The Method of Accounting of Bounded External Disturbances for the Synthesis of Feedbacks with Multi-purpose Structure

Mikhail N. Smirnov, Maria A. Smirnova, and Nikolay V. Smirnov

Abstract—**The problem of suppression of external disturbances is one of the main problems in control theory. In many practical cases external impacts are just bounded and any other information is absent. In many practical cases external impacts are just bounded and any other information is absent. In such situations we need to choose the control law, that will give the best possible result on the quality of the dynamics under the worst bounded disturbances. The paper proposes a method of forming an multi-purpose automatic control law, which provides for the best compensation of bounded external disturbances, the desired degree of stability of the closed-loop system and astaticism on yaw.**

Index Terms—**bounded external disturbances, control, stabilization, the degree of stability.**

I. Introduction

Modern marine ships perform a wide spectrum of various tasks, including the most difficult research efforts and rescue expeditions. In view of rapid development of computer technologies digital systems of automatic control are installed on the modern ships for performance of various maneuvers at optimal trajectories taking into account the features of the ship and active disturbances. In this connection, great number of problems that deal with construction of automatic control systems, such as minimizing the transition time, searching for the optimal trajectory, the suppression of various types of exogenous disturbances like wind and rough sea, arises [1–4, 12–17].

Earlier in literature different scientists consider systems without disturbances or systems with disturbances of a specified form or decreasing ones with time. In the work, the problem of suppression of exogenous disturbance, about which we have no information except its boundedness, is considered. In this situation, it requires to choose the parameters of the controller, which will give the best possible result under the worst bounded disturbance.

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Another essential requirement for a control system is astaticism on the regulated coordinates, i.e. the ability to lead a regulation mistake to zero under a constant external influence.

The paper proposes a method of forming an automatic control law, which provides for these three properties of a closed system: the best compensation of bounded external disturbances, the desired degree of stability of the closed-loop system and astaticism on yaw [6–11].

In case of successful solution of this problem with the use of modern computer technologies obtained control system is easily integrated into on-board computer complex structure and provides the desired dynamics of the control processes.

Particularly, in the work much prominence is given to the questions, associated with computer synthesis and modeling of the control laws those suppress bounded exogenous disturbances.

II. Problem Statement

Let us consider the mathematical model of marine ship motion control system:

$$\begin{aligned} \dot{x} &= Ax + Bu + Hd(t), \\ y &= Cx, \end{aligned} \tag{1}$$

where x is a state vector, y is an output vector, u is a control, A, B, H, C are constant matrices with corresponding dimensions, $d(t)$ is a bounded external disturbance, that satisfies the condition

$$\|d(t)\|_\infty \le 1, \quad 0 \le t < \infty. \tag{2}$$

Consider the state feedback control

$$u = K\,x \tag{3}$$

where gain matrix K is need to be found.

Let's denote the desired degree of stability of the characteristic polynomial of the linear closed-loop system by α_p, the actual degree of stability – by $\alpha=\alpha(K)$

$$\alpha = -\max_{i=\overline{1,n}} [Re\,\lambda_i], \tag{4}$$

where $\lambda_1,\dots,\lambda_n$ are eigenvalues of the characteristic polynomial (Fig. 1).

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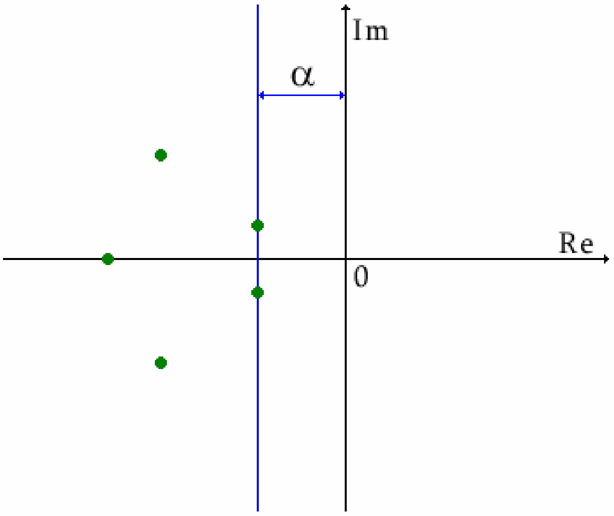

Fig. 1. The degree of stability.

The first part of the problem is to find such automatic control law in the form (3), that will compensate bounded external disturbances, providing the specified limit of the output y and the desired degree of stability of the characteristic polynomial of the closed-loop system.

Using this notation the formulated problem takes the following form: it is necessary to find a coefficient matrix K of the controller $u=Kx$ such that

$$\|y(t,K)\|_\infty \le \mu\ ,\ \alpha(K) \ge \alpha_p\ . \quad (5)$$

The linear stationary system with external input d and output y is output astatic if under stepwise input signal $d(t) \equiv d_0 \cdot \mathbf{1}(t)$ for any real d_0 numerical value of the output, that correspond to the equilibrium position of the system with constant input d_0, y_0 satisfies the condition $y_0=0$.

To provide the astaticism of the system it is proposed is to use special velocity controller with the structure

$$u = \mu\dot{x} + \nu y \quad (6)$$

where μ, ν are constant matrices with corresponding dimensions.

It is necessary to find the control law to compensate bounded external disturbances, to provide the desired degree of stability of the closed-loop system and the property of astaticism on yaw.

The solution of this problem is divided into two main stages:

1) to find the coefficients of the basic law
$$u = K\,x\ , \quad (7)$$
suppressing bounded external disturbances (2) and providing the desired degree of stability of the characteristic polynomial of the closed-loop system;
2) to form the velocity controller based on the control law (7), that provides astaticism of the closed-loop system.

Let's consider each of these stages.

As a base approach we use an external disturbances compensation method, which is based on invariant ellipsoids [18].

The ellipsoid ε_x with the center in the origin and configuration matrix P

$$\varepsilon_x = \left\{ x \in R^n :\ \ x^T P^{-1} x \le 1 \right\},\quad P > 0$$

is said to be state-invariant for the system (1), (2) if the condition $x(0) \in \varepsilon_x$ implies $x(t) \in \varepsilon_x$ for all $t \ge 0$ (Fig. 2).

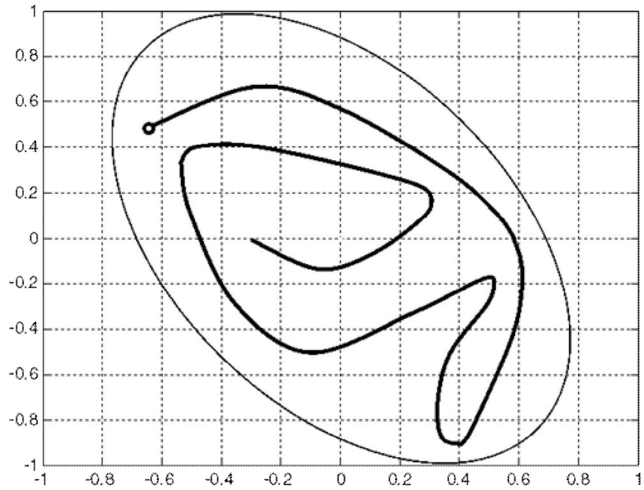

Fig. 2. Invariant ellipsoid.

Here and hereafter the positive definite matrix is denoted as $P > 0$.

In the other words, any trajectory of the system starting in the invariant ellipsoid ε_x stays in it for all $t > 0$.

Similarly, the output-invariant ellipsoid is

$$\varepsilon_y = \left\{ y \in R^m :\ \ y^T (CPC^T)^{-1} y \le 1 \right\},$$

where $P > 0$ is a configuration matrix of the ellipsoid ε_x.

The set of output-invariant ellipsoids ε_y allows us to estimate the impact of external disturbances to the system's output. Thus, selecting from this set the minimal one for some criterion, we will limit the impact of external influences to the system's output $y(t)$.

As a criterion for selecting the minimum invariant ellipsoid the objective function $f(P)=\mathrm{tr}(CPC^T)$, that determines the sum of squares of the semi-axes of the output-invariant ellipsoid of the system (1), is used.

The considered method for solving the problem of compensation of external influences, based on minimization of the l_1-norm of an exit management system is proposed in [5]. Its essence is reduced to the solution of the minimization problem with one parameter

$$\min_{\alpha > 0} \mathrm{tr}(CP(\alpha)C^T)\ ,$$

where $P = P(\alpha)$ is the solution of the Lyapunov inequality ($\alpha > 0$)

$$AP + PA^T + \alpha P + \alpha^{-1} HH^T \le 0,\ \ P > 0\ .$$

After solving this problem, we obtain the ellipsoid, which has minimal trace among all ellipsoids, containing the attainable set. However, this does not guarantee a sufficient reserve of stability, on the value of which the system performance is depends.

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In order to provide the desired degree of stability it is proposed to use the method based on the construction of an auxiliary polynomial with a prescribed degree of stability, described in [19].

According to this method, such n_d-degree polynomial has the form

$$\Delta^*(s,\gamma)=\begin{cases}\tilde{\Delta}^*(s,\gamma), \text{if } n_d \text{ is even;}\\ (s+a_{d+1}(\gamma,\alpha))\tilde{\Delta}^*(s,\gamma), \text{if } n_d \text{ is odd,}\end{cases} \quad (8)$$

where

$$\tilde{\Delta}^*(s,\gamma)=\prod_{i=1}^{d}\left(s^2+a_i^1(\gamma,\alpha)s+a_i^0(\gamma,\alpha)\right), \quad (9)$$

$$d=[n_d/2], \; a_i^1(\gamma,\alpha)=2\alpha+\gamma_{i1}^2, \\ a_i^0(\gamma,\alpha)=\alpha^2+\gamma_{i1}^2\alpha+\gamma_{i2}^2, \; i=\overline{1,d}, \quad (10)$$

$$a_{d+1}(\gamma,\alpha)=\gamma_{d0}^2+\alpha, \\ \gamma=\{\gamma_{11},\gamma_{12},\gamma_{21},\gamma_{22},...,\gamma_{d1},\gamma_{d2},\gamma_{d0}\}. \quad (11)$$

Here γ is n_d-dimensional vector, $\alpha > 0$ is the desired degree of stability.

Thus, specifying the arbitrary vector γ, using (8) – (11) we can construct the polynomial with the prescribed degree of stability.

For the construction of automatic control system it is proposed to modify the external disturbances compensation method [18] by combining it with the method of providing the desired degree of stability.

The developed method consists in:

- Solve the equation
$$AP+PA^T-\gamma BB^T+\alpha P+\alpha^{-1}HH^T=0, \quad P>0$$
with respect to the matrix variable P.
- Find $K(\alpha,\gamma)$ using ratios
$$u=-\gamma B^T P^{-1}(\alpha,\gamma)x=Kx \quad \Rightarrow$$
$$\Rightarrow K(\alpha,\gamma)=-\gamma B^T P^{-1}(\alpha,\gamma).$$
- Use $u = K(\alpha,\gamma)x$ to obtain the closed-loop system.
- Find the characteristic polynomial of the closed-loop system depending on α,γ
$$d(s)=det(sE-A-BK(\alpha,\gamma)C).$$
- Define the desired degree of stability η and construct the auxiliary polynomial $\Delta^*(s,\gamma)$.
- Equate the coefficients of the corresponding powers in characteristic and auxiliary polynomials.
- Solve the system of equations obtained and find the parameters α,γ.
- Solve the minimization problem
$$\min_{\alpha>0} \mathrm{tr}(CP(\alpha)C^T)$$
taking into account the additional restrictions for parameters α,γ from the previous step.
- Find vector K of the controller's coefficients
$$K=-\gamma B^T P^{-1}.$$

Calculating the coefficients of the basic controller, consider the second stage, which is to transform a velocity controller, providing the integral action on regulated coordinate. This controller is uniquely constructed on the base of the control (7).

Let us show that the velocity controller provides the property of astaticism, assuming that its coefficients guarantee stability. To this end, consider the equations of the closed-loop system

$$\dot{x}=Ax+B\delta+Hd, \\ \dot{\delta}=u, \\ y=Cx, \\ u=\mu\dot{x}+\nu y.$$

From these equations it follows that in static, if the restrictions on deviation of rudders allows it, we have

$$Ax+B\delta+Hd=0, \\ y=0.$$

Thus, the velocity controller structure guarantees astatism subject to the stability of the closed-loop system. Then the system (1) takes the form

$$\dot{x}=A_c x+H_c d(t), \\ y=Cx,$$

where $A_c = A+BKC = A+BK$, $H_c = H$.

Transform the basic linear regulator to the velocity form (6) subject to zero external impacts and determine the coefficients μ, ν.

Thus, using the control law (7), providing the fulfillment of the requirements specified above, we can construct a velocity control law (6) that guarantees the astaticism on the systems' output.

III. Conclusion

In the paper the algorithm for optimal compensation of bounded external disturbances with additional requirements – property of astaticism and desired degree of stability, is proposed. The developed algorithm is implemented in an integrated environment MATLAB.

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