilepsy, the normal pattern of neuronal activity is disturbed [1]. Epileptic seizures are manifestations of epilepsy [3]. The electroencephalogram (EEG) is a signal that quantifies the electrical activity of the brain, usually from scalp recordings and is commonly used to assess and detect brain abnormalities, and is crucial for the diagnosis of epilepsy [1]. Electrocorticography (ECoG) is an invasive approach to record the electrical activity of the brain that is conventionally used for the epilepsy treatment.

Concepts and computational methods derived from the contemporary study of complex systems including chaos theory, nonlinear dynamics and fractals have gained increasing interest for applications in biology and medicine because physiological signals and systems can exhibit an extraordinary range of patterns and behaviors [4]. The mathematical concept of a fractal is commonly associated with irregular objects that exhibit a property called scale-invariance or self-similarity [4], [5]. $1/f$ processes are an important class of statistical self-similar random processes [6].

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In [7], a wavelet-based representation for $1/f$ processes was developed where the spectral exponent (γ) is estimated from the slope of the $\log$-var of the wavelet coefficients versus the level, specifies the distribution of power from low to high frequencies. In the previous studies [8], [9], the wavelet-based approach, referred to as the wavelet-based fractal analysis, was applied to analyze epileptic ECoG/EEG data and it was found that the ECoG/EEG data corresponding to various physiological and pathological states of the brain exhibit different scale-invariant characteristics. Furthermore, in [13], [14], the computational results obtained using the wavelet-based fractal analysis were comparable to the results obtained using the correlation dimension [10] and Hurst exponent [11], [12].

In this study, the wavelet-based fractal analysis is applied to analyze two data sets of epochs of ECoG data recorded from an epilepsy patient. The first data set contains the epileptic ECoG epochs obtained during non-seizure period while another data set contains the epileptic ECoG epochs obtained during epileptic seizure events. The spectral exponents of epileptic ECoG epochs estimated using various intervals of levels are investigated. The most distinguishing feature between the spectral exponents of epileptic ECoG epochs of those two data sets can be identified.

II. Methods

A. Wavelet-Based Fractal Analysis

Models of $1/f$ processes are generally represented using a frequency-domain characterization. The dynamics of $1/f$ processes exhibit power-law behaviors [16] and can be characterized in the form of [7]

$$S_x(\omega) \sim \frac{\sigma_x^2}{|\omega|^\gamma} \tag{1}$$

over several decades of the frequency ω, where $S_x(\omega)$ is the Fourier transform of the signal $x(t)$ and γ denotes the spectral exponent.

In [7], [17], it was proved that a random process $x(t)$ constructed by the wavelet basis expansions

$$x(t) = \sum_m \sum_n d_{m,n} \psi_{m,n}(t) \tag{2}$$

where $\psi_{m,n}(t)$ is an orthonormal wavelet basis and $d_{m,n}$ are the wavelet coefficients has a time-averaged spectrum

$$S_x(\omega) = \sigma^2 \sum_m 2^{-\gamma m} \left| \Psi\left(2^{-m}\omega\right)\right|^2 \tag{3}$$

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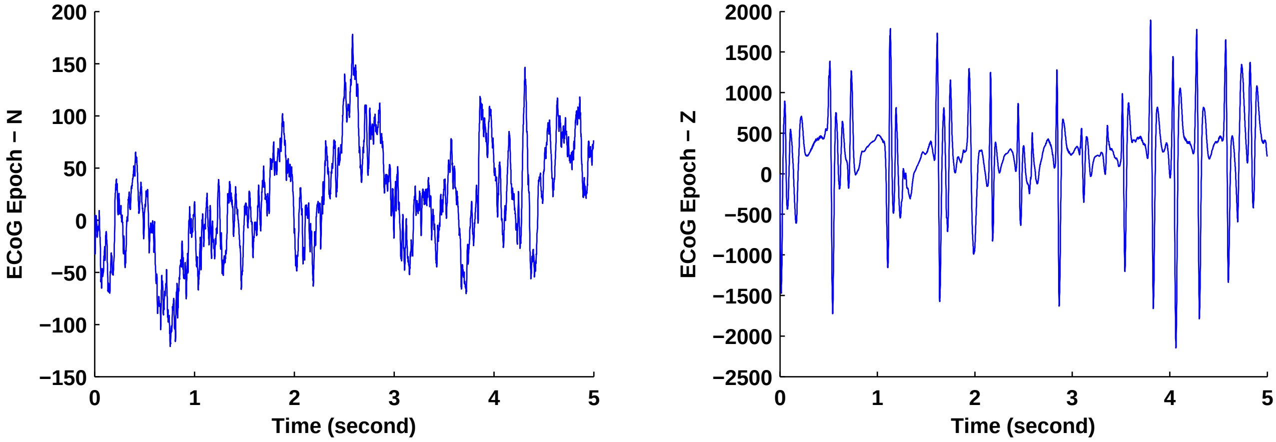

(a) Data set N (b) Data set Z

Fig. 1. The examplary epileptic ECoG epochs of data sets N and Z.

that is nearly-$1/f$, i.e.,

$$\frac{\sigma_L^2}{|\omega|^\gamma} \leq X(\omega) \leq \frac{\sigma_U^2}{|\omega|^\gamma} \tag{4}$$

for some $0 < \sigma_L^2 \leq \sigma_U^2 < \infty$. Variances of the wavelet coefficients $d_{m,n}$ that are a collection of mutually uncorrelated, zero-mean random variables are

$$\text{var}(d_{m,n}) = \sigma^2 2^{-\gamma m}. \tag{5}$$

The spectral exponent γ of a $1/f$ process can therefore be determined from the linear relationship between $\log_2\ \text{var}(d_{m,n})$ and levels m, i.e.,

$$\gamma = \frac{\Delta \log_2\ \text{var}(d_{m,n})}{\Delta m}. \tag{6}$$

The steps for computing the spectral exponent γ of the time series x using the wavelet-based fractal analysis are as follows:

1) Decompose the time series x into M levels using the wavelet-basis expansions to obtain the wavelet coefficients $d_{m,n}$ where levels $m = 1, 2, \ldots, M$.
2) Compute the variance of wavelet coefficients $d_{m,n}$ corresponding to each level m, $\text{var}(d_{m,n})$.
3) Take the logarithm to base 2 of the corresponding variances of wavelet coefficients, $\log_2 \text{var}(d_{m,n})$.
4) Compute the spectral exponent γ by estimating the slope of a $\log_2 \text{var}(d_{m,n})$-$m$ graph between the specified levels m.

B. Data and Analysis

ECoG data analyzed in this study are long-term ECoG recordings of an epilepsy patient at University Hospitals of Cleveland, Case Medical Center in Cleveland, Ohio, USA before surgery. With the consent of the patient, the ECoG data were recorded for few days using a Nihon-Kohden EEG system with a sampling rate of 1,000 Hz. The epileptic ECoG data were partitioned into 5-second epochs. Furthermore, the epochs of epileptic ECoG data were divided into two data sets, referred to as sets N and Z.

The data set N contains 200 epochs of ECoG data obtained during non-seizure period (interictal state) while the data set Z contains 50 epochs of ECoG data obtained during epileptic seizure event (ictal state). There were none of overlapping segments of epochs in both sets N and Z. The epochs of set Z were obtained from four epileptic seizure events. The exemplary epileptic ECoG epochs of data sets N and Z are depicted in Figs. 1(a)–(b), respectively.

C. Analytic Framework

The epochs of epileptic ECoG data are decomposed into 6 levels using the 10th order of Daubechies wavelet (Db10). The spectral subbands corresponding to the levels $m = 1, 2, \ldots, 7$ range approximately between 250.0–500.0, 125.0–250.0, 62.5–125.0, 31.25–62.5, 15.625–31.25, 7.8125–15.625, and 3.9062–7.8125, respectively. The spectral exponents γ of epileptic ECoG epochs are estimated using a linear least-squares regression technique from various intervals of levels m. Six intervals of levels m examined in the spectral exponent estimation include $m = 1, 2, \ldots, 7$, $m = 1, 2, 3$, $m = 2, 3, 4$, $m = 3, 4, 5$, $m = 4, 5, 6$, and $m = 5, 6, 7$. The Mahalanobis distances [15] between the spectral exponents γ of the epileptic ECoG epochs of data sets N and Z are determined.

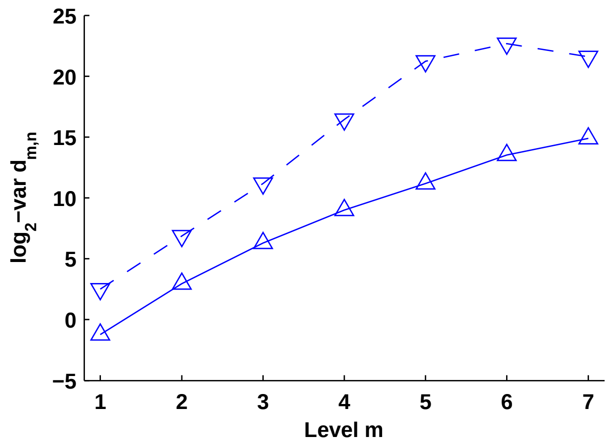

Fig. 2. The log-var of the wavelet coefficients of the exemplary ECoG epochs of data sets N (plotted in '$\triangle$') and Z (plotted in '$\bigtriangledown$').

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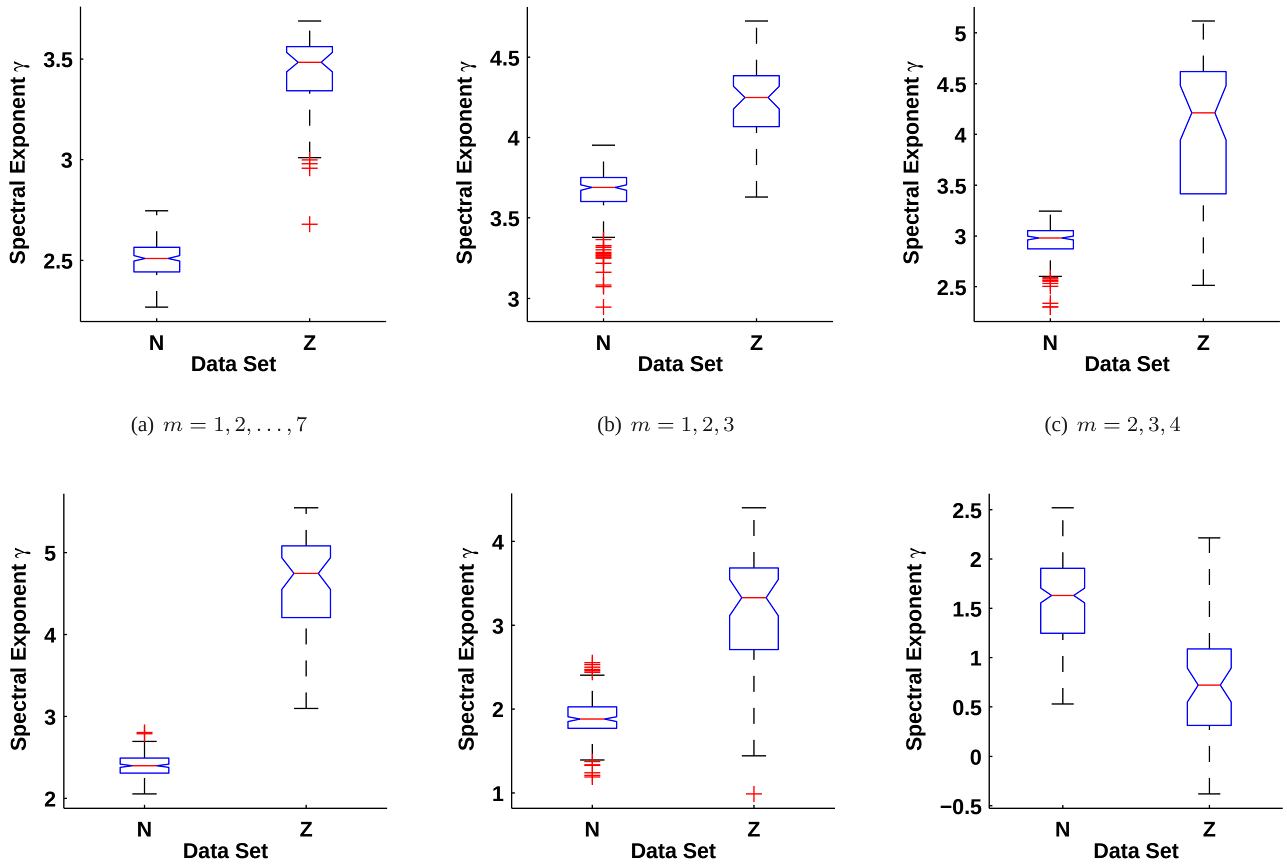

(a) $m = 1, 2, \ldots, 7$ (b) $m = 1, 2, 3$ (c) $m = 2, 3, 4$

(d) $m = 3, 4, 5$ (e) $m = 4, 5, 6$ (f) $m = 5, 6, 7$

Fig. 3. Comparison of the spectral exponents of the epileptic ECoG epochs of data sets N and Z estimated from various intervals of levels m.

III. Results

The log-var of the wavelet coefficients of the exemplary ECoG epochs of data sets N and Z shown in Fig. 1 are shown in Fig. 2. It is observed that the slopes of $\log_2 \mathrm{var}(d_{m,n})$-$m$ graphs of the exemplary ECoG epochs of data sets N and Z are different. The spectral exponents of the exemplary ECoG epochs of data sets N and Z estimated from the levels $m = 1, 2, \ldots, 7$, $m = 1, 2, 3$, $m = 2, 3, 4$, $m = 3, 4, 5$, $m = 4, 5, 6$, and $m = 5, 6, 7$ are 2.6555 and 3.5341, 3.7521 and 4.3378, 3.0208 and 4.7803, 2.4501 and 5.0253, 2.2589 and 3.1185, and 1.8535 and 0.1884, respectively.

The spectral exponents of the epileptic ECoG epochs of data sets N and Z estimated from the levels $m = 1, 2, \ldots, 7$, $m = 1, 2, 3$, $m = 2, 3, 4$, $m = 3, 4, 5$, $m = 4, 5, 6$, and $m = 5, 6, 7$ are compared in Figs. 3(a)–(f). The means and the standard deviations of Mahalanobis distance of the spectral exponents of the epileptic ECoG epochs of data set Z from the spectral exponents of the epileptic ECoG epochs of data set N are summarized in Table I. The interval of levels $m = 3, 4, 5$ provides the farthest distance between the spectral exponents of the epileptic ECoG epochs of data sets N and Z.

TABLE I

Statistical values of the Mahalanobis distances between the spectral exponents of the epileptic ECoG epochs of data sets N and Z estimated from various intervals of levels m.

Levels m	Mean	S.D.
$m = 1, 2, \ldots 7$	94.8486	34.8861
$m = 1, 2, 3$	13.0932	9.4164
$m = 2, 3, 4$	59.7991	52.6181
$m = 3, 4, 5$	295.9030	133.5067
$m = 4, 5, 6$	34.7678	25.6424
$m = 5, 6, 7$	5.3628	4.6753

IV. Conclusions

From the computational results, the slope of $\log_2 \mathrm{var}(d_{m,n})$-$m$ graphs of the epileptic ECoG epochs varies according to the levels m. Therefore, the estimated spectral exponents of the epileptic ECoG epochs depend on the levels m used in the wavelet-based fractal analysis. At the levels $m = 1, 2, \ldots, 7$, $m = 1, 2, 3$, $m = 2, 3, 4$, $m = 3, 4, 5$, and $m = 4, 5, 6$, the spectral exponents of the epileptic ECoG epochs obtained during non-seizure period and epileptic seizure events are relatively similar. The spectral exponent of the epileptic ECoG epochs obtained during epileptic seizure event tends to be higher than that of the epileptic ECoG epochs obtained during non-seizure period. On the other hand, at the levels $m = 5, 6, 7$ the spectral exponent of the epileptic ECoG epochs obtained during epileptic seizure event tends to be lower than that of the epileptic ECoG epochs obtained during non-seizure period.

In addition, the farthest distance between the spectral exponents of the epileptic ECoG epochs obtained during non-seizure period and epileptic seizure events is obtained achieved at the levels $m = 3, 4, 5$. This thus suggests that the components of epileptic ECoG epochs obtained during epileptic seizure events are most different from those of epileptic ECoG epochs obtained during non-seizure period at the 125.0–15.625 frequency band. This intriguing feature of epileptic ECoG epochs is potentially useful for epileptic seizure detection.

REFERENCES

[1] Seizure and Epilepsy: Hope through Research National Institute of Neurological Disorders and Stroke (NINDS), Bethesda, MD, 2004 [Online]. Available: http://www.ninds.nih.gov/disorders/epilepsy/detail_epilepsy.htm
[2] B. Litt and J. Echauz, "Prediction of epileptic seizures," *Lancet Neurology*, vol. 1, pp. 22–30, 2002.
[3] A. Subasi, "Epileptic seizure detection using dynamic wavelet network," *Expert Systems with Applications*, vol. 29, pp. 343–355, 2005.
[4] A. L. Goldberger, "Complex systems," *Proc. Am. Thorac. Soc.*, vol. 3, pp. 467–472, 2006.
[5] B. B. Mandelbrot, *The Fractal Geometry of Nature*, San Francisco: WH Freeman, 1982.
[6] G. W. Wornell, *Signal Processing with Fractals: A Wavelet-Based Approach*, New Jersey: Prentice Hall, 1995.
[7] G. W. Wornell, "Wavelet-based representations for the $1/f$ family of fractal processes," *Proceedings of the IEEE*, vol. 81, pp. 1428–1450, 1993.
[8] S. Janjarasjitt and K. A. Loparo, "Wavelet-based fractal analysis of the epileptic EEG signal," in *Proc. ISPACS*, 2009, pp. 127–130.
[9] S. Janjarasjitt and K. A. Loparo, "Wavelet-based fractal analysis of multi-channel epileptic ECoG," in *Proc. TENCON*, 2010, pp. 373–378.
[10] P. Grassberger and I. Procaccia, "Characterization of strange attractors," *Phys. Rev. Lett.*, vol. 50, pp. 346–349, 1983.
[11] H. E. Hurst, "Long-term storage capacity of reservoirs," *T. Am. Soc. Civ. Eng.*, vol. 116, pp. 770–808, 1951.
[12] C.-K. Peng, S. V. Buldyrev, S. Havlin, M. Simons, H. E. Stanley and A. L. Goldberger, "Mosaic organization of DNA nucleotides," *Phys. Rev. E*, vol. 49, pp. 1685–1689, 1994.
[13] S. Janjarasjitt and K. A. Loparo, "Comparison of complexity measures using two complex system analysis methods applied to the epileptic ECoG," *J. Korean Phys. Soc.*, vol. 63, pp. 1659–1665, 2013.
[14] S. Janjarasjitt and K. A. Loparo, "Scale-invariant behavior of epileptic ECoG," *J. Med. Biol. Eng.*, doi: 10.5405/jmbe.1433.
[15] P. C. Mahalanobis, "On the generalized distance in statistics," *Proc. Natl. Inst. Sci. India*, vol. 2, pp. 49–55, 1936.
[16] P. A. Watters, "Fractal structure in the electroencephalogram," *Complexity International*, vol. 5, 1998.
[17] G. W. Wornell, "Synthesis, analysis, and processing of fractal signals," Ph.D. Thesis, Massachusetts Institute of Technology, Massachusetts, U.S.A., 1991.

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