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Optimum Life Test Plans of Electrical Insulation for Thermal Stress

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***Abstract*—We search for the optimum life test plans of electrical insulation for thermal stress assuming that the Arrhenius law holds between the thermal stress and the lifetime, and that the logarithmic lifetime follows some consistent probability distributions at a constant stress. The optimization target is to find the optimum number of test specimens at each test stress level, and we consider the case of the number of stress level is three. The criterion for optimality is measured by the root mean squared error for the lifetime in use condition. To take into account the reality, we used the parameter values in a real experimental case. Comparing the optimum results with those using the conventional test method where test specimens are equally allocated to each test stress level, we have found that the confidence interval for the predicted value in the optimum case becomes around 80-85% of that in the conventional test. However, there is only a small difference between the optimum test result and the conventional test result if linearity of the Arrhenius plot is required. It would be useful to know the semi-optimum test plan in which the efficiency is close to that in the optimum one and the test condition is simple. In that sense, we have found that we may regard the conventional test plan as one of the semi-optimum test plans.**

***Index Terms*—optimum test plan, thermal deterioration, Arrhenius law, method of least squares, maximum likelihood estimation method.**

I. Introduction

It is well known that the Arrhenius law is dominant as the aging model due to the thermal stress in electrical insulation. Many researchers, such as Montsinger, Dakin, Simoni, Montanari, and Nelson referred to those modeling ([1], [2], [3], [7], [8], [9], [11], [12], [15]). However, there are not so many references describing the probability distribution model for the thermal deterioration due to the thermal stress. Recently, Hirose and Sakumura [4] proposed the mathematical deterioration models due to the thermal stress, where three probability distribution models are combined with the Arrhenius law. In the mathematical models, we assume that the Arrhenius law holds between the thermal stress and the lifetime, and that the logarithmic lifetime follows some probability distributions at a constant stress. We considered the Pareto distribution, the generalized logistic distribution, and the normal distribution for such probability distribution models.

When the probability distribution models for lifetime are established, we can find the reliability of the target in use condition in a life model, such as the confidence intervals of the estimates for life model parameters. In addition, we may pursue the optimum accelerated life test design so that we can use less test time and less cost, i.e., the efficient test plan. In this paper, we search for the optimum number of test specimens at each test stress level in the accelerated lifetime test when the Arrhenius law and some probability distribution model is appropriately assumed. The criterion for optimality is measured by the root mean squared error for the lifetime in use condition. Such optimum test plans for thermal stress are investigated by Nelson, Meeker, and others ([5], [6], [13], [14], [16]). In [12], [13] and [14], optimum plan for two stress levels are discussed; in [6], three stress levels are incorporated but the number of test specimens at one level is fixed. We consider the case of the number of stress level is three. This is a new challenge.

However, if the optimum test obtained is too complex to perform, practitioners would be reluctant to use the method. Therefore, in this paper, we also aim at finding the semi-optimum test plan in which the efficiency is close to that in the optimum one and the test condition is simple. To do that, we compare the optimum results with those using the conventional test method where test specimens are equally allocated to each test stress level.

To take into account the reality, we first obtained the estimates of parameters as a typical model using a real experimental case. Then, using the parameters just obtained, we investigated the efficiency by using the simulation study for the three probability distribution models.

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II. Mathematical Model

A. Arrhenius Law

We assume that under a constant thermal stress of T [K] there is a relationship between the thermal stress T and the chemical reaction rate k such that

$$k = A\exp(-\frac{E}{RT}), \tag{1}$$

where, E, R, and A are the activation energy, Boltzmann constant, and a constant. Then, the time to failure, t, can be given by

$$t = \frac{C}{k} = A' \exp\left(\frac{E}{RT}\right). \tag{2}$$

Transforming this to the logarithmic formula, we obtain the logarithmic lifetime $y = \log t$, such that

$$y = \log t = \frac{E}{R}\frac{1}{T} - \log B. \tag{3}$$

Then, we have a linear relationship between $y = \log t$ and $1/T$. This is called the well-known Arrhenius Law.

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B. Probability Distribution Model for Deterioration

We assume some probability distribution models for thermal deterioration. These are, 1) the generalized Pareto distribution model, 2) generalized logistic distribution model, and 3) the normal distribution model as shown in [4]. We usually observe the degradation phenomenon by the logarithmic time scale, we may transform y such that $y = \log t$ where t is time to failure.

1) Generalized logistic distribution model: We assume that the logarithmic time to failure, $y = \log t$, follows that the generalized logistic distribution function $F(y)$ such as

$$F(y) = \frac{1}{\{1+\exp(-z)\}^{\beta}}, \tag{4}$$

where $z = (y-\mu)/\sigma, \ (\sigma > 0)$.

2) Generalized Pareto distribution model: We assume that the logarithmic time to failure, $y = \log t$, follows that the generalized Pareto distribution function $F(y)$ such as

$$F(y) = 1 - \frac{1}{(1+\xi z)^{1/\xi}}, \ (1+\xi z > 0), \tag{5}$$

where $z = (y-\mu)/\sigma, \ (\sigma > 0)$.

3) Normal distribution model: We assume that the logarithmic time to failure, $y = \log t$, follows that the normal distribution function $F(y)$ such as

$$F(y) = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(s-\mu)^2}{2\sigma^2}\right\} ds. \tag{6}$$

C. Arrhenius Combined Mathematical Models

The Arrhenius law is combined with one of the probability distribution models mentioned above. These are 1) Arrhenius Pareto model, 2) Arrhenius logistic model, and 3) Arrhenius normal model.

III. Parameter Estimation Method

When we assume that underlying probability distribution for $y = \log t$ follows the generalized Pareto distribution, the generalized logistic distribution, or the normal distribution, we use one of the two methods to find the unknown parameters to be estimated; one is the maximum likelihood estimation method (MLE), and the other is the method of least squares (LS).

A. Maximum Likelihood Estimation Method

Assuming that $T_i (i = 1, \ldots, m)$ are the thermal stress levels, c_i are the censoring times under stress T_i, $t_{i,j}$ are the time to failure under stress T_i, and n_i are the number of specimens under stress T_i. We define $r_{i,j}$ such that $r_{i,j} = 0$ if the specimen failed, and $r_{i,j} = 1$ if the specimen did not fail until c_i.

Then, the maximum likelihood function L is given by

$$L \ \propto \ \prod_{i=1}^{m}\prod_{j=1}^{n_i} \left\{ g(t_{i,j})^{r_{i,j}} \cdot (1-G(c_i))^{1-r_{i,j}} \right\}, \tag{7}$$

where, $g(x)$ and $G(x)$ are the density function and the cumulative probability distribution function, respectively, for the normal, the generalized logistic, or the generalized Pareto distribution models. We can obtain the unknown parameters when L becomes the maximum. In the probability distribution models, we assume that the Arrhenius model is incorporated.

B. Method of Least Squares

When censoring is not planned, the method of (non-linear) least squares is a useful estimation tool to obtain the unknown parameters. The optimization method we used here is the downhill simplex method [10]. We find the parameters so that RSS shown below becomes the minimum.

$$RSS = \sum_{i=1}^{m}\sum_{j=1}^{n_i} \left(\hat{G}(x_{i,j}) - G(x_{i,j})\right)^2, \tag{8}$$

where $\hat{G}(x_{i,j})$ is the estimated value for $G(x_{i,j})$ in the normal distribution, the generalized logistic distribution, or the generalized Pareto distribution.

IV. An Experimental Case

For some insulation material, we have an experimental test case. Test thermal stresses in testing are 250, 270, 290 [deg]. The thermal stresses in use are assumed to be 150, 180, 200 [deg]. Figure 1 shows the times to failure in experiments. The number of test specimens is 25 to each test stress level. Table 1 shows the estimates for the parameters in the mathematical models using this real test data case.

We do not know whether this test design was efficient or not. Thus, we next pursue the optimum test plan by mimicking this situation; that is, we use the similar test condition to one mentioned above, but a little bit different from it. The exception is the number of test specimens to each stress level. In conventional test cases, we use the same number of test specimens to each level. Let us check if the conventional test is efficient or not in the next section.

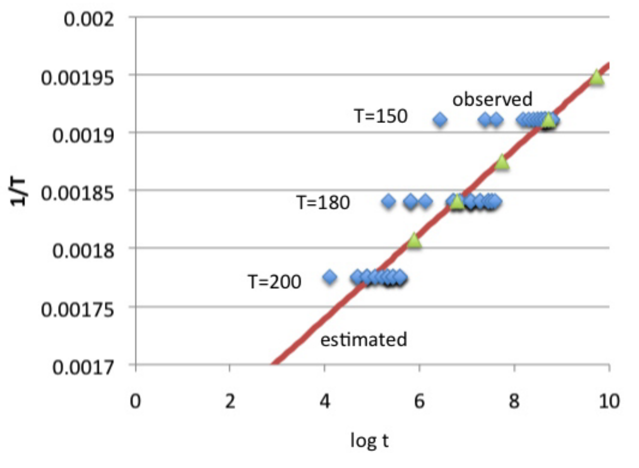

Fig. 1. Arrhenius Plot of an Experimental Case.

TABLE I
MAXIMUM LIKELIHOOD ESTIMATES FOR PARAMETERS

model	$\hat{E}$	$\log \hat{B}$	$\hat{\sigma}$	$\hat{\xi}$	$\hat{\beta}$
Pareto	2.12	40.6	4.45	-1.53	-
logistic	2.06	37.0	0.126	-	0.306
normal	2.06	37.3	0.502	-	-

V. Optimum Test Analysis

A. Test Condition

First, by mimicking the real case mentioned above, we set the test stress levels T such that $(T_H, T_M, T_L) =$

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$(290, 270, 250)$, the stress levels in use T_U are 150, 180, and 200, and the parameters such as $\hat{E}$ and $\log \hat{B}$ are the same to those in Table 2. However, the total number of specimens N is set to be 30 which is smaller than that in the experimental test case because we assume less test specimens. The number of specimens at each levels (T_H, T_M, T_L) are denoted by (n_H, n_M, n_L), and we consider all the combinatorial cases where the number of specimens (n_H, n_M, n_L) consists of integer combinations with exceptions that two of $\{n_H, n_M, n_L\}$ are zero. That is, we consider $493 = (31 \times 30)/2 - 3$ cases.

To evaluate the test efficiency, we use the root mean square error ($RMSE$) for T_U by using the Monte Carlo simulations with 10000 trials to each case. For comparison, we choose the case of $(n_H, n_M, n_L) = (10, 10, 10)$ as a standard, which is commonly used in the conventional cases.

B. *Efficiency Analysis*

Figures 1-3 show the $RMSE$ of $y(= \log t)$ for $T_U = 150, 180, 200$ for the combinatorial cases $\{n_H, n_M, n_L\}$ mentioned above in the generalized Pareto distribution, the generalized logistic distribution, and the normal distribution, respectively. In the figures, horizontal axis means the ratio p, the number of specimens to the total at T_H, and vertical axis means the ratio q, the number of specimens to the total at T_M. The figure indicates that the optimum cases are observed when $q = 0$ and $p < 15$ which means that $n_H < n_L$. In the figure, the points for $(n_H, n_M, n_L) = (10, 10, 10), (5, 10, 15), (15, 10, 5)$ are also shown as indices for simple comparisons.

To compare the efficiency of the optimum cases to the conventional cases, we computed the ratio of the $RMSE$ of the optimum case to that of the conventional case. Table 2 shows the ratio of the optimum $RMSE$ value of $y = \log t$ for T_U to the $RMSE$ value in the conventional case. The table reveals us that the $RMSE$ value of $y = \log t$ in the optimum case is around 80-85% of $RMSE$ value in the conventional test.

TABLE II
RATIO OF $RMSE$ TO THAT OF THE CONVENTIONAL CASE.

model	case	$T_U = 150$	$T_U = 180$	$T_U = 200$
Pareto	optimum	0.858	0.835	0.816
	(10, 10, 10)	1	1	1
	(15, 10, 5)	1.21	1.23	1.25
	(5, 10, 15)	1.01	0.984	0.962
logistic	optimum	0.839	0.848	0.860
	(10, 10, 10)	1	1	1
	(15, 10, 5)	1.13	1.15	1.16
	(5, 10, 15)	1.09	1.06	1.04
normal	optimum	0.811	0.806	0.800
	(10, 10, 10)	1	1	1
	(15, 10, 5)	1.15	1.17	1.19
	(5, 10, 15)	1.04	1.02	1.00

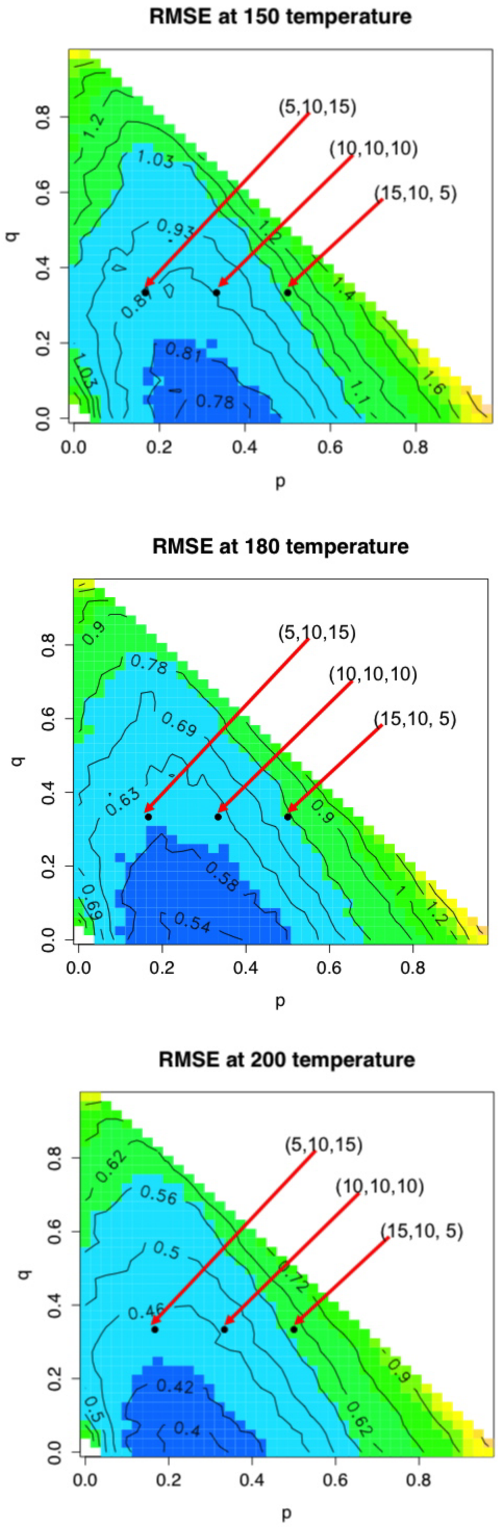

Fig. 2. $RMSE$ of $y(= \log t)$ for $T_U = 150, 180, 200$ for the Combinatorial Cases $\{n_H, n_M, n_L\}$ When the Underlying Distribution is Assumed to be the Generalized Pareto Distribution.

VI. DISCUSSION

A. *Do We Meed More than Two Different Thermal Stresses?*

As indicated in Figures 2-4, the optimum cases do not require the test specimens at T_M. It is obvious that only two different x values are sufficient to make the linear regression model to be consistent. The simplest and the most efficient allocation of the specimen is to locate the specimens at T_H and T_L. The removal of specimens at T_M means the stability increase of the straight line of the Arrhenius plot. When we are interested in predicting the lifetime at T_U in use at lower temperature, it is naturally imagined that we need more specimens to T_L level than to T_H level. This tendency is indicated in Figures 2-4. However, we cannot assume the linearity without absolute many evidences. It would be

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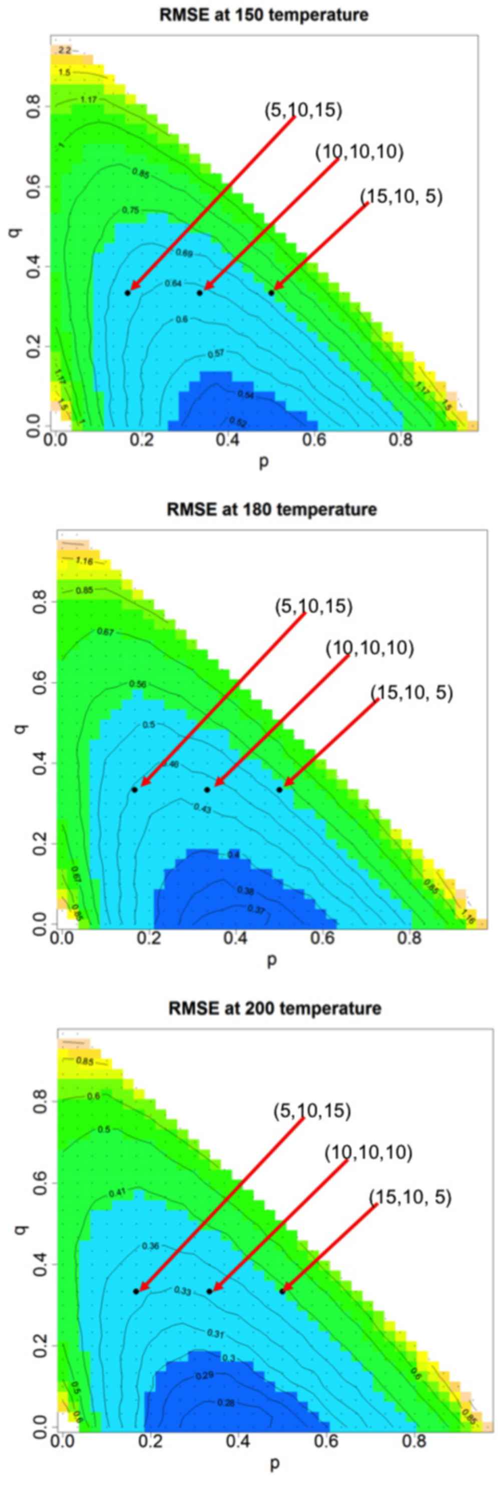

Fig. 3. $RMSE$ of $y(=\log t)$ for $T_U = 150, 180, 200$ for the Combinatorial Cases $\{n_H, n_M, n_L\}$ When the Underlying Distribution is Assumed to be the Generalized Logistic Distribution.

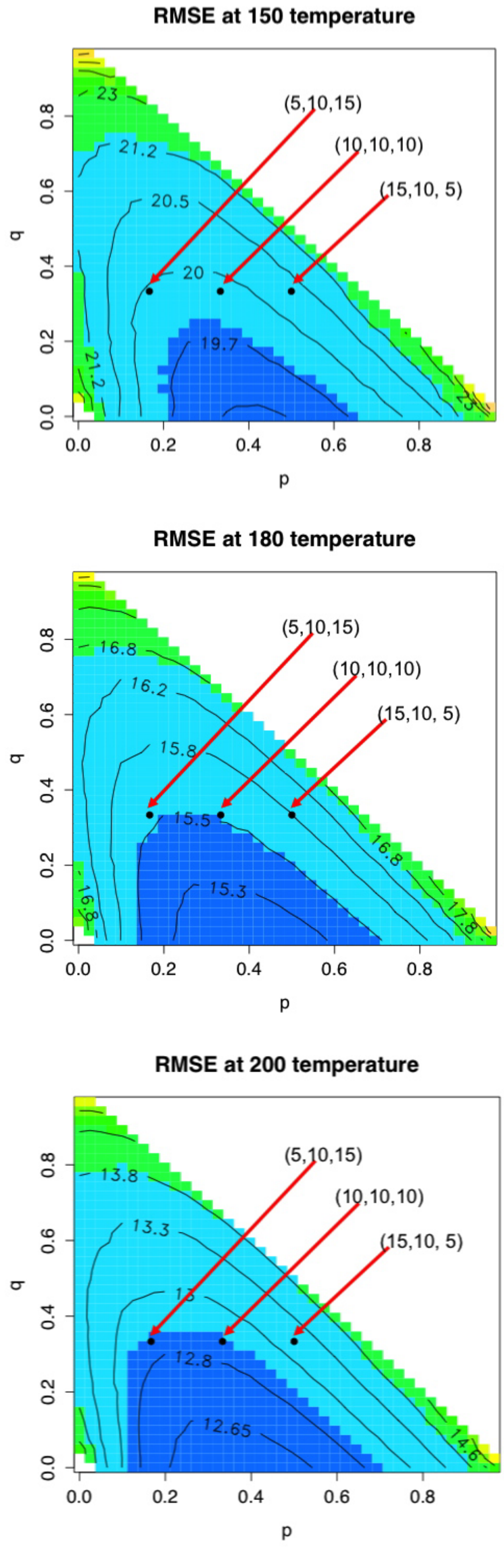

Fig. 4. $RMSE$ of $y(=\log t)$ for $T_U = 150, 180, 200$ for the Combinatorial Cases $\{n_H, n_M, n_L\}$ When the Underlying Distribution is Assumed to be the Normal Distribution.

convenient that we can check if the linearity holds using the hypothesis testing. Then, the allocation of test specimens at T_M $(n_M \neq 0)$ makes sense. In that sense, the conventional method that each number of specimens at each stress level is equivalently allocated is a good choice. Because we do not lose the efficiency much and the linearity can be assessed. For linearity check, we need the hypothesis testing.

B. Optimum Test and Semi-optimum Test

We have pursued the optimum test plan for the thermal stress accelerated test using the real experimental case. The practitioners use the conventional test such that the allocation of the test specimens to each stress level is equivalent. The optimum test using T_L and T_H becomes efficient regarding the standard deviation of the predicted (logarithmic) lifetime is around 80-85%. This means that the total specimens in the optimum test can be saved about 30-35% comparing to the conventional case. More concretely, the conventional test using 30 specimens is equivalent to the optimum test using 20 specimens. When the number of test specimens at T_L and T_H are 18 and 12, the optimality condition is attained. We may allocate, in addition, 10 test specimens to T_M, which will make the linearity check. Therefore, we can call this (conventional method) the semi-optimum test.

C. Are the Results Obtained above Applicable to Other Situations?

You may wonder if the results obtained above are not applicable to other situations because the parameters used here relied on only one experimental case. However, you do not have to worry about that. We have experienced this tendency in other situations.

D. How to Assess the Prediction Accuracy of Lifetime?

When we measure the $s.d.$ or $RMSE$ in logarithmic scale $y = \log t$, we can transform this to scale t on some assumption. When y is small,

$$\log y \approx y - 1.$$

Using this, the variance or the MSE in logarithmic scale y can be transformed to

$$\begin{aligned} S_l &= \frac{1}{n-2}\sum_i (\log t_i - \log \tilde{t})^2 \\ &= \frac{1}{n-2}\sum_i (\log(t_i/\tilde{t}))^2 \\ &\approx \frac{1}{n-2}\sum_i (t_i/\tilde{t} - 1)^2 \\ &= \frac{1}{(n-2)\tilde{t}^2}\sum_i (t_i - \tilde{t})^2 \\ &= \frac{1}{\tilde{t}^2} S, \end{aligned} \quad (9)$$

where, S measures the variance or the MSE in scale t. We have used the ratio for the $s.d.$ or $RMSE$ in logarithmic scale $y = \log t$ here.

VII. Conclusions

Assuming that the Arrhenius law holds between the thermal stress and the lifetime, and that the logarithmic lifetime follows some consistent probability distributions at a constant stress. Under such life models, we have investigated the optimum life test plan. The optimization target is to find the optimum number of test specimens at each test stress level, and we consider the case of the number of stress level is three. The criterion for optimality is measured by the root mean squared error for the lifetime in use condition. To take into account the reality, we used the parameter values in a real experimental case. Comparing the optimum results with those using the conventional test method where test specimens are equally allocated to each test stress level, we have found that the $RMSE$ value of in the optimum case is about 80-85% of $RMSE$ value in the conventional test. However, there is only a small difference between the optimum test result and the conventional test result if linearity of the Arrhenius plot is required. It would be useful to know the semi-optimum test plan in which the efficiency is close to that in the optimum one and the test condition is simple. In that sense, we have found that we may regard the conventional test plan as one of the semi-optimum test plans.

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