

Theoretical Study of Radiation Characteristics of Short Dipole Antenna

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ABSTRACT— This paper theoretically examines the key dipole antenna parameters that define their performances in terms of current distribution, radiation resistance, input impedance, radiation efficiency, size, bandwidth, quality factor and directivity. We have investigated how these parameters are related to each other and also how variation of one alters the overall performance of the antenna.

Index Terms – Bandwidth, Input Impedance, Quality factor, Radiation Resistance

I. INTRODUCTION

Whether an antenna is electrically small or large is decided on the basis of it's dimension relative to wavelength and frequency of operation. According to H.A. Wheeler [1], an electrically small antenna is one whose maximum dimension is smaller than the Radian length $\lambda/2\pi$. An antenna enclosed inside a hypothetical sphere of radius equal to one Radian length is referred to as a *Radian sphere*. The surface of the radian sphere forms the boundary between the near field and the far field. Interior to the sphere, the stored energy dominates over the radiating energy.

Wheeler presents formulas that relate the antenna quality factor (Q) to the physical volume of the antenna. Chu [2] associates the spherical-wave solution with an equivalent lumped-element circuit. The simplest of all the equivalent circuits corresponds to that of an electrically small dipole. This simple circuit can be used to derive the equation for the fundamental theoretical lower bound for Q of an electrically small antenna. According to C.A. Balanis [3], an antenna is short if it's length is in the range of $\lambda/50 < l \leq \lambda/10$, and radius a is very thin ($a \ll \lambda$). R. C. Hansen [4] states that an electrically small antenna (ESA) is a dipole-type antenna with $L < \lambda/2$. Alfred R. Lopez [5] defines electrically small antenna is one whose greatest dimension $L < \lambda/10$. Lopez presents Wheelers formula in revised form and used WIPL-D computer code to validate the formula.

There is extensive work in last six decades to develop theoretical limits on developing specific antenna designs in an attempt to optimize the relationship between size and bandwidth (BW).

In the field of small antennas, there have been many attempts to optimize antenna designs to get as close as possible to the theoretical limits [6].

The objective of the proposed work is to model a short dipole antenna and to simulate it to study the current distribution and associated radiation pattern, radiation resistance, radiation efficiency, input resistance and reactance, Q, BW and directivity of the system.

II. METHOD OF ANALYSIS

The current distribution is an important parameter which defines the radiation properties of the antenna. Thus for analysis purpose, we consider a simple current model approximated by a triangular distribution which is illustrated in Fig.1. We have introduced phase terms with the current equations and are given by [3]:

$$I_e(x', y', z') = \hat{a}_z I_0 \left(1 - \frac{z'}{l}\right) \quad 0 \leq z' \leq \frac{l}{2} \quad (1)$$

$$I_e(x', y', z') = \hat{a}_z I_0 \left(1 + \frac{z'}{l}\right) \quad -\frac{l}{2} \leq z' \leq 0 \quad (2)$$

A. Antenna parameters

The far zone antenna parameters, namely radiation resistance R_r and directivity D for short dipole is given by [3]:

$$R_r = 20\pi^2 (L/\lambda)^2 \quad (3)$$

$$D = 1.5 \sin^2 \theta \quad (4)$$

For center fed dipole antenna the radiation resistance is expressed as [3]:

$$R_r = \frac{\eta}{2\pi} \int_0^\pi \frac{[\cos(k\frac{L}{2}\cos\theta) - \cos(k\frac{L}{2})]^2}{\sin\theta} d\theta \quad (5)$$

In general, the input impedance of small antennas is typically characterized by low resistance and high reactance. As the antenna size decreases the R_r decreases, causing the antenna reactance X_{in} to dominate. Solution of integral (eqn.5) involve sine and cosine integral equation given by [3]

$$R_r = \frac{2P_{rad}}{|I_0|^2} = \frac{\eta}{2\pi} \left\{ \gamma + \ln(kl) - C_i(kl) + \frac{1}{2} \sin(kl) \times [S_i(2kl) - 2S_i(kl)] + \frac{1}{2} \cos(kl) \times \left[\gamma + \ln\left(\frac{kl}{2}\right) + C_i(2kl) - 2C_i(kl) \right] \right\} \quad (5a)$$

Where $\gamma=0.5772$ (Euler's constant) and $C_i(x)$ and $S_i(x)$ are cosine and sine integral. Input resistance is given by:

$$R_{in} = R_r / \sin^2(kl/2) \quad (5b)$$

Manuscript received January 12, 2014; revised February 8, 2014.

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Similarly reactance can be expressed as:

$$X_m = \eta/4\pi\{2S_i(kl) + \cos(kl)[2S_i(2kl) - S_i(2kl)] - \sin(kl)[2C_i(2kl) - C_i(2kl) - C_i(2ka^2/2)]\} \quad (5c)$$

And input reactance can be written as:

$$X_{in} = X_m/\sin^2(kl/2) \quad (5d)$$

In the expressions for input resistance and reactance above, the total length L does not include *gap* between its two elements. Certainly, such an antenna cannot be obtained either in practice or in modeling. In practice, the nominal length of the dipole antenna includes the gap. We introduced a gap of length d , which is a fraction of the dipole length, the necessary length correction is given by [15],

$$L_{eff} = (L/(L - d)) \times L \quad (5e)$$

It can be seen that input resistance with gap, is greater than, input resistance without gap as shown in Fig.4.

B. Antenna Radiation Efficiency

The radiation efficiency η of a given antenna can be expressed as:

$$\eta = R_r/(R_r + R_L) \quad (6)$$

where R_r is radiation resistance and R_L is loss resistance.

The resistance representing loss due to the finite-conductivity of the antenna is given by:

$$R_L = L/2\pi a \sqrt{(\pi f \mu_0)/\sigma} \quad (6a)$$

In the above equation L is the length of dipole, a is radius of dipole wire, μ_0 is permeability of free space and σ represents the conductivity of the material of dipole. Assuming that the antenna is made of copper with $\sigma=5.8 \times 10^7$ S/m.

The plot of loss resistance in eqn(6a) with frequency for different a and corresponding radiation efficiency given by eqn (6) with frequency is shown in Fig.6-7.

C. Antenna Quality Factor (Q) comparison

The expression for minimum Q is given in [7] as:

$$Q_{min} = 1/(ka)^3 \quad (7)$$

$$Q_{Chu} = (1 + 2(ka)^2)/[(ka)^3[1 + (ka)^2]] \quad (8)$$

Similarly, approximate expressions for Q of different authors given in [7], Hansen eqn (9), McLean eqn(10), eqn(10a) Geyi eqn (11), Thal eqn (12) and Gustafsson et. al. as in eqn (13) :

$$Q_{Hansen} = 6 \left[\ln\left(\frac{L}{d}\right) - 1 \right] / (kl/2)^2 \tan\left(\frac{kl}{2}\right) \quad (9)$$

$$Q_{McLean} = (1 + (ka)^2)/(ka)^3 \quad (10)$$

$$Q_{McLean}^{CP} = (1 + 2(ka)^2/2(ka)^3 \quad (10a)$$

$$Q_{Geyi} = 6 \left[\ln\left(\frac{L}{2a_0}\right) - 1 \right] / (kl/2)^3 \quad (11)$$

$$Q_{TM1m,Thal} \approx \frac{1}{\sqrt{2}(kl/2)} + \frac{3}{2(kl/2)^3} \quad (12)$$

where a_0 is the radius of dipole antenna wire, $a=L/2$ and $k=2\pi/\lambda$.

$$Q_{min,Gustafson} = D/(ka)^3(\gamma_1^{norm}) \quad (13)$$

Where D is directivity and γ_1^{norm} is normalized eigen value. The normalized eigen values given in [7] for sphere, circular ring, disk, prolate spheroid, and circular cylinder with diameter to length ratio=0.001 are as follows:

$$\gamma_{1sphere}^{norm} = 1, \quad \gamma_{1Ring}^{norm} = 0.24,$$

$$\gamma_{1Disk}^{norm} = 0.42, \quad \gamma_{1Prolate Spheroid}^{norm} = 0.056,$$

$$\gamma_{1Cylinder Circular}^{norm} = 0.050,$$

We have plotted eqn(13) for different eigen values given above with length as shown in Fig.9.

D. Input Impedance and Matching

In general, the input impedance of small antennas is typically characterized by low resistance and high reactance [3]. As the antenna size decreases, the radiation resistance R_{rad} decreases, causing the antenna reactance X_A to dominate.

For the narrowband antennas, the fundamental bounds on lossless passive matching networks were derived by Fano [13] as:

$$BWQ \leq \pi/\ln\left(\frac{1}{\Gamma_{max}}\right) \quad (14)$$

$$BW \leq \pi/Q \ln\left(\frac{VSWR+1}{VSWR-1}\right) \quad (14a)$$

where BW is the 3 dB fractional BW, Γ_{max} is the maximum allowable reflection coefficient in the pass-band, VSWR is voltage standing wave ratio and Q is the quality factor of the load to be matched.

Greater BW can only be achieved at the cost of increased maximum reflection coefficient. Since input resistance decreases quadratically with electrical size, so, appropriate matching circuits are needed for improving the small antenna efficiency over a wide range of frequencies. But, this imposes a significant practical design challenge as the matching network must also be physically small. It is, therefore, important to design self-resonant antennas with high radiation resistances which can be matched to standard transmission lines. From eqn(14a) using eqn(10), we have:

$$BW = \pi(ka)^3 / [\ln\left(\frac{VSWR+1}{VSWR-1}\right) (1 + (ka)^2)] \quad (15)$$

From the numerical computations of a wideband disk-loaded antenna, according to [12] Chu's Q was never violated through the entire frequency band and the measured BW falls below as given in eqn. (15) when evaluating at the center frequency.

E. Fundamental Limitations on Fractional BW

An upper bound for the matched VSWR fractional BW is provided by Best [14]. This is similar to the minimum achievable value for Q. We begin by the lower bound on Q for an antenna radiating only TE (Transverse Electric) or

TM (Transverse Magnetic) modes including the η given by :

$$Q_{min} = \eta \left(\frac{1}{ka} + \frac{1}{(ka)^3} \right) \quad (16)$$

The upper bound of BW is given in [17]

$$BW_{UB} = \frac{\eta}{Q_{Mclean}} \frac{VSWR-1}{\sqrt{VSWR}} \quad (17)$$

The Q of the antenna itself does not change however, the operating BW can change depending on the external Q.

III. RESULT AND DISCUSSIONS

We have plotted reactance and input reactance for a dipole radius from 3×10^{-5} m to 3×10^{-2} m given by eqn(.5c) and eqn(5d) as shown in Fig.2-3. it is seen that for small length and radius, input reactance approaches very large value, indicating that small length antenna behaves as capacitive while about half wavelength input reactance approaches zero.

The plot of input resistance eqn(5b) with zero gap, 10% and 5% of gap in terms of dipole length is shown in Fig.4. For dipole length of 1.5m the input impedance is found to approximately (73+j42.5) ohms.

The first resonance ($X_{in}=0$) occurs when the dipole length is slightly less than $\lambda/2$. The exact resonant length depends on the wire radius and feed gap. With 10% and 5% of dipole length as gap corresponding input impedance increased as 101.5+j106(Ω) and 85.4+j71.3 (Ω).

We have plotted R_L and η for frequency 0-100MHz as shown in Fig.5-6. It is seen that R_L is maximum for thin wire dipole while radiation efficiency is maximum for thick wire. In all the equations involving Q, instead of varying ka ($k(2\pi/\lambda)$ and half length ($L/2$)) we have replaced a by half length $L/2$ (in λ) to visualize how variation of electrical length can change antenna Q.

We have plotted Q values equations of different authors eqn(7)-(8), as in Fig.7, similarly combined graphs of eqn(9), eqn(10), eqn(10a) and eqn(12) is shown in Fig.8.

In the same way we have plotted eqn(13) for various shapes such as sphere, ring, dick, prolate spheroid and circular cylinders with associated eigen values, by varying L of the dipole ($L \leq 0.5\lambda$). It is observed that minimum value of Q can be achieved with the increase in size of the antenna and how well an equivalent antenna can utilize *Chu sphere*, (a sphere of radius equal to radian length $\{2\pi/\lambda\}$), and ultimately Wheeler-Chu-Maclean expressions qualify for minimum Q.

We have plotted eqn(15) and eqn(17) for VSWR=2 and 1.5 as shown in Fig.10-11. It is observed that maximum BW can be achieved at minimum Q and large VSWR value.

Also using eqn(14a) and eqn(17) and replacing different Q of eqn(10),eqn(8),eqn(10a) and eqn(12) for VSWR=2 and 100% efficiency, we have examined, best possible Q expression to qualify for large BW as shown in Fig.12-13. It is thus clear from the graph that Maclean's Q gives maximum BW.

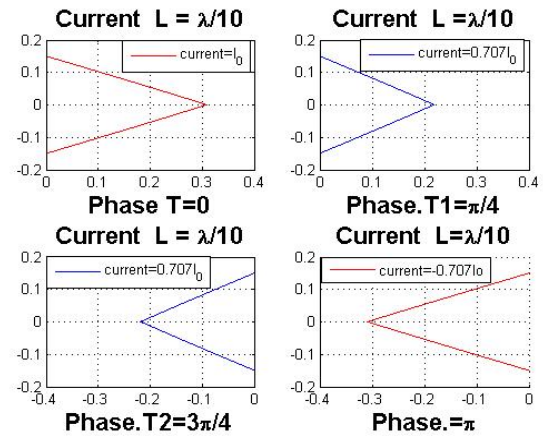


Fig.1 current for small dipole ($\Phi=0, \frac{\pi}{4}, \frac{3\pi}{4}$ and π)

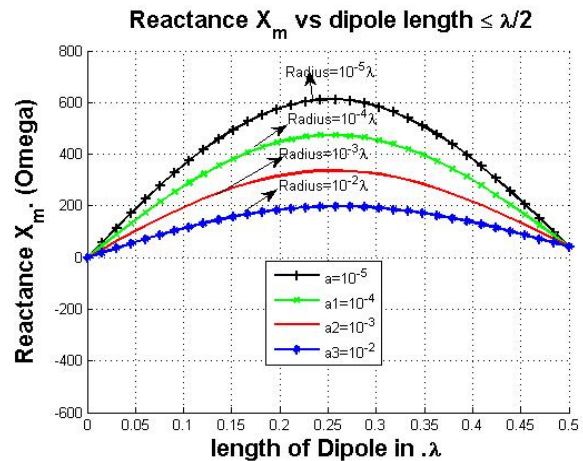


Fig.2 Reactance (X_m) vs Length with different wire radius ($l \leq \lambda/2$).

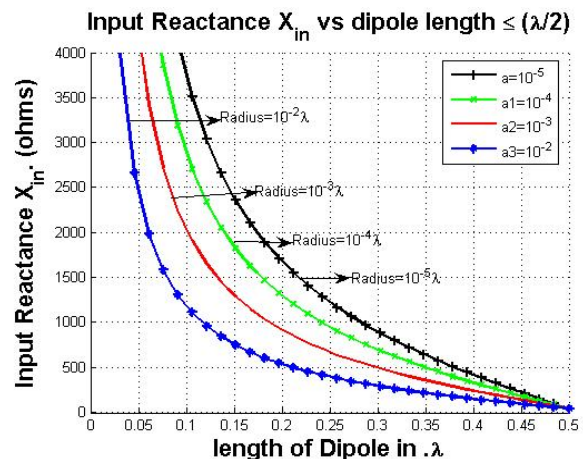


Fig.3. Input reactance (X_{in}) variation with length for different wire radius for $l \leq \lambda/2$

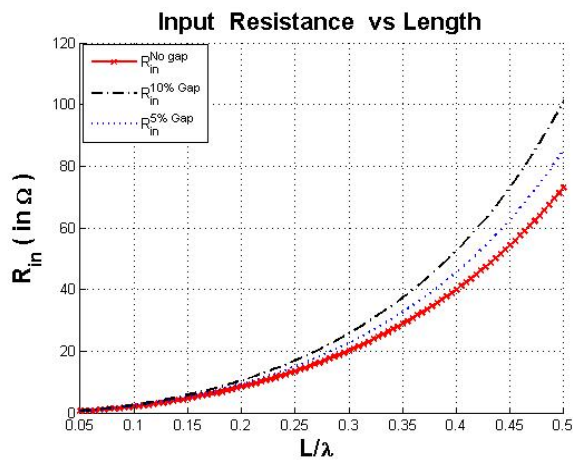


Fig.4. Input resistance of the dipole antenna without gap and the input resistance of a dipole with a gap of 10 % and 5% versus the length factor (L/λ).

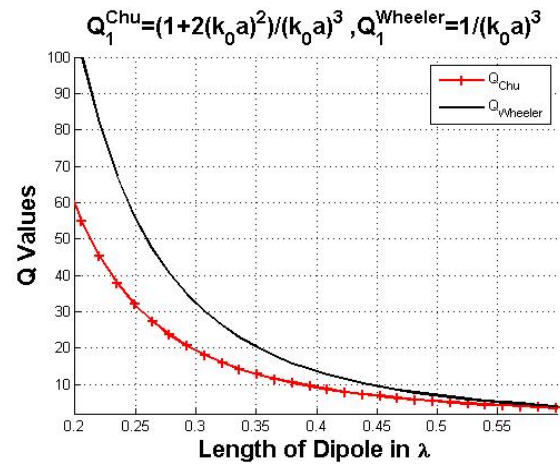


Fig.7. Q comparison (7),(8) by varying length of short dipole.

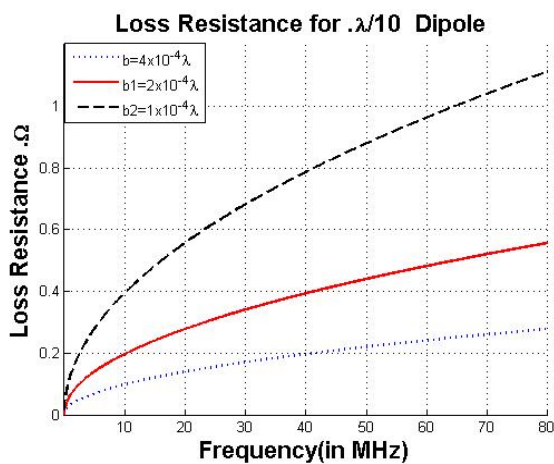


Fig.5. Loss resistance (R_l) for different radius.

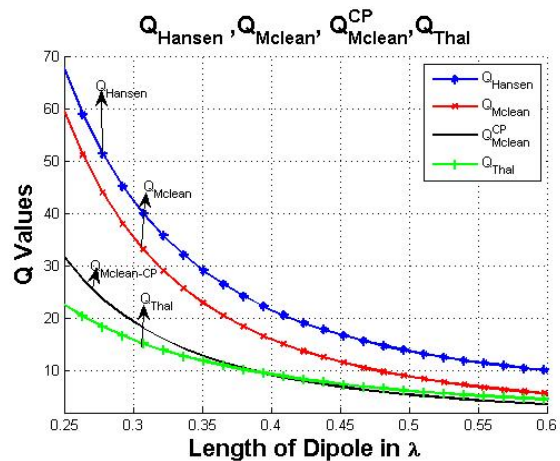


Fig.8. Q comparison (9), (10),(10a) and (12) by varying length of short dipole.

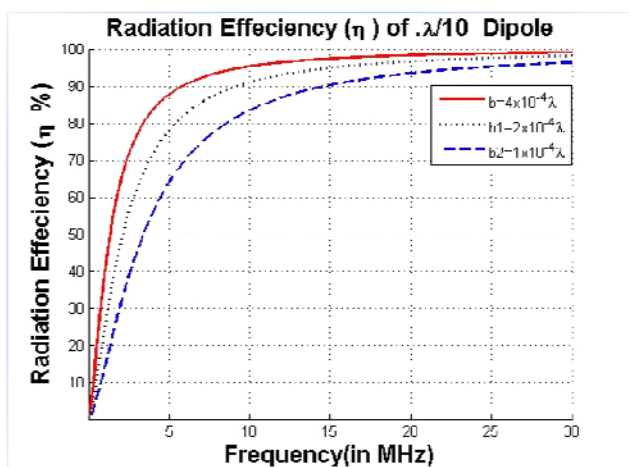


Fig.6 Radiation efficiency (η) for different radius.

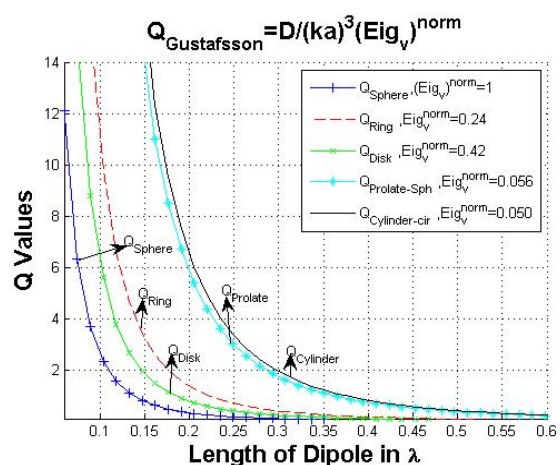


Fig.9. Q comparison equation(13) with different eigen values (directivity $D=1.5$) by varying length of antenna, modeled as scatterers,

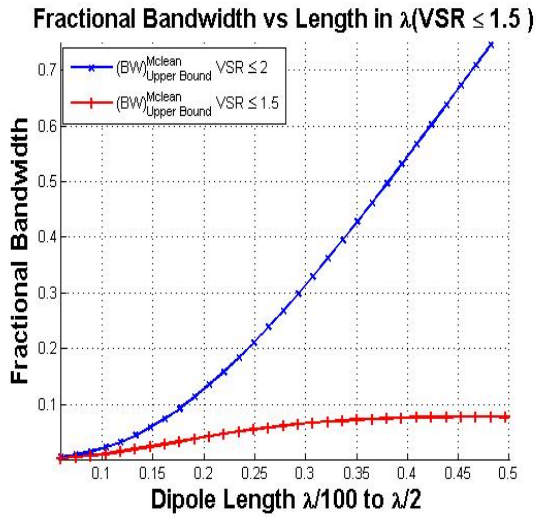


Fig.10. Bandwidth-Q comparison (15) of short dipole for VSR=2 and 1. 5

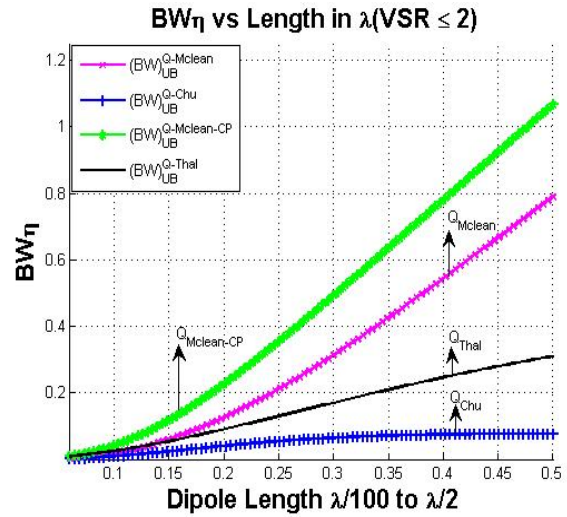


Fig.13. BW-Q comparison equation (17) of short dipole for VSR=2 with different Q of equations (10),(8),(10a) and (12).

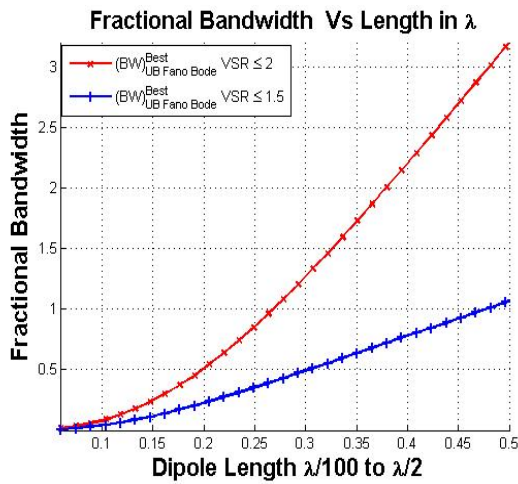


Fig.11. Bandwidth-Q comparison (17) of short dipole for VSR=2 and 1. 5.

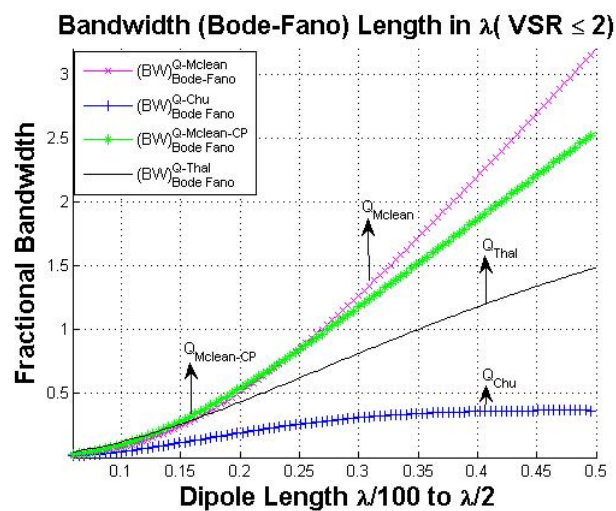


Fig.12. BW-Q comparison equation (14a) of short dipole for VSR=2 with different Q of equations (10),(8),(10a) and (12).

IV. CONCLUSION

Expressions for the current distribution, radiation resistance, loss resistance, radiation efficiency, directivity, quality factor, fractional BW, radiation resistance, reactance and input reactance for centre fed short dipole antennas have been developed.

In this paper, we have graphically represented radiation resistance, reactance, directivity, Q-factor and BW-VSWR-Q relation for short dipole. The maximum directivity of short dipole antenna in unbound medium is found to be 1.5 but in presence of ground plane, it may be increased substantially at low height in Perfect Electric Conducting (PEC) medium [3].

Thus, we may conclude by examining the graphical analysis (Fig.10-13) that there are no wideband electrically small antennas, unless efficiency is sacrificed for BW.

ACKNOWLEDGMENT

Authors are thankful to the Head, Dept of Electronics & Communication Technology, Gauhati University for valuable suggestions during the work. One of the authors P Banerjee is also thankful to Sohail Ahmed, Dept of English, Jagiroad College, Morigaon, Assam, India, for his helpful suggestions.

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