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Astaticism in the Motion Control Systems of Marine Vessels

Maria A. Smirnova, Mikhail N. Smirnov, and Tatiana E. Smirnova

Abstract—**The problem of construction of systems for automatic motion control of marine vessels is one of the main problems considered in the scientific publications of both theoretical and practical orientation. One of the most important requirements for such systems is astaticism on regulated coordinates. The known analytical and numerical methods of construction of automatic motion control systems generally aim to improve local dynamic characteristics of control processes, while the practical application of these systems requires a multi-purpose orientation of used approaches. The aim of the paper is the development of methods to provide the astaticism using the multi-purpose structure control laws.**

Index Terms—**astaticism, control, stabilization.**

I. Introduction

Nowadays the problems of construction of automatic motion control systems for marine vessels received considerable attention in the scientific publications of both theoretical and practical orientation. On the one hand, this is due to the permanent extension of a range of requirements to such systems. On the other hand, it is determined by the continuously increasing capabilities of onboard digital devices, implementing the control laws.

The known analytical and numerical methods of construction of automatic motion control systems generally aim to improve local dynamic characteristics of control processes, while the practical application of these systems requires a multi-purpose orientation of used approaches.

Works [1–5, 12–17] present the theory of multi-purpose synthesis of motion control systems, taking into account the complex set of conditions, requirements and restrictions which certainly should be performed in all operation modes of the vessel. One of the most important requirements for a control system is the property of astaticism on regulated coordinates, i.e. the ability of the system to provide zero static error under the constant external disturbances. Some methods to provide the astaticism are described in [6–11].

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M. A. Smirnova is with the Faculty of Applied Mathematics and Control Processes, Saint-Petersburg State University, Saint-Petersburg, Russia (e-mail: smirnova-ma@bk.ru).

M. N. Smirnov is with Faculty of Applied Mathematics and Control Processes, Saint-Petersburg State University, Saint-Petersburg, Russia (e-mail: smirnov-mn@mail.ru).

T. E. Smirnova is with the Faculty of Applied Mathematics and Control Processes, Saint-Petersburg State University, Saint-Petersburg, Russia (e-mail: t.smirnova@spbu.ru).

Particularly, in the paper much prominence is given to the complex of questions, associated with computer synthesis and modeling of control laws, that provide the astaticism of the closed-loop system on yaw. An example of modeling control system for the marine vessel is performed.

II. Problem Statement

Consider mathematical model of the dynamics of marine vessel represented by the following system of differential equations [1, 2]

$$\begin{aligned} \dot{x} &= F_x(x,\delta) + Bd(t), \\ \dot{\delta} &= F_\delta(\delta,u), \\ y &= Cx, \end{aligned} \tag{1}$$

where functions F_x and F_δ define nonlinearities of the vessel and drive accordingly, x is a state vector, δ is a control vector, y is an output vector, u is a vector of control signals, A, B, H, C are constant matrices with corresponding dimensions, $d(t)$ is a vector of external disturbances like wind, stream and waves. We assume that at a constant speed matrices B and C have constant components.

Along with the system (1), which presents the controlled object, consider the feed-back equation (regulator)

$$\begin{aligned} \dot{z} &= F_z(z,\delta,y), \\ u &= F_u(z,\delta,y) \end{aligned} \tag{2}$$

with state vector z.

The closed-loop system (1), (2) is output astatic, if under input signal $d(t) \equiv d_0 \cdot \mathbf{1}(t)$ with stepwise components for any real vector d_0 from specified set M_d the system has zero equilibrium position, i.e. the following condition is fulfilled

$$\lim_{t\to\infty} y(t) = 0.$$

The main problem is to design feedback (2) in such a way that the specified equilibrium position will be asymptotically stable, and a closed-loop system (1), (2) – output astatic.

To provide the astaticism of the system it is proposed is to use special velocity controller with the structure

$$u = \mu\dot{x} + \nu y \tag{3}$$

where μ, ν are constant matrices with corresponding dimensions.

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III. Astaticism in Tracking Control Systems

For illustration of the idea of implementation of the tracking control consider the mathematical model of linear stationary object as an example

$$\begin{aligned} &\dot{\xi} = A\xi + Bu, \quad \xi(0) = \xi_0, \\ &y = C\xi + Du, \end{aligned} \tag{4}$$

where $\xi \in E^{\nu}$ is a state vector, $u \in E^{\mu}$ is a control vector, $y \in E^{\kappa}$ is a vector of regulated coordinates, *A, B, C, B* are constant matrices with corresponding dimensions.

Equations (4) determine the linear stationary operator

$$\Re_p(\xi_0): U \to Y, \; y = \Re_p(\xi_0)u, \tag{5}$$

that at given initial conditions $\xi(0) = \xi_0$ establishes a one-to-one correspondence between each control *u* from the set *U* and each output *y* from the set *Y*. Further, we will assume that the corresponding inverse operator $\Re_p^{-1}(\xi_0)$ is defined.

Let the stabilizing feedback with LTI (Linear Time Invariant) mathematical model is given

$$\begin{aligned} &\dot{\zeta} = A_c\zeta + B_c y, \\ &u = C_c\zeta + D_c y, \end{aligned} \tag{6}$$

where $\zeta \in E^{\nu 1}$ is a state vector of controller, A_c, B_c, C_c, D_c, are constant matrices with corresponding dimensions. Note that the initial conditions for vector ζ are assumed to be zero.

As for the controlled object, the linear stationary feedback operator $\Re_c : Y \to U$ corresponds to the model (6)

$$u = \Re_c y \tag{7}$$

that establishes a one-to-one correspondence between each output *y* from the set *Y* and each control *u* from the set *U*.

At the closure of the controlled object (4) with the feedback (6) in accordance with the relations (5) and (7) we have

$$y = \Re_p(\xi_0)\Re_c y \tag{8}$$

i.e. equation, the solution of which leads to a linear stationary operator $\Re_3(\xi_0)$ of the closed-loop homogeneous system

$$\Re_3(\xi_0)y = 0 \tag{9}$$

Since the feedback is stabilizing, zero equilibrium position of system (9) is asymptotically stable by Lyapunov, i.e. the condition (10) is fulfilled

$$y(t) \to 0 \text{ at } t \to \infty \text{ for any } \xi_0 \in E^{\nu}. \tag{10}$$

Now, instead of feedback (7) we form the control action as the following sum

$$u = \Re_p^{-1}(\xi_0)y_d + \Re_c(y - y_d) \tag{11}$$

where the first term can be interpreted as command signal $u^*(t) = \Re_p^{-1}(\xi_0)y_d(t)$, fed to the closed-loop system, and the second term $\widetilde{u} = \Re_c(y - y_d)$ represents the feedback with the tracking error $e(t) = y(t) - y_d(t)$.

Closing the object (5) with the feedback (11) subject to the linearity of the operator $\Re_p(\xi_0)$, we obtain $y = y_d + \Re_p(\xi_0)\Re_c(y - y_d) \Leftrightarrow y - y_d = \Re_p(\xi_0)\Re_c(y - y_d)$ or $e = \Re_p(\xi_0)\Re_c e$. According to (8) and (9) we have the closed-loop homogeneous system

$$\Re_3(\xi_0)e = 0 \tag{12}$$

with respect to the tracking error.

Taking into account the properties of the operator $\Re_3(\xi_0)$ we have

$$e(t) \to 0 \text{ at } t \to \infty \text{ for any } \xi_0 \in E^{\nu}. \tag{13}$$

which implies the condition

$$y(t) \to y_d(t) \text{ at } t \to \infty.$$

Given the above considerations, we discuss the problem of ensuring the astaticism of the marine vessel by tracking error. For the speed controller, which structure is described in [9–11], the rule of transformation of given stabilizing controller for its use for realization of the desired motion of the vessel on yaw is as follows:

$$\begin{aligned} &\dot{z} = Az + b\delta + g(\varphi - cz), \\ &u = \mu\dot{z} + m\varphi, \end{aligned} \Rightarrow \tag{14}$$

$$\Rightarrow \begin{aligned} &\dot{z} = Az + b\delta + g(\varphi - cz), \\ &u = H^{-1}(p)\varphi_d(t) + \mu\dot{z} + m[\varphi - \varphi_d(t)], \end{aligned} \tag{15}$$

where

$$H^{-1}(p) = A_m(p)/B(p),$$

$$A_m(s) = \left| \begin{array}{ccc|c} & Es - A & & -b \\ \hline -k_1 & -k_2 & 0 & s - k_4 \end{array} \right|,$$

k_1 and k_2 are the elements of the string

$$k_z = \mu(A - gc) = (k_1 \quad k_2 \quad k_{03}), \; k_4 = \mu b, \; m = k_3,$$

$$B(s) = -\begin{vmatrix} s - a_{11} & -a_{12} & -b_1 \\ -a_{21} & s - a_{22} & -b_2 \\ 0 & -1 & 0 \end{vmatrix}.$$

Note that the first term can be interpreted as feed-forward reference command signal $u^*(t) = H^{-1}(p)\varphi_d(t)$, fed to the closed-loop system, and the second term $\widetilde{u} = \mu\dot{z} + m[\varphi - \varphi_d(t)]$ represents the feedback with the tracking error $e(t) \equiv \varphi(t) - \varphi_d(t)$. Astatic property by the tracking error taking into account the linear equation of the drive is obvious.

IV. Example

Now consider the system of automatic tracking control with respect to the requirement of astaticism.

Consider the mathematical model of marine vessel given in [2]:

$$\begin{aligned} &\dot{\beta} = a_{11}\beta + a_{12}\omega + a_{10}\beta|\beta| + b_1\delta + c_1F(t); \\ &\dot{\omega} = a_{21}\beta + a_{22}\omega + b_2\delta + c_2M(t); \\ &\dot{\varphi} = \omega. \end{aligned} \tag{16}$$

Here ω is the angular velocity relative to the vertical axis, φ is a yaw, δ is the deviation angle of vertical rudders, β is the drift angle, *u* is a control, $F = h_1 f(t)$ is

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a side force, $M = h_2 f(t)$ is a moment, *f(t)* is an external stepwise disturbance, determined by wind and waves (Fig.1).

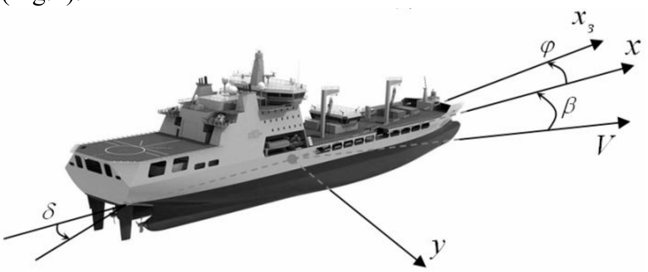

Fig. 1. Parameters of the model.

Let's take harmonic oscillation $\varphi_d(t) = A_d \sin\omega_d t$ with the given amplitude and frequency as program motion. Then, according to (15), the transformation rule for the given stabilizing control

$$u = \mu_1\dot{\beta} + \mu_2\dot{\omega} + \mu_3\omega + \nu\varphi$$

is as follows

$$\dot{z} = Az + b\delta + g(\varphi - cz),$$

$$u = H^{-1}(p)\varphi_d(t) + \mu\dot{z} + \nu[\varphi - \varphi_d(t)],$$

$$\mu = [\mu_1 \quad \mu_2 \quad \mu_3],$$

where coefficients are

$$\mu_1 = -1.7068, \mu_2 = -6.9414, \mu_3 = -4.2482, \nu = -0.4062.$$

The corresponding dynamic process for specified program motion φ_d is presented in Fig. 2, 3. The graphs show that when using speed controller the vessel reach the desired trajectory.

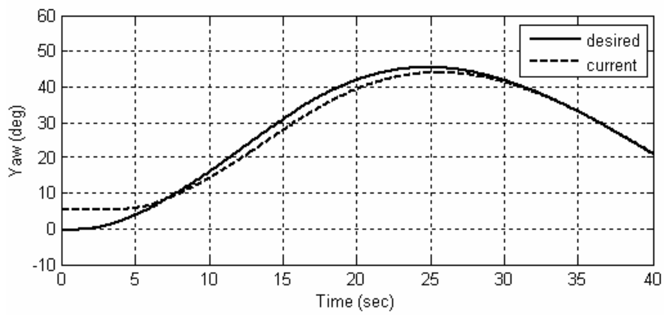

Fig. 2. Adjustment of the program motion.

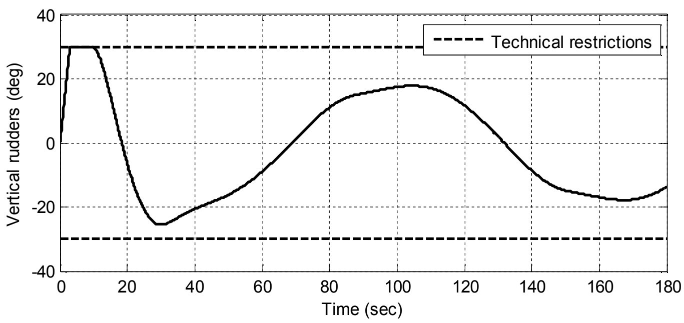

Fig. 3. Rudder deviations during the transition process.

At Fig. 2 solid line represents the desired trajectory and dashed line – current position of the marine vessel. As shown in the Fig. 2, the vessel needs only 25 seconds to reach the given trajectory φ_d. At the same time rudders reach the upper limit, caused by technical restrictions, just once at the beginning of the transition process. As follows from the example the control law (15) provides zero tracking error, i.e. the system with controller (15) is astatic on yaw.

Represented algorithm is implemented in the environment MATLAB with the subsystem Simulink. MATLAB is one of the most effective tools to form and use in the researches computer models of dynamic systems. So the realization of this algorithm can be easily used for any controlled object.

V. Conclusion

Following the given trajectory is related to the necessity to avoid obstacles, vessels' movement in narrow waters, performing the maneuvers of divergence and group motion of marine vessels and etc. In these situations the program of yaw motion $\varphi_d(t)$ is specified, and the control problem is to provide the proximity of current yaw values $\varphi(t)$ and desired yaw $\varphi_d(t)$ at every time moment $t > 0$. At the same time the property of astaticism is very important to provide zero stabilization error in the process of any complicated motion.

In the paper the method to provide the astaticism in tracking control system is proposed, described in details and examined on the example.

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