Intel-22nm Squelch Yield Analysis and Optimization

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Abstract— **It's always been a tough problem to make conservative estimate of yield due to limited silicon test samples. Besides, lack of understanding of relationship between yield and design parameters gives low confidence to designer. This paper gives rigorous mathematical treatment to the subject of yield analysis and optimization. It outlines the approach for conservative estimate of yield even for smaller sample size, n < 25. It bridges the gap between our subjective knowledge to objective conclusions. Finally it analyses Intel-22nm USB2 Squelch circuit for yield and sets yield optimization guidelines.**

Keywords **—Probability, Statistics, DPM, Estimation, Normal density, Chi-square density, Student t density.**

I. Introduction

Probability and Statistics is a separate discipline of mathematics which is close to natural phenomena's of this real world. Statistical methods are used to test scientific theories, hypothesis and to determine physical constants. It's used to solve numerically a variety of deterministic problems with the help of computers. For example, value of a constant π is determined by a well known random experiment "Buffon's needle". It plays major role in the improvement of quality of product or service. This paper discusses the statistics of estimation of unknown parameters from the known distribution and observations. It analyses the yield of Intel 22nm Squelch circuit and discusses optimization flow for better yield result. The commonly used measure to quantify yield is "defects per million" (a.k.a dpm) [1]. This results due to manufacturing variability [2] [3]. In the yield analysis we try to find dpm number based on observations from silicon and known distribution model. On the other side, in optimization problem we replace our silicon observations with observations from Monte Carlo simulations and try to find out the required standard deviation to meet target dpm number. This requires few design iterations to converge to the final solution. To prepare reader towards this goal we start with statistics revision in sections II-IV. Equipped with this knowledge we proceed for final goal of yield analysis and optimization of Squelch circuit as test case.

This paper is organised in 6 sections. Section II discusses typical distributions widely used in probability and statistics [4]. Section III defines sample mean and variance of observation vector of random variables. Section IV discusses estimation of unknown parameters of model, especially mean and variance under different given constraints. Section V analyses yield of Squelch circuit and sets guidelines for optimization. Finally section VI concludes this work.

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II. Typical Distributions

A. Normal distribution

Normal/Gaussian distribution is one of the most important distributions in statistics because various natural phenomena's follow this distribution. In fact normal distribution arises as an outcome of central limit theorem which states that under certain general conditions the limiting distribution of the average of large number of independent identically distributed random variables is normal. This distribution has the form,

$$e^{-x^2/2\sigma^2} \tag{1}$$

This function being continuous area under this curve is given by Riemann integral,

$$A = \int_{-\infty}^{\infty} e^{-x^2/2\sigma^2}\, dx \tag{2}$$

Therefore,

$$A^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)/2\sigma^2}\, dx\, dy \tag{3}$$

Changing the domain of the integration from Cartesian co-ordinate to polar co-ordinate system by change of variables,

$x = r\cos\theta$, $y = r\sin\theta$

=> Jacobian,

$$|J(r,\theta)| = \left\| \begin{matrix} \partial x/\partial r & \partial x/\partial \theta \\ \partial y/\partial r & \partial y/\partial \theta \end{matrix} \right\| = r \tag{4}$$

and

$$\begin{aligned} A^2 &= \int_0^{2\pi}\int_0^{\infty} e^{-r^2/2\sigma^2}\, r\, dr\, d\theta \\ &= 2\pi\sigma^2 \int_0^{\infty} e^{-u}\, du = 2\pi\sigma^2 \end{aligned} \tag{5}$$

Therefore area under function,

$$e^{-x^2/2\sigma^2} \quad \text{is} \quad \sqrt{2\pi\sigma^2}$$

One of the 3-axioms of probability states that the probability of any event lies between 0 and 1. i.e. $0 \le P(A) \le 1$. Hence normalizing the distribution to maintain area under curve to unity we get, Normal density function,

$$f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\eta)^2/2\sigma^2} \tag{6}$$

with the parameters of distribution as mean $=\eta$ and variance $=\sigma^2$ as shown in fig.1. Here the parameter mean is a measure of central tendency and variance is a measure of dispersion from mean. The corresponding distribution function is given by,

$$F_x(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(y-\eta)^2/2\sigma^2} dy \tag{7}$$

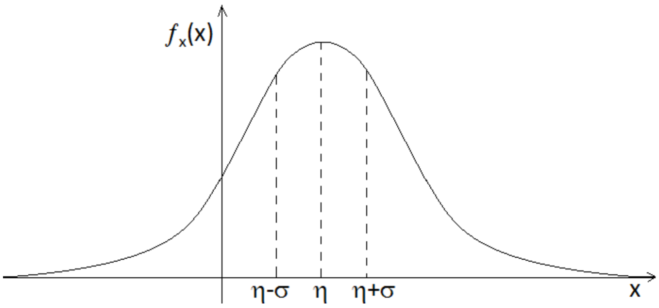

Fig. 1. Normal probability density (pdf) function

A special case of normal distribution is standard normal distribution denoted as z ~N(0,1) where $\eta=0$ and $\sigma^2=1$. Here standard normal variable can be formed as

$$z = \frac{(x-\eta)}{\sigma}$$

=>

$$F_z(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy \triangleq G(z) = G\left(\frac{(x-\eta)}{\sigma}\right) \tag{8}$$

Where the function,

$$G(z) \triangleq \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy \tag{9}$$

can be rewritten as,

$$G(z) \triangleq \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy = \int_{0}^{z} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy + \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy \tag{10}$$

Therefore,

$$G(z) = erf(z) + \frac{1}{2} \tag{11}$$

Generally the error function erf*f*(z) is available in tabulated form as shown in fig. 2 and hence G(z) can be evaluated by use of (11). First column of this tabulated form represents z and first row its hundredth digit. The intersection of column

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9958	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

Fig. 2. Error function in tabulated form

and row represents the value of erf*f*(z). For example z = 2.15 => erf*f*(z) = erf*f*(2.15) = 0.4842. The error function er*f*(z) and normal probability density function $f_z(z)$ plots are as shown in fig.3.

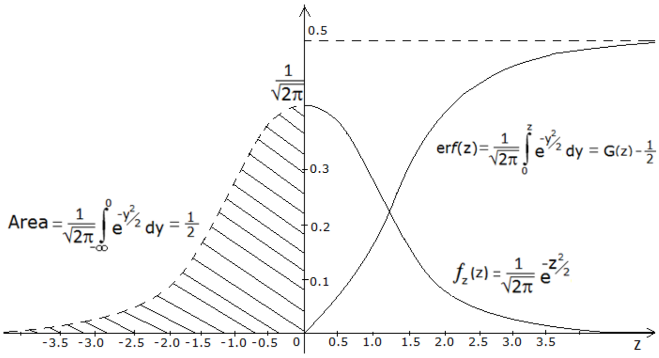

Fig. 3. Normal pdf and error function plots

Percentiles:

The u percentile of random variable z is the smallest number z_u such that u = p{z ≤ z_u }= $F_z(z_u)$. Thus z_u is the inverse of the function u = F(z). It's domain is the interval 0 ≤ u ≤ 1 and it's range is horizontal-axis. Thus inverse function, $F_z^{-1}(u) = z_u$ can be found by interchanging horizontal and vertical axis. Here z_u is called the u percentile of the standard normal density.

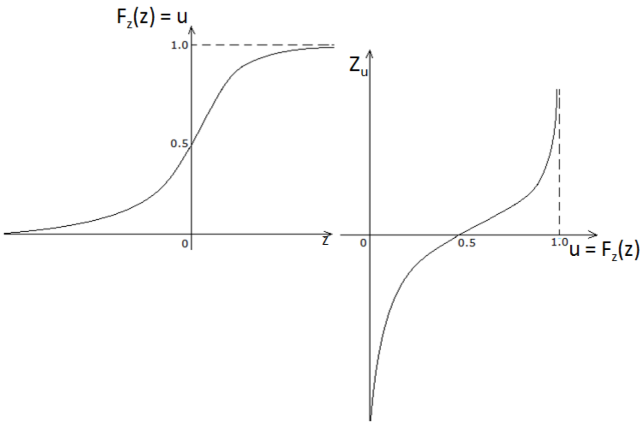

Fig. 4. Normal distribution and its inverse function

Thus $z_u = -z_{1-u}$ and $G(-z_{1-u}) = G(z_u) = u$. The normal distribution function and it's inverse plots are as shown in fig. 4.

B. Chi-square distribution

The random variable x is said to be $\chi^2(n)$ with n degrees of freedom if

$$f_x(x) = \begin{cases} \dfrac{x^{n/2-1}}{2^{n/2}\,\Gamma(n/2)} e^{-x/2} & x \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

The graphs of this $\chi^2(n)$ density function for various values of 'n' are as shown in fig. 5.

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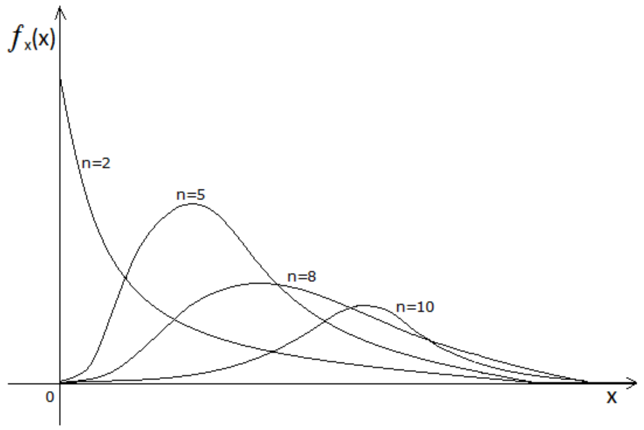

Fig. 5. Chi-square density function for various degrees of freedom

	P										
DF	0.995	0.975	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.002	0.001
25	10.520	13.120	30.675	34.382	37.652	40.646	41.566	44.314	46.928	50.223	52.620
26	11.160	13.844	31.795	35.563	38.885	41.923	42.856	45.642	48.290	51.627	54.052
27	11.808	14.573	32.912	36.741	40.113	43.195	44.140	46.963	49.645	53.023	55.476
28	12.461	15.308	34.027	37.916	41.337	44.461	45.419	48.278	50.993	54.411	56.892
29	13.121	16.047	35.139	39.087	42.557	45.722	46.693	49.588	52.336	55.792	58.301
30	13.787	16.791	36.250	40.256	43.773	46.979	47.962	50.892	53.672	57.167	59.703

Fig. 6. Chi-square density function in tabulated form

This density function is also available in tabulated form as shown in fig. 6. Here degree of freedom (DF) can be read from first column and probability of random variable (P) is from second row. For example $P(x > 40.646) = 0.025$ for DF = 25.

C. Student t distribution

A random variable z has student t distribution t(n) with n degrees of freedom if $-\infty < z < \infty$ and

$$f_z(z) = \frac{\gamma_1}{\sqrt{(1+z^2/n)^{n+1}}} \quad \text{where} \quad \gamma_1 = \frac{\Gamma((n+1)/2)}{\sqrt{\pi n}\;\Gamma(n/2)} \tag{13}$$

This random variable z has student t distribution such that

$$z = \frac{x}{\sqrt{y/n}}$$

where x and y are two independent random variables, x is N(0,1) and y is $\chi^2(n)$.

$$f_x(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \quad \text{and} \quad f_y(y) = \frac{1}{2^{n/2}\Gamma(n/2)} y^{n/2-1} e^{-y/2} U(y) \tag{14}$$

i.e. student t distribution represents the ratio of a normal random variable to the square root of an independent χ^2 random variable divided by its degree of freedom. This density function for various values of 'n' is s shown in fig.7 whereby t(n) distribution approaches to normal as $n \to \infty$. This pdf is also available in tabulated form as shown in fig. 8. Here degree of freedom, DF = n-1 where n = sample size. 'A' represents the probability that the random variable lies between –t and t whereas 'P' represents the probability that it lies outside –t and t range. For example $A = P(-2.063 < x < 2.063) = 0.95$ and $P = P(-2.063 > x > 2.063) = 0.05$.

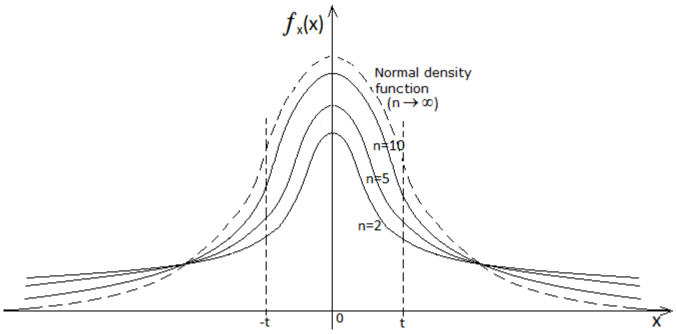

Fig. 7. Student t density function for various degrees of freedom

DF	A P	0.80 0.20	0.90 0.10	0.95 0.05	0.98 0.02	0.99 0.01	0.995 0.005	0.998 0.002	0.999 0.001
25		1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26		1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27		1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28		1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29		1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30		1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646

Fig. 8. Student t density function for various degrees of freedom

III. Sample Mean & Variance

The sample mean and the sample variance of random variable x_i are given by,

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \quad \text{and} \quad s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \tag{15}$$

If random variables x_i are uncorrelated with the same mean $E\{x_i\} = \eta$ and variance $\sigma_i = \sigma^2$ the mean of $\bar{x}$ is

$$E\{\bar{x}\} = \frac{1}{n}\sum_{i=1}^{n} E\{x_i\} = \eta \tag{16}$$

and variance,

$$E\{(\bar{x}-\eta)^2\} = E\{(\bar{x}-\eta)(\bar{x}-\eta)\} \tag{17}$$

$$= E\{[(x_1+x_2+ \cdots + x_n)/n - \eta].[(x_1+x_2+ \cdots + x_n)/n - \eta]\} \tag{18}$$

Since uncorrelatedness property of random variables, x_i leads to covariance coefficient,

$$C_{oxy} = E\{(x_i - \eta)\cdot(x - \eta)\} = 0 \tag{19}$$

Therefore we get

$$E\{(\bar{x}-\eta)^2\} = \frac{1}{n^2}\,[E\{(x_1-\eta)^2\} + E\{(x_2-\eta)^2\} + \cdots + E\{(x_n-\eta)^2\}] \tag{20}$$

and
$$\hat{\sigma}_x^2 = \sigma^2/n \tag{21}$$

Thus $\bar{x}$ is a normal random variable with parameters, η and $\sigma/\sqrt{n}$ represented as $\bar{x} \sim N(\eta, \sigma/\sqrt{n})$.

Similarly the expected value of s^2 is given by,

$$E\{s^2\} = \frac{1}{n-1}\sum_{i=1}^{n} E\{(x_i - \bar{x})^2\} \tag{22}$$

$$= \frac{1}{n-1}\sum_{i=1}^{n} E\{[(x_i - \eta) - (\bar{x} - \eta)]^2\} \tag{23}$$

Where

$$E\{(x_i-\eta)(\bar{x}-\eta)\} = \frac{1}{n}\, E\{(x_i - \eta)[(x_1-\eta)+(x_2-\eta)+ \cdots +(x_n-\eta)]\} \tag{24}$$

$$= \frac{1}{n} E\{(x_i - \eta)(x_j - \eta)\} = \sigma^2/n \tag{25}$$

Therefore,

$$E\{s^2\} = \frac{1}{n-1} \sum_{i=1}^{n} (\sigma^2 - 2\sigma^2/n + \sigma^2/n) \tag{26}$$

$$E\{s^2\} = \sigma^2 \tag{27}$$

For given n i.i.d $N(\eta,\sigma)$ random variables x_i the unbiased sample variance is given by,

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \tag{28}$$

Where newly formed random variable from (28) is

$$\frac{(n-1)s^2}{\sigma^2} = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{\sigma^2} \tag{29}$$

This is a $\chi^2(n\text{-}1)$ random variable. This can be shown as follows,

$$(x_i - \eta)^2 = (x_i - \bar{x} + \bar{x} - \eta)^2 = (x_i - \bar{x})^2 + (\bar{x} - \eta)^2 + 2(x_i - \bar{x})(\bar{x} - \eta) \tag{30}$$

Summing this identity from 1 to n yields,

$$\sum_{i=1}^{n} \frac{(x_i - \eta)^2}{\sigma^2} = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{\sigma^2} + \frac{(\bar{x} - \eta)^2}{\sigma^2} \quad \text{since} \sum_{i=1}^{n} (x_i - \bar{x}) = 0 \tag{31}$$

Therefore,

$$\sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{\sigma^2} = \sum_{i=1}^{n} \frac{(x_i - \eta)^2}{\sigma^2} - \left(\frac{\bar{x} - \eta}{\sigma/\sqrt{n}}\right)^2 \tag{32}$$

For random variable $\bar{X}$ with normal density function, $N(\eta,\sigma/\sqrt{n})$. Let's form other random variables,

$$y = \left(\frac{\bar{x} - \eta}{\sigma/\sqrt{n}}\right)^2 \text{ and } x' = \left(\frac{\bar{x} - \eta}{\sigma/\sqrt{n}}\right) \tag{33}$$

Hence x'~N(0,1), standard normal variable and

$$y = \left(\frac{\bar{x} - \eta}{\sigma/\sqrt{n}}\right)^2 = (x')^2 \tag{34}$$

Therefore,

$$f_y(y) = \frac{1}{2\sqrt{y}} [f_{x'}(\sqrt{y}) + f_{x'}(-\sqrt{y})] \; ; \; y > 0 \tag{35}$$

$$= \frac{f_{x'}(\sqrt{y})}{\sqrt{y}} \tag{36}$$

where $f_{x'}(x')$ being symmetrical density function,

$$f_y(y) = \frac{\frac{1}{\sqrt{2\pi}} e^{-x'^2/2}}{\sqrt{y}} = \frac{1}{\sqrt{2\pi y}} e^{-y/2} \tag{37}$$

Further implies $f_y(y)$ is Chi-square density with n=1 from (12). Therefore random variable

$$\left(\frac{\bar{x} - \eta}{\sigma/\sqrt{n}}\right) \text{ is } \chi^2(1) \tag{38}$$

For two independent random variables with Chi-square distributions, $\chi^2(m)$ and $\chi^2(n)$ respectively, the sum also results in $\chi^2(m+n)$ random variable. Hence the random variable,

$$\sum_{i=1}^{n} \left(\frac{x_i - \eta}{\sigma}\right)^2 \text{ is } \chi^2(n) \tag{39}$$

and hence it proves from (12), (38) and (39) that

$$\frac{(n-1)s^2}{\sigma^2} = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{\sigma^2} \text{ is } \chi^2(n-1) \tag{40}$$

IV. Estimation

The problem of estimation is very fundamental in the application of probability where main idea is estimation of a random variable y in terms of another random variable x and the optimality criteria is to minimize the mean square (MS) value of the estimation error. Estimation approaches are classified as classical and baysian and we'll focus on classical approach here. For any random variable we can find point estimate or interval estimate.

A point estimate is a function $\hat{\theta} = g(x)$ of the observation vector $X = [x_1, x_2, x_3, \ldots, x_n]$. The corresponding random variable $\hat{\theta} = g(x)$ is the point estimator of θ. An interval estimate of a parameter θ is an interval (θ_1, θ_2), the end points of which are functions $\theta_1 = g_1(x)$ and $\theta_2 = g_2(x)$ of the observation vector X. The corresponding random interval (θ_1, θ_2) is the interval estimator of θ. We say that (θ_1, θ_2) is a γ confidence interval of

$$\theta \text{ if } P\{\theta_1 < \theta < \theta_2\} = \gamma \tag{41}$$

The constant γ is the confidence coefficient of the estimate and the difference $\delta = 1-\gamma$ is the confidence level. Thus γ is a subjective measure of our confidence that the unknown θ is in the interval (θ_1, θ_2). If γ is close to 1 we can expect with near certainty that estimate is true. Our estimate is correct in 100γ percent of the cases. The objective of interval estimation is the determination of the functions $g_1(x)$ and $g_2(x)$ so as to minimize the length $\theta_2 - \theta_1$ of the interval (θ_1, θ_2) subject to constraint 41. However choice of the confidence coefficient γ is dictated by two conflicting requirements. If γ is close to 1, the estimation is reliable but the interval (θ_1, θ_2) size is large. If γ is reduced, interval size is small but estimate becomes less reliable. Thus final choice of γ is a compromise and is based on the application.

A. Mean

Let's estimate the mean η of the random variable X. The point estimate of η is the value

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \tag{42}$$

of the sample mean $\bar{X}$ of x. An interval estimate of η is difficult problem since $\bar{X}$ is sum of random variables X_i hence its resulting density function involves multiple convolutions. To simplify this problem we assume that $\bar{X}$ is normal. This assumption is true if x is normal and approximately true for any x with arbitrary distribution if n is large. This follows from central limit theorem. Here we estimate the interval under two scenarios.

i. Known variance

Suppose the variance σ^2 of x is known. The normality assumption leads to the conclusion that the point estimator $\bar{X}$ of η is $N(\eta, \sigma/\sqrt{n})$. Therefore we conclude that

$$p\{\eta - z_{1-\delta/2}\sigma/\sqrt{n} < \bar{x} < \eta + z_{1-\delta/2}\sigma/\sqrt{n}\} = G(z_{1-\delta/2}) - G(-z_{1-\delta/2}) = 1-\delta/2-\delta/2 \quad (43)$$

But $Z_u = -Z_{1-u}$ and $G(-Z_{1-u}) = G(Z_u) = u$ where $u = \delta/2$, This yields

$$p\{\bar{x} - z_{1-\delta/2}\sigma/\sqrt{n} < \eta < \bar{x} + z_{1-\delta/2}\sigma/\sqrt{n}\} = 1-\delta = \gamma \quad (44)$$

We can thus state with confidence coefficient γ that η is in the interval $\bar{X} \pm Z_{1-\delta/2}\ \sigma/\sqrt{n}$.

ii. Unknown variance

If σ^2 is unknown, we can not use (44). To estimate η, we form the sample variance

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \quad (45)$$

This is an unbiased estimate of σ^2 as $n \to \infty$. Hence for large n, we can use the approximation $s \cong \sigma$ in (44). This yields to the approximate confidence interval

$$\bar{x} - z_{1-\delta/2}\, s/\sqrt{n} < \eta < \bar{x} + z_{1-\delta/2}\, s/\sqrt{n} \quad (46)$$

The exact confidence interval under the normality assumption of x can be found forming random variable

$$\frac{\bar{x}-\eta}{s/\sqrt{n}}$$

Where

$$\frac{\bar{x}-\eta}{s/\sqrt{n}} = \frac{\left(\frac{\bar{x}-\eta}{\sigma/\sqrt{n}}\right)}{\sqrt{\frac{s^2}{n}\cdot\frac{n}{\sigma^2}}} = \frac{\left(\frac{\bar{x}-\eta}{\sigma/\sqrt{n}}\right)}{\sqrt{\frac{n-1}{\sigma^2}s^2\cdot\frac{1}{n-1}}} = \frac{z}{\sqrt{w/(n-1)}} \quad (47)$$

Where

$$z = \left(\frac{\bar{x}-\eta}{\sigma/\sqrt{n}}\right)$$

is standard normal random variable, $Z \sim N(0,1)$ and

$$w = \frac{(n-1)s^2}{\sigma^2}$$

is $\chi^2(n-1)$ from (29)-(40). Therefore from (13) and (14) the random variable

$$\frac{\bar{x}-\eta}{s/\sqrt{n}}$$

has a Student t distribution with (n-1) degrees of freedom. Denoting by t_u its u percentile, we conclude

$$p\left\{-t_u < \frac{x-\eta}{s/\sqrt{n}} < t_u\right\} = 2u - 1 = \gamma \quad (48)$$

This yields the exact confidence interval to be [3]

$$\bar{x} - t_{1-\delta/2}\, s/\sqrt{n} < \eta < \bar{x} + t_{1-\delta/2}\, s/\sqrt{n} \quad (49)$$

B. Variance

Let's now find point estimate and interval estimate of variance of normal random variable x in terms of the n-samples x_i of x. This estimate again for two cases as described below.

i. Known mean

First we assume η of x is known and we use as the point estimate of v the average

$$\hat{v} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \eta)^2 \quad (50)$$

Therefore

$$E\{\hat{v}\} = \frac{1}{n}\sum_{i=1}^{n}E\{(x_i - \eta)^2\} \quad (51)$$

$$E\{\hat{v}\} = \sigma^2 = v \quad (52)$$

Since
$$\sigma_{\hat{v}}^2 = 2\sigma^4/n \xrightarrow{n\to\infty} 0 \quad (53)$$

where $\bar{v}$ is a consistent estimator of σ^2. Next to find interval estimate we form random variable

$$\frac{n\hat{v}}{\sigma^2} = \sum_{i=1}^{n}\frac{(x_i-\eta)^2}{\sigma^2} \quad (54)$$

which from (51) is a $\chi^2(n)$ density. This density is not symmetrical hence the interval estimate of σ^2 is not centered at σ^2. To determine it, we introduce two constants c_1 and c_2 such that

$$p\left\{\frac{n\hat{v}}{\sigma^2} < c_1\right\} = \delta/2 \text{ and } p\left\{\frac{n\hat{v}}{\sigma^2} > c_2\right\} = \delta/2 \quad (55)$$

as shown in fig.9.

Therefore

$$p\left\{c_1 < \frac{n\hat{v}}{\sigma^2} < c_2\right\} = 1-\delta = \gamma \quad \text{implies} \quad (56)$$

$$p\left\{\frac{\chi_{\delta/2}(n)}{n\hat{v}} < \frac{1}{\sigma^2} < \frac{\chi_{1-\delta/2}(n)}{n\hat{v}}\right\} = \gamma \quad (57)$$

Where c_1 is $\chi^2_{\delta/2}(n)$ and c_2 is $\chi^2_{1-\delta/2}(n)$.

Therefore

$$p\left\{\frac{n\hat{v}}{\chi_{1-\delta/2}(n)} < \sigma^2 < \frac{n\hat{v}}{\chi_{\delta/2}(n)}\right\} = \gamma \quad (58)$$

Hence for chosen confidence γ the interval estimate is

$$\frac{n\hat{v}}{\chi_{1-\delta/2}(n)} < \sigma^2 < \frac{n\hat{v}}{\chi_{\delta/2}(n)} \quad (59)$$

ii. Unknown mean

If η is unknown, we use as the point estimate of σ^2 the sample variance s^2. The random variable $(n-1)s^2/\sigma^2$ has $\chi^2(n-1)$ distribution. Hence

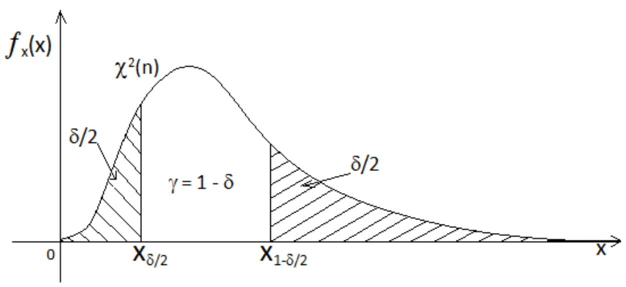

Fig. 9. Chi-square density function.

$$p\left\{\chi^2_{\delta/2}(n-1) < \frac{(n-1)s^2}{\sigma^2} < \chi^2_{1-\delta/2}(n-1)\right\} = \gamma \quad (60)$$

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This yields the interval estimate to be [3]

$$\frac{(n-1)s^2}{\chi^2_{1-\delta/2}(n-1)} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{\delta/2}(n-1)} \tag{61}$$

V. Yield Analysis & Optimization

This section analyses Intel 22nm USB2 Squelch circuit for yield and sets guidelines for optimization. The interest here is not the details of squelch circuit architecture but statistical analysis over its trip point variation. Hence circuit details are omitted focusing only on statistical analysis of trip point. We have performed lab measurement of Squelch trip point on 3999 silicon samples available. First we will be analyzing for very limited number of silicon samples, n = 25 based upon approach outlined in sections I through IV. Later we will compare these statistical results with results produced by JMP statistical analysis software using large sample size (n=3999). In the first approach sample size is small and conservative results are obtained due to accurate statistical approach. Fig. 10 a) shows squelch trip point data for 25 samples and b) shows yield analysis for given dpm target of 25 with confidence coefficient of 0.95 which is very common and widely used. The purpose of this analysis is to find upper specification limit (USL) and

Sr. No.	1	2	3	4	5	6	7	8
Xi	0.106	0.092	0.112	0.106	0.1	0.11	0.108	0.118

9	10	11	12	13	14	15	16	17
0.098	0.11	0.104	0.108	0.108	0.104	0.122	0.112	0.106

18	19	20	21	22	23	24	25
0.116	0.106	0.112	0.104	0.108	0.096	0.112	0.1

a)

dpm target	25	
sample size, n	25	
confidence coefficient, γ	0.95	
confidence level, δ	0.05	Eq.(56)
mean, $\bar{x}$	0.10712 V	Eq.(42)
variance, s^2	4.6026E-05 V^2	Eq.(45)
SD, s	0.00678429 V	
η_u	0.10992042 V	Eq.(49)
η_l	0.10431958 V	Eq.(49)
σ_u^2	8.9075E-05 V^2	Eq.(61)
σ_u	0.00943798 V	
student t score	2.063898547	Student t-table from fig. 8
chi-square χ score	12.40115026	Chi-Square table from fig. 6
dpm z score	4.055626981	Normal table from fig. 2
Upper Specification Limit	0.14819737 V	$USL=\eta_u+Z\cdot\sigma_u$
Lower Specification Limit	0.06604262 V	$LSL=\eta_l-Z\cdot\sigma_u$

b)

Fig. 10. a) Squelch trip point data for 25 samples b) Yield analysis for Squelch circuit.

lower specification limit (LSL) to meet target dpm count of 25, at confidence coefficient of 0.95. Further to check USL and LSL against respective USB2 Squelch maximum and minimum trip point specifications. The analysis is as follows. Target dpm = 25, Number of samples, n=25. Statistical results expected within confidence coefficient of 0.95. Therefore γ = 0.95. From (56), this gives confidence level, $\delta=1-\gamma$ = (1-0.95) = 0.05.

From (42), along with sample data in fig. 10 a) sample mean is given by,

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Therefore

$$\bar{x} = \frac{1}{25}\sum_{i=1}^{25} x_i = 107.12\text{mV}$$

Similarly from (45), along with sample data in fig. 10 a) sample variance, s^2 is given by,

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Therefore

$$s^2 = \frac{1}{(25-1)}\sum_{i=1}^{25}(x_i - 107.12\text{mV})^2 = 4.6\text{x}10^{-5}\,V^2$$

and standard deviation, SD is given by,

$$s = \sqrt{s^2} = 6.78\text{mV}$$

Student t score is found from standard student t table in fig. 8.

$$t_{1-\delta/2} = t_{(1-0.05/2)} = t_{0.975} = 2.064$$

From this, estimate of upper interval limit of mean is calculated using data in fig. 10 a). We do not know the true statistical variance hence we estimate for mean interval as per section IV→ A→ ii→ (49) as follows.

$$\eta_u = \bar{x} + t_{1-\delta/2}\, s/\sqrt{n} = 107.12\text{mV} + 2.064(6.78\text{mV}/\sqrt{25}) = 109.92\text{mV}$$

Similarly estimate of lower interval limit of mean is given by,

$$\eta_l = \bar{x} - t_{1-\delta/2}\, s/\sqrt{n} = 107.12\text{mV} - 2.064(6.78\text{mV}/\sqrt{25}) = 104.32\text{mV}$$

Now, Chi-square score can be calculated from table in fig. 6.

$$\chi^2_{1-\delta/2}(n-1) = \chi^2_{(1-0.05/2)}(25-1) = \chi^2_{0.975}(24) = 12.4$$

From this, we estimate upper interval limit of variance. Here true statistical mean is not known hence we use section IV→ B→ ii→(61) for its estimation.

$$\sigma_u^2 = \frac{(n-1)s^2}{\chi^2_{\delta/2}(n-1)} = \frac{(25-1)\text{x}4.6\text{x}10^{-5}}{12.4} = 8.9\text{x}10^{-5}\,V^2$$

Therefore estimate of upper interval limit of standard deviation is given by,

$$\sigma_u = \sqrt{\sigma_u^2} = 9.44\text{mV}$$

From central limit theorem expected distribution for squelch trip point variation is Gaussian as shown in fig. 11. This is proved by silicon results. Hence we use z score for calculation of USL & LSL. For given dpm target of 25, dpm z score can be calculated from table in fig. 2.

Therefore Z{1-probability of defect density}

$Z\{1-(25/1\text{x}10^6)\}$ = 4.055

Now estimated upper specification limit of trip point is

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$USL = \eta_u + Z \cdot \sigma_u = 109.92mV + 4.055(9.44mV) = 148.2mV$

and estimated lower specification limit of trip point is

$LSL = \eta_l - Z \cdot \sigma_u = 104.32mV - 4.055(9.44mV) = 66mV$

USB2 standard specifies squelch trip point maximum as 150mV and minimum as 100mV. The center of this range can be calculated as (150mV+100mV)/2 = 125mV. From calculated USL & LSL it's found that the lower side specification is violated. This situation arises because trip point design of 107.12mV (mean) is not at the center of specification range. For trip point design of 107.12mV, back calculation to meet LSL = 100mV gives σ_u = 1mV.

$LSL = 100mV = \eta_l - Z \cdot \sigma_u = 104.32mV - 4.055 \cdot \sigma_u \Rightarrow \sigma_u = 1mV$.

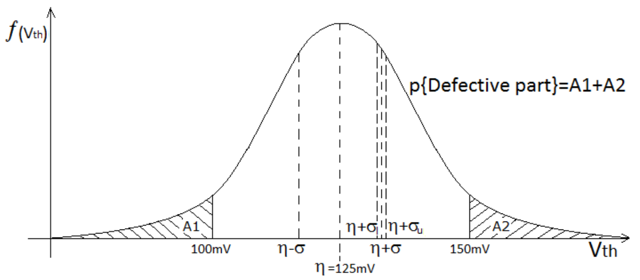

Fig. 11. Squelch circuit trip point Pdf.

This is very stringent requirement to meet. Solution to this issue is to adjust trip point at the center of the specification range i.e. 125mV and optimize design for standard deviation of ~4mV.

Therefore assuming $\eta_u \approx 128mV$ and $\eta_l \approx 122mV$ gives

$USL = \eta_u + Z \cdot \sigma_u = 128mV + 4.055(4mV) = 144mV$ (meeting spec)

$LSL = \eta_l - Z \cdot \sigma_u = 122mV - 4.055(4mV) = 106mV$ (meeting spec)

Yield optimization steps in the design phase of squelch circuit are as shown in fig. 13. In general this approach can be adopted for any circuit design to meet yield requirement.

Fig. 12 shows statistical simulation results calculated using JMP statistical analysis software. These results are obtained with very large sample size of 3999. There is a close match of mean and standard deviation provided by this JMP software with respective mean and standard deviation produced by our analysis using limited sample size. This proves that conservative statistical results can be obtained with limited sample size by following proposed approach.

Six-Sigma quality: On the 6-σ quality of this design [5], we calculate process capability, process potential index, and process capability index from the statistical results in fig. 12.

Process capability = 6-σ = 6 x 6.294 mV = 37.76mV

Process potential index, $C_p = (USL - LSL)/ 6\sigma = (150mV - 100mV)/37.76mV = 1.32$

Process capability index, $C_{pk} = C_p(1-k) = 0.5$

Where $k = |T-\mu|/0.5(USL-LSL) = |125mV - 109.52mV|/0.5x(150mV - 100mV) = 0.62$ and

T = Specification target = 125mV

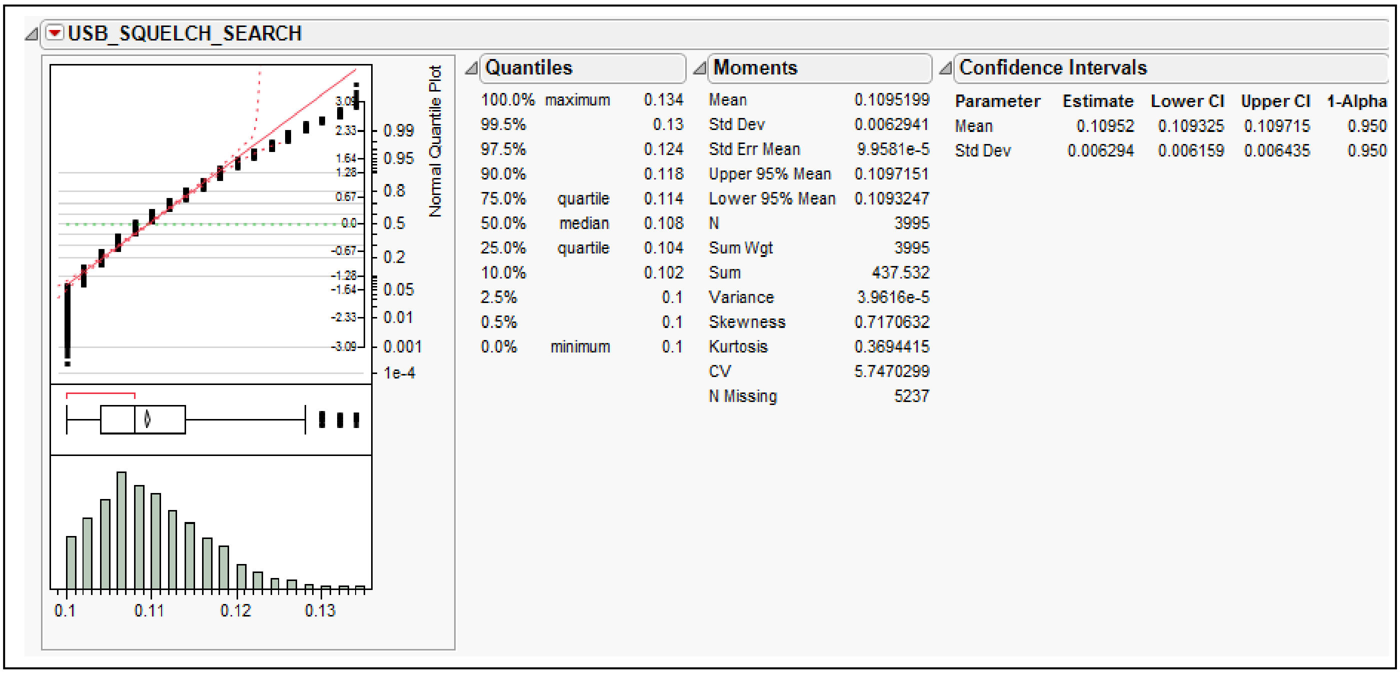

Fig. 12. Squelch trip point analysis using JMP statistical analysis software.

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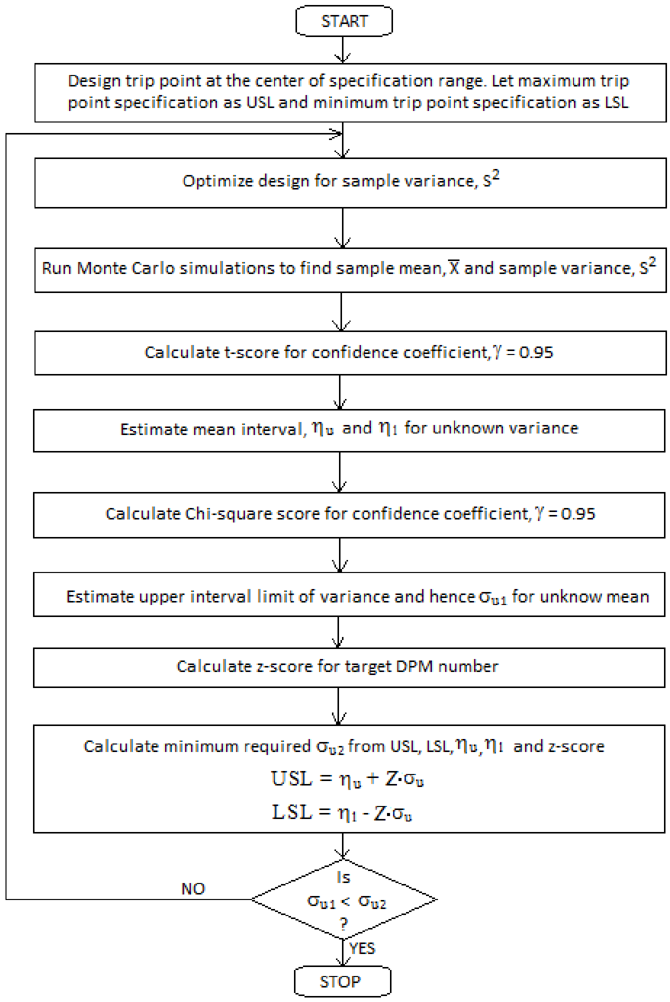

Fig. 13. Design steps for yield optimization of Squelch circuit.

Center of the statistical distribution = 109.52mV which is away from center of upper and lower test limits by more than 1.5σ. As per six-sigma quality standards, low defect rates (< 3.4 defective parts per million) are achieved for C_p values greater than 2 and C_{pk} values greater than 1.5. This quality standard also mandates center of the statistical distribution is no more than 1.5σ away from the center of the upper and lower test limits. This shows 22nm process capability to the Squelch circuit not meeting six-sigma quality standard due to tight specification limits.

VI. Conclusion

This paper analyses Intel 22nm Squelch circuit yield and lists design steps for yield optimization. To prepare readers towards this goal, it develops basic statistical framework in sections I through IV. It proves that conservative statistical results can be obtained with limited sample size. Finally, it analyses 22nm process capability for squelch design on six-sigma quality standard.

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