

# Recurrent-Fuzzy-Neural System-based Adaptive Controller for Nonlinear Uncertain Systems

Tzu-Wei Hu and Ching-Hung Lee, *Member, IAENG*

**Abstract—** This paper aims to treat the adaptive control of nonlinear system with un-model uncertainty and bounded disturbance by using a novel recurrent fuzzy neural system. The used recurrent interval type-2 fuzzy neural network with asymmetric membership functions (RT2FNN-A) combines the interval asymmetric type-2 fuzzy sets and fuzzy logic system and implements in a five-layer neural network structure which contains four layer forward network and a feedback layer. The RT2FNN-A is modified to provide memory elements for capturing the system's dynamic information and has the properties of high approximation accuracy and small network structure (fewer rules and tuning parameters) from the simulation results. Based on the Lyapunov theorem, adaptive update laws of RT2FNN-A derived and the stability of closed-loop system is guaranteed for control of the nonlinear uncertain systems. Simulation result is also introduced to show the performance and effectiveness of RT2FNN-A system.

**Index Terms—** Nonlinear control, uncertainty, recurrent, interval type-2 fuzzy system

## I. INTRODUCTION

Recently, the fuzzy systems and control are regarded as the most widely used application of fuzzy logic system [8-11, 20, 23, 26, 31, 32]. In traditional fuzzy system models, the structure is characterized by using type 1 fuzzy sets, which are defined on a universe of discourse, map an element of the universe of discourse onto a precise number in the unit interval [0, 1]. The concept of type-2 fuzzy sets was initially proposed by Zadeh as an extension of typical fuzzy sets (called type-1) [34]. Mendel and Karnik developed a complete theory of type-2 fuzzy logic systems (T2FLSs) [12, 21, 26]. Recently, T2FLSs have attracted more attention in many literatures and special issues [5, 9, 15, 21, 26].

T2FLSs are more complex than type-1 ones. The major difference being the present of type-2 is their antecedent and consequent sets. T2FLSs result in better performance than type-1 fuzzy logic systems (T1FLSs) on the applications of function approximation, modeling, and control. Besides, neural networks have found numerous practical applications, especially in the areas of prediction, classification, and control [18, 23]. The main aspect of neural networks lies in the connection weights which are obtained by training process. Based on the advantages of T2FLSs and neural networks, the type-2 fuzzy neural network (T2FNN) systems

are presented to handle the system uncertainty and reduce the rule number and computation [15, 26]. In addition, recurrent fuzzy neural network (RFNN) has been successfully used in many areas, such as the nonlinear dynamic system identification and control problems [11, 18, 24]. One of the most important features of RFNN is its feedback path in the circuit. The feedback paths of RFNN make it have the advantages of storing past information and speeding up convergence [18].

The design of a fuzzy partition and rules engine normally affects system performance. Symmetric and fixed membership functions (MFs) (e.g., Gaussian or triangular) are commonly used to simplify the design procedure. Therefore, a large number of rules should be used to achieve the specified approximation accuracy [25]. Several approaches have been introduced to optimize fuzzy MFs and choose an efficient scheme for structure and parameter learning. This problem has been discussed and analyzed using asymmetric fuzzy MFs (AFMFs) [1, 13, 15, 22, 24, 28]. The results showed that using AFMFs can improve the modeling capability. According to the results above, our purpose is to introduce a recurrent interval type-2 fuzzy neural network with asymmetric membership functions (RT2FNN-A).

Recently, there are many literatures addressing in nonlinear dynamic system identification and control using neural fuzzy systems [5, 15, 16, 18, 20, 24, 32]. Literature [30] introduced the design method of nonlinear control, whereas in [32], the adaptive fuzzy control approach was introduced. The parameter update laws can be obtained by Lyapunov theorem. In [18], there were successful application cases in nonlinear system identification and control by using RFNN, but lots of rules should be used. In [15, 16], the T2FNN was successfully applied in many cases. However, the network structure was a static model and lots of rule numbers should be used. In [14], the T2FNN with AFMFs was proposed to improve the system performance. The AFMFs improved approximation accuracy and reduced the fuzzy rules, but the network structure still is static. In [15, 16], the T2FNN has been successfully applied in time-series prediction problem, but lots of rules should be used. Literature [29] adopted wavelet network for nonlinear system modeling, but the initial values of translations and dilations required more care in procedure design.

In this paper, we proposed a combining interval type-2 fuzzy asymmetric membership functions with recurrent neural network system, called RT2FNN-A. The proposed RT2FNN-A is a modified version of the T2FNN [15-17] which provides memorized elements to capture system dynamic response [18]. The RT2FNN-A system capability for temporarily storing information allowed us to extend the application domain to include temporal problem. Simulations

C. H. Lee is with the Department of Mechanical Engineering, National Chung Hsing University, Taichung, 402, Taiwan, R.O.C. (Tel: +886-422840433, ext: 417; fax: 886-4-22877170; e-mail: chleenchu@dragon.nchu.edu.tw). This work was supported in part by the National Science Council, Taiwan, R.O.C., under contracts NSC-100-2221-E-005-093-MY2.

are shown to illustrate the effectiveness of the RT2FNN-A system.

This paper is organized as follows. Section II introduces the problem formulation of nonlinear control problem. The RT2FNN-A system and the corresponding adaptive control scheme is described in Section III. Several simulations for nonlinear system identification and control problems are done and introduced in Section IV. Finally, conclusion is given.

## II. PROBLEM FORMULATION

Consider an  $n$ th-order nonlinear dynamic system in the companion form or controllability canonical form

$$\begin{aligned} \dot{\mathbf{x}}^{(n)} &= F(\mathbf{x}) + G(\mathbf{x})u + D \\ y &= x \end{aligned} \quad (1)$$

where  $u \in \mathfrak{R}$  and  $y \in \mathfrak{R}$  are the control input and output of the nonlinear system.  $\mathbf{x} = [x \ \dot{x} \ \dots \ x^{(n-1)}]^T \in \mathfrak{R}^n$ ,  $F(\cdot)$ , and  $G(\cdot)$  are unknown nonlinear and continuous functions;  $D$  is the external bounded disturbance or system uncertainty. If (1) is controllable, the  $G(\mathbf{x})$  needs to be invertible for all  $\mathbf{x} \in U_c \subset \mathfrak{R}^n$ .

Our purpose is to design a robust adaptive control scheme to guarantee boundedness of all closed-loop variables and tracking of a given bounded reference trajectory  $y_r$ . We define the tracking error  $e$  as  $y_r - y$ , and

$$\mathbf{E} = [e \ \dot{e} \ \dots \ e^{(n-1)}]^T. \quad (2)$$

If all the parameters of the plant dynamics are exactly known, the ideal control law  $u^*$  can be design by *feedback linearization* approach [30]

$$u^* = G(\mathbf{x})^{-1} [y_r^{(n)} - F(\mathbf{x}) - D(t) + \mathbf{K}\mathbf{E}] \quad (3)$$

where  $\mathbf{K} = [k_n \ k_{n-1} \ \dots \ k_1] \in \mathfrak{R}^n$  and  $k_i > 0$ ,  $i = 1, \dots, n$ . Substituting (46) into (44) and yields

$$e^{(n)} + k_1 e^{(n-1)} + \dots + k_n e = 0 \quad (4)$$

which implies that  $\lim_{t \rightarrow \infty} e(t) = 0$ . This can be done by choosing proper  $\mathbf{K}$  so that all roots of the polynomial  $s^n + k_1 s^{(n-1)} + \dots + k_n = 0$  are located in the open left-half plane. However, the nonlinear functions  $F(\mathbf{x})$  and  $G(\mathbf{x})$  are not exactly known in general. Therefore, we cannot implement the ideal control law (3). In order to solve this problem, the adaptive RT2FNN-A control system is proposed to approximate to the ideal control law  $u^*$ . The adopted recurrent fuzzy neural system is introduced in next section.

## III. RECURRENT FUZZY NEURAL SYSTEM BASED ADAPTIVE CONTROLLER DESIGN

### A. Recurrent Fuzzy Neural System

We first introduce the recurrent type-2 neural fuzzy inference system with asymmetric fuzzy MFs (RT2FNN-A) that was modified and extended from previous results [10, 14]. The RT2FNN-A uses the interval asymmetric type-2 fuzzy sets and it implements the FLS in a five-layer neural network structure which contains four-layer forward network and a feedback layer. Layer-1 nodes are input nodes representing input linguistic variables, and layer-4 nodes are

output nodes representing output linguistic variables. The nodes in layer 2 are term nodes that act as MFs, where each membership node is responsible for mapping an input linguistic variable into a corresponding linguistic value for that variable. All of the layer-3 nodes together formulate a fuzzy rule basis, and the links between layers 3 and 4 function as a connectionist inference engine. The rule nodes reside in layer 3, and layer 5 is the recurrent part in type-2 fuzzy sets.

### B. Construction of Type-2 Asymmetric Fuzzy Member Functions

In general, given an system input data set  $x_i, i=1, 2, \dots, n$ , and the desired output  $y_p, p=1, 2, \dots, m$ , the representation of  $j$ th rule for RT2FNN-A is

Rule  $j$ : IF  $x_1$  is  $\tilde{G}_{1j}$  and  $\dots$   $x_n$  is  $\tilde{G}_{nj}$  and  $g_j$  is  $\tilde{G}_j^F$

THEN  $y_1$  is  $\tilde{w}_1^j$  and  $\dots$   $y_m$  is  $\tilde{w}_m^j$ ,

$g_1$  is  $\tilde{a}_1^j, g_2$  is  $\tilde{a}_2^j, \dots$ , and  $g_M$  is  $\tilde{a}_M^j$ .

where  $\tilde{G}_{ij}$  represents the linguistic term of the antecedent part,  $\tilde{w}_p^j$  and  $\tilde{a}_p^j$  represents the interval real number of the consequent part; and  $M$  is the rule number. Here the fuzzy MFs of the antecedent part  $\tilde{G}_{ij}$  are asymmetric interval type-2 fuzzy sets, which represent the different from typical Gaussian MFs.

The MFs of the precondition part discussed in this article are of asymmetric type as described below. Each type-2 fuzzy MF is constructed by parts of four Gaussian functions. Each upper and lower MFs is constructed by two Gaussian MFs and one segment. Herein, we use superscript  $l$  and  $r$  to denote the left and right curves of Gaussian MF. The parameters of lower and upper MFs are denoted by  $\underline{\cdot}$  and  $\overline{\cdot}$ , respectively. Thus, the upper MF is constructed as

$$\bar{\mu}_{\tilde{G}}(x) = \begin{cases} \exp\left[-\frac{1}{2}\left(\frac{x-\bar{m}^l}{\bar{\sigma}^l}\right)^2\right], & \text{for } x \leq \bar{m}^l \\ 1, & \text{for } \bar{m}^l < x < \bar{m}^r \\ \exp\left[-\frac{1}{2}\left(\frac{x-\bar{m}^r}{\bar{\sigma}^r}\right)^2\right], & \text{for } \bar{m}^r \leq x \end{cases} \quad (5)$$

where  $\bar{m}^l$  and  $\bar{m}^r$  denote the uncertain means of two Gaussian MFs satisfying  $\bar{m}^l \leq \bar{m}^r$ ,  $\bar{\sigma}^l$  and  $\bar{\sigma}^r$  denote the uncertain deviations (width) of two Gaussian MFs. Similarly, the lower MF is defined as

$$\underline{\mu}_{\tilde{G}}(x) = \begin{cases} \gamma \cdot \exp\left[-\frac{1}{2}\left(\frac{x-\underline{m}^l}{\underline{\sigma}^l}\right)^2\right], & \text{for } x \leq \underline{m}^l \\ \gamma, & \text{for } \underline{m}^l < x < \underline{m}^r \\ \gamma \cdot \exp\left[-\frac{1}{2}\left(\frac{x-\underline{m}^r}{\underline{\sigma}^r}\right)^2\right], & \text{for } \underline{m}^r \leq x \end{cases} \quad (6)$$

where  $\underline{m}^l \leq \underline{m}^r$ . The corresponding width of MFs are  $\underline{\sigma}^l$  and  $\underline{\sigma}^r$ . To avoid the activation value is too small,  $\gamma$  is chosen between 0.5 and 1. Thus, the following restrictions should be added to avoid the unreasonable MFs

$$\begin{cases} \underline{m}' \leq \underline{m}' \leq \underline{m}' \leq \underline{m}' \\ \underline{\sigma}' \leq \underline{\sigma}', \underline{\sigma}' \leq \underline{\sigma}' \\ 0.5 \leq \gamma \leq 1. \end{cases} \quad (7)$$

This introduces the properties of uncertain mean and variance [16]. Additionally, we can construct other type-2 asymmetric MFs by tuning the parameters. The corresponding tuning algorithm is derived to improve system accuracy and approximation ability.

### C. Network Structure

In this section, the structure of recurrent type-2 fuzzy neural network with AFMFs (RT2FNN-A) is introduced. The MISO case is considered here for convenience. The RT2FNN-A is implemented as the four-layer network with feedback layer shown in Fig. 1. We first indicate the signal propagation and the operation functions of the nodes in each layer. In the following description,  $O_i^{(l)}$  denotes the  $i$ th output of a node in the  $l$ th layer.

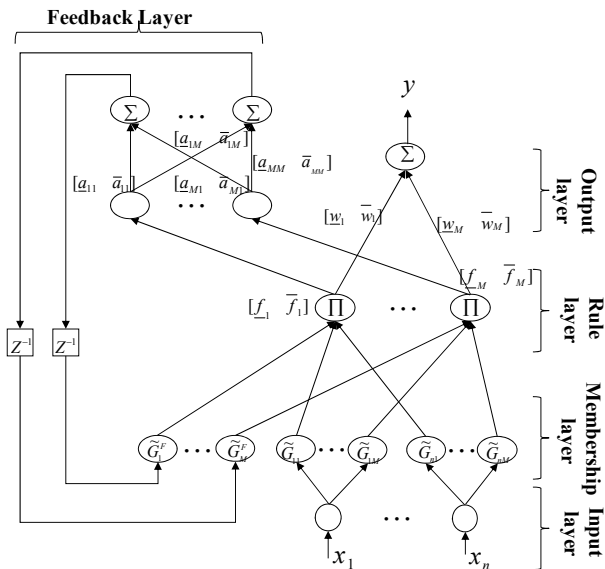


Figure 1: Diagram of MISO recurrent type-2 fuzzy neural network with AFMFs (RT2FNN-A).

#### Layer 1: Input Layer

For the  $i$ th node of layer 1, the net input and output are represented as

$$O_i^{(1)} = x_i^{(1)}. \quad (8)$$

#### Layer 2: Membership Layer

In layer 2, each node performs a type-2 fuzzy MF introduced by (5)–(7). The following simplified notation is adopted

$$O_{ij}^{(2)} = \tilde{\mu}_{\tilde{G}_i}(O_i^{(1)}). \quad (9)$$

It is clear that there are two parts in this layer, regular nodes and feedback nodes. Their input are  $O_j^{(1)}$  and  $g_j(k)$ . Therefore, for network input  $x_i$ , the output is

$$O_{ij}^{(2)} = [\underline{O}_{ij}^{(2)} \quad \overline{O}_{ij}^{(2)}]^T = [\underline{\mu}_{\tilde{G}_i}(O_i^{(1)}) \quad \overline{\mu}_{\tilde{G}_i}(O_i^{(1)})]^T. \quad (10)$$

For internal or feedback variable  $g_j$ ,

$$O_j^{F(2)} = [\underline{O}_j^{F(2)} \quad \overline{O}_j^{F(2)}]^T = [\underline{\mu}_{\tilde{G}_j}(g_j(k)) \quad \overline{\mu}_{\tilde{G}_j}(g_j(k))]^T \quad (11)$$

where the subscript  $ij$  indicates the  $j$ th term of the  $i$ th input  $O_i^{(1)}$ , where  $j=1, \dots, M$ . The superscript  $F$  indicates the feedback layer.

#### Layer 3: Rule Layer

The links in this layer are used to implement the antecedent matching and these are equal to the work in rule layer. Using the product  $t$ -norm, the firing strength associated with the  $j$ th rule is

$$\underline{f}^j = \underline{\mu}_{\tilde{F}_1}(x_1) * \dots * \underline{\mu}_{\tilde{F}_n}(x_n) * \underline{\mu}_{\tilde{G}_j}(\cdot) \quad (12a)$$

$$\overline{f}^j = \overline{\mu}_{\tilde{F}_1}(x_1) * \dots * \overline{\mu}_{\tilde{F}_n}(x_n) * \overline{\mu}_{\tilde{G}_j}(\cdot), \quad (12b)$$

where  $\underline{\mu}_{\tilde{G}_i}(\cdot)$  and  $\overline{\mu}_{\tilde{G}_i}(\cdot)$  are the lower and upper membership grades of  $\mu_{\tilde{G}_i}(\cdot)$ . Therefore, a simple PRODUCT operation is used. Then, for the  $j$ th input rule node

$$\begin{aligned} O_j^{(3)} &= [\underline{O}_j^{(3)} \quad \overline{O}_j^{(3)}]^T \\ &= \left[ \underline{O}_j^{F(2)} \prod_{i=1}^n w_{ij}^{(3)} \cdot \underline{O}_{ij}^{(2)} \quad \overline{O}_j^{F(2)} \prod_{i=1}^n w_{ij}^{(3)} \cdot \overline{O}_{ij}^{(2)} \right]^T, \end{aligned} \quad (13)$$

where the weights  $w_{ij}^{(3)}$  are assumed to be unity.

#### Layer 4: Output Layer

Without loss of generality, the consequent part of interval T2FLS is  $\tilde{w}_j = [\underline{w}_j \quad \overline{w}_j]^T$ ,  $\underline{w}_j \leq \overline{w}_j$ . The vector notations  $\underline{w} = [\underline{w}_1 \quad \dots \quad \underline{w}_M]^T$  and  $\overline{w} = [\overline{w}_1 \quad \dots \quad \overline{w}_M]^T$  are used for clarity. The remaining works are type reduction and defuzzification. For type-reduction, we should calculate the lower and upper bounds  $[y_l, y_r]$  [12, 26]. Modifying the Karnik-Mendel procedure [12, 26], we let

$$O_{TR}^{(4)} = \int_{w_l \in [\underline{w}_l, \overline{w}_l]} \dots \int_{w_M \in [\underline{w}_M, \overline{w}_M]} \int_{f_l \in [\underline{f}_l, \overline{f}_l]} \dots \int_{f_M \in [\underline{f}_M, \overline{f}_M]} / \sum_{i=1}^M f_i w_i. \quad (14)$$

Note that the normalization  $(\sum_{i=1}^M f_i)$  is removed to simplify the type-reduction procedure, computation, and the derivation of learning algorithm by gradient descent method.

We denote the maximum and minimum of  $\sum_{i=1}^M f_i w_i$  as  $\overline{O}^{(4)}$  and  $\underline{O}^{(4)}$ , as follows

$$\underline{O}^{(4)} = \underline{w}^T f_L = \sum_{j=1}^L (\overline{O}_j^{(3)} \underline{w}_j) + \sum_{j=L+1}^M (\underline{O}_j^{(3)} \underline{w}_j), \quad (15a)$$

$$\overline{O}^{(4)} = \overline{w}^T f_R = \sum_{j=1}^R (\underline{O}_j^{(3)} \overline{w}_j) + \sum_{j=R+1}^M (\overline{O}_j^{(3)} \overline{w}_j), \quad (15b)$$

where

$$f_L = [\underline{f}_1, \dots, \underline{f}_L, \underline{f}_{L+1}, \dots, \underline{f}_M]^T = [\underline{O}_1^{(3)}, \dots, \underline{O}_L^{(3)}, \underline{O}_{L+1}^{(3)}, \dots, \underline{O}_M^{(3)}]^T,$$

$$f_R = [\underline{f}_1, \dots, \underline{f}_R, \underline{f}_{R+1}, \dots, \underline{f}_M]^T = [\underline{O}_1^{(3)}, \dots, \underline{O}_R^{(3)}, \underline{O}_{R+1}^{(3)}, \dots, \underline{O}_M^{(3)}]^T.$$

It is obvious that  $R$  and  $L$  should be calculated first. The weights are arranged in order as  $\underline{w}_1 \leq \underline{w}_2 \leq \dots \underline{w}_M$  and  $\overline{w}_1 \leq \overline{w}_2 \leq \dots \overline{w}_M$ . According to the Karnik-Mendel procedure [12, 26],  $L$  and  $R$  are

$$L = \arg \min_{j \in [1, \dots, M-1]} (\underline{O}^{(4)}), \quad R = \arg \max_{j \in [1, \dots, M-1]} (\overline{O}^{(4)}). \quad (16)$$

Finally, the crisp output is

$$O^{(4)} = \frac{\underline{O}^{(4)} + \overline{O}^{(4)}}{2}. \quad (17)$$

#### Layer 5: Feedback Layer

This layer contains the context nodes which is used to produce the internal or feedback variable  $g_j$ . Each rule is associated with a particular internal variable. Hence, the number of the context nodes is equal to the number of rules. The same operations (type-reduction and defuzzification) as layer 4 are performed here.

$$\underline{O}_j^{(5)}(k+1) = \underline{a}_j^T f_L = \sum_{h=1}^{L_f^f} (\bar{O}_h^{(3)} \underline{a}_{jh}) + \sum_{h=L_f^f+1}^M (\underline{O}_h^{(3)} \underline{a}_{jh}), \quad (18a)$$

$$\bar{O}_j^{(5)}(k+1) = \bar{a}_j^T f_R = \sum_{h=1}^{R_f^f} (\underline{O}_h^{(3)} \bar{a}_{jh}) + \sum_{h=R_f^f+1}^M (\bar{O}_h^{(3)} \bar{a}_{jh}), \quad (18b)$$

and

$$L_j^F = \arg \min_{h \in [1, \dots, M-1]} (\underline{O}_j^{(5)}(k+1)), \quad R_j^F = \arg \max_{h \in [1, \dots, M-1]} (\bar{O}_j^{(5)}(k+1)). \quad (19)$$

Finally, the crisp output of this layer is

$$g_j(k+1) = O_j^{(5)}(k+1) = \frac{1}{2} [\underline{O}_j^{(5)}(k+1) + \bar{O}_j^{(5)}(k+1)] \quad (20)$$

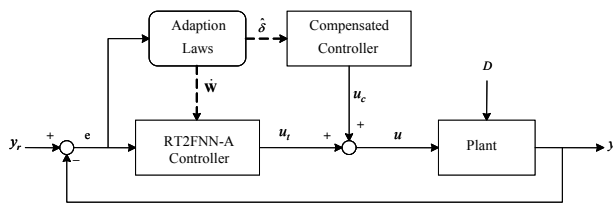


Figure 2: Diagram of robust adaptive control scheme using RT2FNN-A.

### C. Adaptive Controller Design for Nonlinear uncertain Systems

The configuration of the proposed RT2FNN-A control system is depicted in Fig. 2. The RT2FNN-A controller  $u_i$  is connected with a compensated controller  $u_c$  to generate the control signal  $u$ , which is

$$u = u_i + u_c. \quad (21)$$

According to (15)-(20), we may define the control input produced by RT2FNN-A as

$$u_i = \frac{1}{2} \underline{w}^T f_L + \frac{1}{2} \bar{w}^T f_R. \quad (22)$$

Based on the universal approximation theorem of [23, 18], there exists optimal parameters  $\underline{w}^*$  and  $\bar{w}^*$  such that  $u_i(\underline{w}^*, \bar{w}^*) \equiv u_i^*$  approximates the ideal controller  $u^*$  defined in (3). Define the minimum approximation error  $\varepsilon$

$$\varepsilon = u^* - u_i^*, \quad (23)$$

where  $|\varepsilon| < \delta$ , and  $\delta > 0$  is the uncertainty bound of approximation error.

The control objective is let  $u_i^*$  be able to approximate  $u^*$  as close as possible, meaning, minimum approximation error  $\varepsilon$  can be made as small as possible. Therefore,  $u_i^*$  can be represented as

$$u_i^* = \frac{1}{2} (\underline{w}^*)^T f_L + \frac{1}{2} (\bar{w}^*)^T f_R + \varepsilon. \quad (24)$$

Thus, the following error dynamic is obtained

$$\begin{aligned} \dot{\mathbf{E}} &= \mathbf{A}\mathbf{E} + B(x)(u^* - u_i - u_c) \\ &= \mathbf{A}\mathbf{E} + B(x) \left[ \varepsilon + \frac{1}{2} (\underline{w}^* - \underline{w})^T f_L + \frac{1}{2} (\bar{w}^* - \bar{w})^T f_R - u_c \right] \end{aligned} \quad (25)$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ -k_n & -k_{n-1} & \dots & -k_1 \end{bmatrix} \in \mathbb{R}^{n \times n} \text{ and } \mathbf{B} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ G(\mathbf{x}) \end{bmatrix} \in \mathbb{R}^n.$$

Thus, we have the following stability Theorem.

**Theorem 1:** Consider the nonlinear system (1) having adaptive control input (21). The adaptive control input  $u_i$  are design by (22). Let the parameter vectors  $\underline{w}$  and  $\bar{w}$  be adjusted by the following adaptive laws

$$\begin{cases} \dot{\underline{w}} = \gamma_w \mathbf{E}^T \mathbf{P} B f_L \\ \dot{\bar{w}} = \gamma_w \mathbf{E}^T \mathbf{P} B f_R \end{cases} \quad (26)$$

where  $\gamma_w$  is positive constant.  $\mathbf{P}$  is a symmetric positive definite matrix satisfies

$$\mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A} = -\mathbf{Q}, \quad (27)$$

where  $\mathbf{Q}$  is a symmetric positive definite matrix and is selected by the designer. The compensated controller  $u_c$  is

$$u_c = \hat{\delta} \operatorname{sgn}(\mathbf{E}^T \mathbf{P} B) \quad (28)$$

where  $\hat{\delta}$  is the estimation error bound by

$$\dot{\hat{\delta}} = \begin{cases} \alpha |\mathbf{E}^T \mathbf{P} B|, & \alpha > 0 \text{ if } \hat{\delta} < \hat{\delta}_{\max} \\ 0, & \text{if } \hat{\delta} \geq \hat{\delta}_{\max} \end{cases} \quad (29)$$

where  $\hat{\delta}_{\max}$  is the maximum of  $\hat{\delta}$  and is selected by the designer.  $\square$

Using Lyapunov theorem, we can only obtain the update laws of consequent part parameters. In order to obtain a better performance of the system, we need to obtain the update laws of antecedent part parameters. We can also use the gradient descent method to obtain the update laws for the antecedent part's parameters for interval type-2 asymmetric membership functions to enhance the performance of the recurrent fuzzy neural system. More details can be found in literature [6].

**Remark 1:** Theorem 1 introduces the compensated controller  $u_c = \hat{\delta} \operatorname{sign}(\mathbf{E}^T \mathbf{P} B)$ , the sign function signal often results in chattering phenomenon. In order to reduce the chattering, we choose the compensated controller as [14]

$$u_c = \hat{\delta} \operatorname{sat}(\mathbf{E}^T \mathbf{P} B, h) \quad (30)$$

where

$$\operatorname{sat}(\mathbf{E}^T \mathbf{P} B, h) = \begin{cases} \operatorname{sign}(\mathbf{E}^T \mathbf{P} B), & |\mathbf{E}^T \mathbf{P} B| \geq h \\ \frac{\mathbf{E}^T \mathbf{P} B}{h}, & |\mathbf{E}^T \mathbf{P} B| < h, \end{cases}$$

in which  $h$  is a small positive number. If  $h$  we chose is small enough ( $h > 0$ ), then  $\mathbf{E}$  can be limited to a small range as the sliding mode control with sliding surface. Although the saturation function can solve the chattering property, it loses the accuracy of system. In this section, we only take  $u_c$  to be a compensated controller. The main function of this controller is to maintain system convergence when the error is large. If the RT2FNN-A system can achieve the accuracy, the whole control framework then can make the error converge to zero [14].

#### IV. SIMULATION RESULTS

The inverted pendulum system is [30, 32]

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= F(x_1, x_2) + G(x_1, x_2)u + D\end{aligned}\quad (31)$$

The nonlinear functions  $F(x_1, x_2)$  and  $G(x_1, x_2)$  in (31) are

$$\begin{aligned}F(x_1, x_2) &= \frac{g \sin x_1 - \frac{mlx_2^2 \cos x_1 \sin x_1}{M_c + m}}{l\left(\frac{4}{3} - \frac{m \cos^2 x_1}{M_c + m}\right)} \quad \text{and} \\ G(x_1, x_2) &= \frac{\cos x_1}{l\left(\frac{4}{3} - \frac{m \cos^2 x_1}{M_c + m}\right)}, \quad \text{where } g = 9.8m/s^2 \text{ is the}\end{aligned}$$

acceleration due to gravity,  $M_c$  is the mass of the cart,  $m$  is the mass of the pole,  $l$  is the half-length of the pole,  $u$  is the applied force (control), and  $D$  is the external bounded disturbance.

Suppose we set  $M_c = 1kg$ ,  $m = 0.1kg$ ,  $l = 0.5m$ , and the reference signal chose as  $[x_{1d} \ x_{2d}]^T = [0 \ 0]^T$  in the following simulations (other choices are possible). The initial condition of state is  $\mathbf{x} = [x_1 \ x_2]^T = [\pi/18 \ 0]^T$ ; compensated controller parameters are chosen as  $\mathbf{K} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$ ,  $\mathbf{P} = \begin{bmatrix} 1 & 0.8 \\ 0.8 & 1 \end{bmatrix}$ ,  $\alpha = 0.01$ ,  $\delta_0 = 2$ ,  $\hat{\delta}_{\max} = 10$ , and  $h = 0.1$ ; the learning parameter is chosen as  $\gamma_w = \gamma = \bar{\gamma} = 1$ . Then, we simulate two cases as follows

**Case 1:** External disturbance free, i.e.,  $D = 0$ ,

**Case 2:** The external disturbance  $D$  as follows exists.

$$D = \begin{cases} 0, & \text{if } t < 8 \\ \sin(t), & \text{if } t \geq 8. \end{cases} \quad (32)$$

The simulation results of our approach for **Case 1** and **Case 2** are shown in Figs. 3 and 4. Figure 3 presents the comparison results of state trajectories for **Case 1**, (solid-line: RT2FNN-A, dashed-line: T2FNN-A, dash-dotted-line: T2FNN, and dotted-line: reference trajectory). Figure 4 shows the corresponding control effort for **Case 1** (solid-line: total control force, dash-dotted-line: compensated controller, and dotted-line: RT2FNN-A controller). The comparison results of state trajectories for **Case 2** are shown in Fig. 5 (solid-line: RT2FNN-A, dashed-line: T2FNN-A, dash-dotted-line: T2FNN, and dotted-line: reference trajectory). Figure 6 shows the corresponding control effort for **Case 2** (solid-line: total control force, dash-dotted-line: compensated controller, and dotted-line: RT2FNN-A controller). Obviously, the RT2FNN-A system performs well with using less adjustable parameters and fuzzy rules. In addition, the constructed fuzzy control rules are

$R^1$ : IF  $x_1$  is  $\tilde{G}_{11}$  and  $x_2$  is  $\tilde{G}_{21}$  and  $g_1$  is  $\tilde{G}_1^F$  THEN  $y$  is  $[-1.9104 \ -2.8653]$ , and  $g_1$  is  $[-0.9323 \ -0.9193]$  and  $g_2$  is  $[-1.1937 \ -1.1348]$ .

$R^2$ : IF  $x_1$  is  $\tilde{G}_{12}$  and  $x_2$  is  $\tilde{G}_{22}$  and  $g_2$  is  $\tilde{G}_2^F$  THEN  $y$  is  $[0.1327 \ 0.2559]$ , and  $g_1$  is  $[0.9666 \ 1.1594]$ , and  $g_2$  is  $[0.9781 \ 0.9146]$ .

where  $\tilde{G}_{ij}$ ,  $i, j = 1, 2$ , are shown in [6].

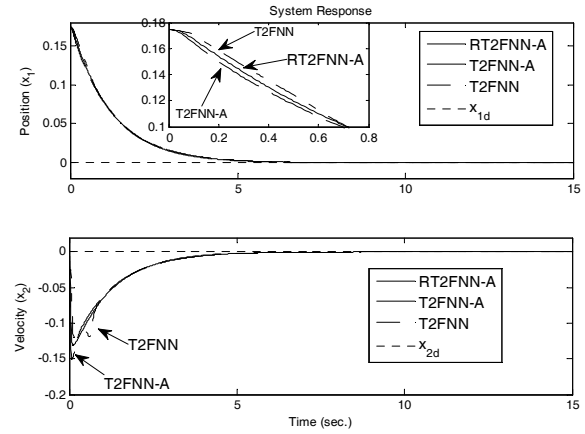


Figure 3: Simulation results in **Case 1** of inverted pendulum system: state trajectories (solid-line: RT2FNN-A, dashed-line: T2FNN-A, dash-dotted-line: T2FNN, and dotted line: reference trajectory).

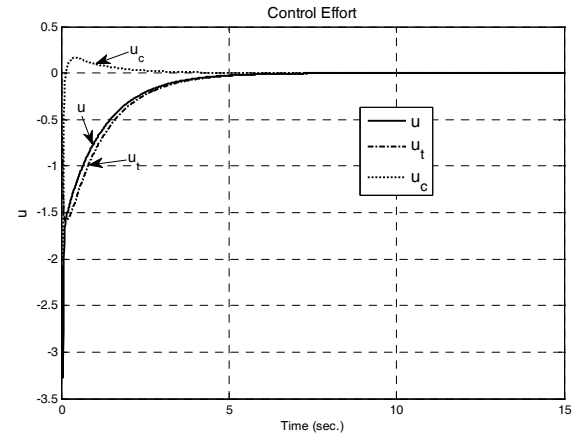


Figure 4: Control effort in **Case 1** of inverted pendulum system (solid-line: the total control force, dash-dotted-line: compensated controller, and dotted-line: RT2FNN-A controller).

#### V. CONCLUSIONS

In this paper, we have proposed a recurrent interval type-2 fuzzy neural network with asymmetric membership functions (RT2FNN-A) for solving the control problem of nonlinear systems with system uncertainty. The RT2FNN-A uses the interval asymmetric type-2 fuzzy sets implements the fuzzy logic system in a five-layer neural network structure which contains four layer forward network and a feedback layer. According to the Lyapunov theorem, the stability of the closed-loop system of nonlinear system is guaranteed and the adaptive laws of RT2FNN-A's parameters is derived. The performance of the proposed approach has been introduced by the illustration example.

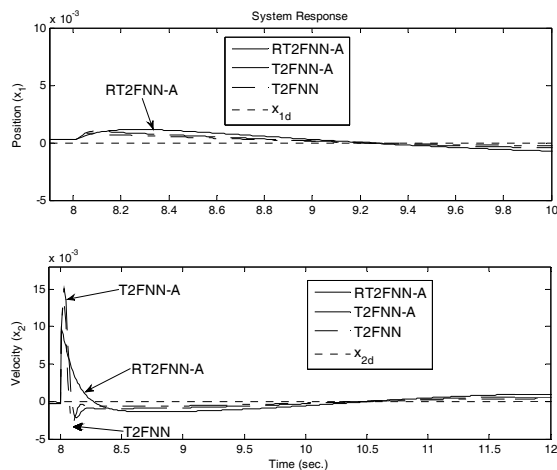


Figure 5: Simulation results in **Case 2** of inverted pendulum system in large sacle at 8-12 seconds: state trajectories (solid-line: RT2FNN-A, dashed-line: T2FNN-A, dash-dotted-line: T2FNN, and dotted-line: reference trajectory).

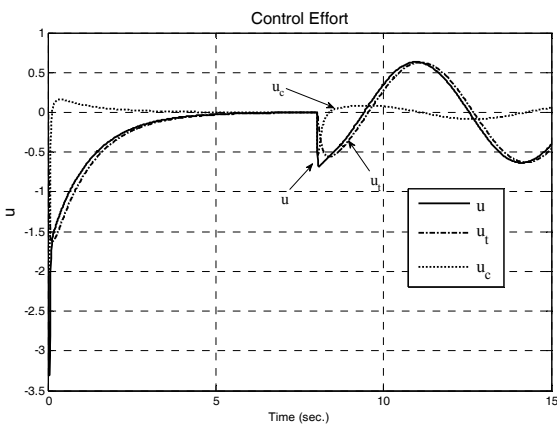


Figure 6: Control effort in **Case 2** of inverted pendulum system (solid-line: total control force, dash-dotted-line: compensated controller, and dotted-line: RT2FNN-A controller).

## REFERENCES

- [1] J. F. Baldwin and S. B. Karake, "Asymmetric Triangular Fuzzy Sets for Classification Models," *Lecture Notes in Artificial Intelligence*, Vol. 2773, pp. 364-370, 2003.
- [2] M. Casdagli, "A Dynamical Systems Approach to Modeling Input-Output Systems," in *Nonlinear Modeling and Forecasting, SFI Studies in the Sciences of Complexity Process*, Vol. 12, pp. 265-281, Addison-Wesley, New York, 1992.
- [3] G. Chen and X. Dong, "Identification and Control of Chaotic Systems: An Artificial Neural Network Approach," *IEEE International Sym. on Circuits and Systems*, Vol. 2, pp. 1177-1182, 1995.
- [4] G. Chen and X. Dong, "On Feedback Control of Chaotic Continuous-time Systems," *IEEE Trans. on Circuits and Systems I: Fundamental Theory and Applications*, Vol. 40, No. 9, pp. 591-601, 1993.
- [5] H. Hagsras, "Type-2 FLCs: A New Generation of Fuzzy Controllers," *IEEE Computational Intelligence Magazine*, Vol. 2, No. 1, pp. 30-43, 2007.
- [6] Z. W. Hu, Master Thesis, *The Study and Applications of Combining Interval Type-2 Fuzzy Asymmetric Membership Functions and Recurrent Neural Network System*, Yuan Ze University, Taiwan, R.O.C.
- [7] J. R. Jang, C. T. Sun, and E. Mizutani, *Neuro-fuzzy and Soft-computing*, Prentice-Hall, Upper Saddle River, NJ, 1997.
- [8] J. S. Jang, "ANFIS: Adaptive-network-based Fuzzy Inference System," *IEEE Trans. on Systems, Man, and Cybernetics*, Vol. 23, No. 3, pp. 665-685, 1993.
- [9] R. John and S. Coupland, "Geometric Type-1 and Type-2 Fuzzy Logic Systems," *IEEE Trans. on Fuzzy Systems, Special Issue on Type-2 Fuzzy Systems*, Vol. 15, No. 1, pp. 3-15, 2007.

- [10] C. F. Juang and C. T. Lin, "Noisy Speech Processing by Recurrently Adaptive Fuzzy Filters," *IEEE Trans. on Fuzzy Systems*, Vol. 9, No. 1, pp. 139-152, 2001.
- [11] C. F. Juang, "A TSK-Type Recurrent Fuzzy Network for Dynamic Systems Processing by Neural Network and Genetic Algorithms," *IEEE Trans. on Fuzzy Systems*, Vol. 10, No. 2, pp. 155-170, 2002.
- [12] N. N. Kamik, J. Mendel, and Q. Liang, "Type-2 Fuzzy Logic Systems," *IEEE Trans. on Fuzzy Systems*, Vol. 7, No. 6, pp. 643-658, 1999.
- [13] M. S. Kim, C. H. Kim, and J. J. Lee, "Evolutionary Optimization of Fuzzy Models with Asymmetric RBF Membership Functions Using Simplified Fitness Sharing," *Lecture Notes in Artificial Intelligence*, Vol. 2715, pp. 628-635, 2003.
- [14] C. H. Lee and H. Y. Pan, "Enhancing the Performance of Neural Fuzzy Systems Using Asymmetric Membership Functions," *Fuzzy Sets and Systems*, vol. 160, issue, 7, pp. 949-971, 2009.
- [15] C. H. Lee and Y. C. Lin, "An Adaptive Type-2 Fuzzy Neural Controller for Nonlinear Uncertain Systems," *Int. J. of Control and Intelligent*, Vol. 12, No. 1, pp. 41-50, 2005.
- [16] C. H. Lee and Y. C. Lin, "Robust Adaptive Control for Nonlinear Uncertain Systems Using Type-2 Fuzzy Neural Network System," *Mathematical Problems in Engineering*, 2011.
- [17] C. H. Lee and W. Y. Lai, "Nonlinear Control of Benchmark Problems Using TSK-Type Fuzzy Neural System," Accepted to appear in *Neural Computing and Applications*, Oct. 2012.
- [18] C. H. Lee and C. C. Teng, "Identification and Control of Dynamic Systems Using Recurrent Fuzzy Neural Networks," *IEEE Trans. on Fuzzy Systems*, Vol. 8, No. 4, pp. 349-366, 2000.
- [19] C. Li, K. H. Cheng, and J. D. Lee, "Hybrid Learning Neuro-fuzzy Approach for Complex Modeling Using Asymmetric Fuzzy Sets," *Proc. of the 17th IEEE Int. Conf. on Tools with Artificial Intelligence*, pp. 397-401, 2005.
- [20] C. Li and C. Y. Lee, "Self-organizing Neuro-fuzzy System for Control of Unknown Plants," *IEEE Trans. on Fuzzy Systems*, Vol. 11, No. 1, pp. 135-150, 2003.
- [21] A. Q. Liang and J. M. Mendel, "Interval Type-2 Fuzzy Logic Systems: Theory and Design," *IEEE Trans. on Fuzzy Systems*, Vol. 8, No. 5, pp. 535-550, 2000.
- [22] C. J. Lin and W. H. Ho, "An Asymmetric-similarity-measure-based Neural Fuzzy Inference System," *Fuzzy Sets and Systems*, Vol. 152, pp. 535-551, 2005.
- [23] C. T. Lin and C. S. G. Lee, *Neural Fuzzy Systems: A Neuro-fuzzy Synergism to Intelligent Systems*, Prentice-Hall, Englewood Cliffs, 1996.
- [24] P. Z. Lin and T. T. Lee, "Robust Self-organizing Fuzzy-neural Control Using Asymmetric Gaussian Membership Functions," *International Journal of Fuzzy Systems*, Vol. 9, No. 2, pp. 77-86, 2007.
- [25] A. Lotfi and A. C. Tsoi, "Learning Fuzzy Inference Systems Using An Adaptive Membership Function Scheme," *IEEE Trans. on Systems, Man, and Cybernetics, Part-B*, Vol. 26, No. 2, pp. 326-331, 1996.
- [26] J. M. Mendel, *Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New Directions*, Upper Saddle River, Prentice-Hall, NJ, 2001.
- [27] M. Mizumoto and K. Tanaka, "Some Properties of Fuzzy Sets of Type 2," *Information and Control*, Vol. 31, No. 4, pp. 312-340, 1976.
- [28] T. Ozen and J. M. Garibaldi, "Effect of Type-2 Fuzzy Membership Function Shape on Modeling Variation in Human Decision Making," *IEEE International Conf. on Fuzzy Systems*, Vol. 2, pp. 971-976, 2004.
- [29] S. Postalcioglu and Y. Becerikli, "Wavelet Networks for Nonlinear System Modeling," *Neural Computing and Applications*, Vol. 16, No. 4-5, pp. 433-441, 2007.
- [30] J. E. Slotine and W. Li, *Applied Nonlinear Control*, Prentice-Hall, Englewood Cliffs, NJ, 1991.
- [31] D. G. Stavrakoudis and J. B. Theocharis, "Pipelined Recurrent Fuzzy Neural Networks for Nonlinear Adaptive Speech Prediction," *IEEE Trans. on Systems, Man, Cybernetics Part-B*, Vol. 37, No. 5, pp. 1305-1320, 2007.
- [32] L. X. Wang, *Adaptive Fuzzy Systems and Control: Design and Stability Analysis*, Prentice-Hall, Englewood Cliffs, NJ, 1994.
- [33] R. J. Williams and D. Zipser, "A Learning Algorithm for Continually Running Recurrent Neural Networks," *Neural Computation*, Vol. 1, No. 2, pp. 270-280, 1989.
- [34] L. A. Zadeh, "The Concept of A Linguistic Variable and Its Application to Approximate Reasoning," *Information Sciences*, Vol. 8, No. 3, pp. 199-249, 1975.