Radiation Effects on Heat and Mass Transfer over an Exponentially Accelerated Infinite Vertical Plate with Chemical Reaction

A. Ahmed, M. N.Sarki, M. Ahmad

Abstract— **In this paper the study of unsteady flow of a viscous incompressible fluid past an exponentially accelerated moving vertical plate had been investigated. The fluid is gray, absorbing –emitting but non scattering medium and Rosseland approximation is used to describe the radiative heat flux in energy equation. The dimensionless governing equations are solved analytically using the Laplace transform technique. The velocity, concentration and temperature fields are studied for different physical parameters like thermal Grashof number, mass Grashof number, Schmidt number and time. Temperature decreases with increased strength of radiation and Prandtl number. It is also observed that the velocity increases with increasing values thermal Grashof or mass Grashof number. Similarly, velocity increases with decreasing values of the Schmidt numbers.**

Keywords: Chemical reaction, Exponential, Heat transfer and Radiation.

I. INTRODUCTION

Radiation effects on heat and mass transfer are of greater importance in many processes and have, therefore, received a considerable amount of attention in recent time. It is applied in engineering fields and physiology such as transpiration, cooling gaseous diffusion and blood flow in arteries. Radiative heat and mass transfer play important roles in the design of spacecraft, filtrations processes, the drying of porous material in textiles industries solar energy collector and nuclear reactors.

Singh and Kumar (1982) studied free convection flow past exponentially accelerated plate. Basant and Ravindra (1990) analysed mass transfer effects on the flow past an accelerated infinite vertical plate with heat sources. Seddeck (2001) investigated the thermal radiation and buoyancy effects on MHD free convective heat generating flow over an accelerating permeable surface with temperature dependent viscosity. Also analysis of thermal radiation on unsteady free convection and mass transfer of an optically thin gray gas was done by Raptis and Perdikis (2003). Makinde and Mhone (2005), described the effects of heat transfer to MHD oscillatory flow in a channel filled with porous medium. Radiative heat and mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction was studied by Muthucumaraswamy and Raj (2006). Analysis of mass transfer effects on exponentially accelerated isothermal vertical plate was made by Muthucumaraswamy and Natarajon (2008). Rajesh and Varma (2009) analyzed the radiation and mass transfer effects on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature. Also Narahari and Ishak (2011), studied radiation effects on free convection flow near a moving vertical plate with Newtonian heating. Chamkha (2007) studied heat and mass transfer for a non-Newtonian fluid flow along a surface embedded in a porous medium. Muthucumaraswamy and Subramanian (2010) studied the heat transfer effects on accelerated vertical plate with variable temperature and mass flux. Das and Jana (2010), analyzed heat and mass transfer effects on unsteady MHD free convection. Patrick and Jane (2010), observed the effect of natural convective heat transfer from a narrow vertical flat plate with a uniform surface heat flux and with different plate edge condition. Experimental studies of Radiation effects on MHD free convection flow over a vertical plate have been described by Abdussamad and Rahman (2006), Sivaiah and Nagarajan (2010).

The aim of this paper is to study the effects of radiation on heat and mass transfer on exponentially accelerated infinite vertical plate with Newtonian heating to include chemical reaction.

II. MATHEMATICAL FORMULATION.

Consider the unsteady flow of a viscous incompressible fluid past a moving vertical plate. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical radiation between the diffusing species and the fluid. The x’-axis is taken along the plate in vertically upward direction and the y’-axis is taken normal to the plate. Initially the plate and the fluid are at the same temperature and concentration. At time t’>0, the plate is exponentially accelerated with velocity $U = u_0 \exp(a' t')$ in its own plane. At the same time, the plate temperature is raised to T_W. As the level of concentration is raised to C'_w, the fluid is considered grey, absorbing-emitting radiation but a non-scattering medium. It is also assumed that there exists a homogeneous first order chemical reaction between the fluid and species concentration. By usual Boussinesq’s approximation, the unsteady flow is governed by the following equations: Muthucumaraswamy and Raj (2006).

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$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + v\frac{\partial^2 u}{\partial y^2} \quad (1)$$

$$\rho Cp\frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \quad (2)$$

$$\frac{\partial C'}{\partial t} = D\frac{\partial^2 C'}{\partial y^2} - K_l C. \quad (3)$$

with the following initial and boundary conditions;

$$\left.\begin{aligned} &t' \le 0 : T = T_\infty,\ C' = C'_\infty \quad for\ all\quad y = 0 \\ &t' > 0 : U = u_0 \exp(a't'), \frac{dT}{dy} = -\frac{h}{k}T, \\ &\qquad C' = C'_w\ at\ y = 0 \\ &U = 0, T \to T_\infty,\ C' = C'_\infty\ as\ y \to \infty \end{aligned}\right\} \quad (4)$$

Where, β volumetric coefficient of the thermal expansion, v the kinematic viscosity, ρ is density, μ the coefficient of viscosity, β^* is volumetric coefficient of expansion with concentration, u_0 the velocity of the plate, y the coordinate axis normal to the plate, g acceleration due to gravity, q_r the radiation heat flux in the y direction, Cp is specific heat at constant pressure, C' is specific concentration in the fluid, C'_∞ the concentration in the fluid far away from the plate, C'_W the concentration on the plate, D the mass diffusion coefficient, t' is time, T the temperature of the fluid near the plate, T_W the temperature of the plate, T_∞ the temperature of the fluid far away from the plate.

The local radiant for the case of an optically thin grey gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^*\sigma(T_\infty^4 - T^4) \quad (5)$$

where a^* is absorption coefficient, σ the Stefan Boltzmann constant, K_l chemical reaction parameter, k the thermal conductivity of the fluid.
We assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in Taylor series about T_∞ and neglecting higher order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (6)$$

By using equations (5) and (6), equation (2) reduces to

$$\rho Cp\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2} + 16a^*\sigma T_\infty^3(T_\infty - T) \quad (7)$$

III. METHOD OF SOLUTIONS

Introducing the following non-dimensional quantities,

$$U = \frac{u}{u_0},\quad \theta = \frac{T - T_\infty}{T_w - T_\infty},\quad t = \frac{t'u_0^2}{V},\quad Y = \frac{yu_0}{V},\quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty},$$
$$Gr = \frac{g\beta V(T_w - T_\infty)}{u_0^3}$$
$$Gc = \frac{Vg\beta^*(C'_w - C'_\infty)}{u_0^3},\quad \Pr = \frac{\mu Cp}{k},\quad R = \frac{16a^*\partial v^2 T_\infty^3}{ku_0^2}$$
$$Sc = \frac{v}{D},\ K = \frac{vK_l}{u_o^2}\ a = \frac{a'n}{u_0^2} \quad (8)$$

Equation (1), (3) and (7) respectively using the non dimensional quantities (8) above, become

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} \quad (9)$$

$$\Pr\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial Y^2} - R\theta \quad (10)$$

((10)

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - KC \quad (11)$$

The initial and boundary conditions in non-dimensional quantities are

$$\left.\begin{aligned} &U = 0,\quad \theta = 0,\quad C = 0\ for\ all\ \ t \le 0 \\ &t \ge 0 : U = \exp(at), \frac{\partial \theta}{\partial Y} = -(1+\theta), C = 1\ at\ \ y = 0 \\ &U = 0,\ \theta \to 0,\ \ C \to 0 \qquad y \to \infty \end{aligned}\right\} \quad (12)$$

Where Gc the mass Grashof number, Gr the thermal Grashof number, C the dimensionless concentration, K the dimensionless chemical reaction parameter, Pr the Prandtl number, R the radiation parameter, Sc the Schmidt number, t the dimensionless time, U the dimensionless velocity, Y the dimensionless coordinate axis normal to the plate, θ the dimensionless temperature.

According to above non dimensional process,
$$\frac{\P q}{\P Y} = -B_i\{g + q\}$$

Where $B_i = \frac{vh}{u_0 k}$ called Biot number, $g = \frac{T_¥}{T_w - T_¥}$. For simplicity we take $B_i = 1 = g$.

The dimensionless governing equations (9) to (11), subject to the boundary conditions (12), are solved by the usual Laplace transform techniques and the solutions are derived as follows;

$$C=\frac{1}{2}\left\{\exp(2\eta\sqrt{ScKt})\,erfc\left(\eta\sqrt{Sc}+\sqrt{Kt}\right)+\exp(-2\eta\sqrt{ScKt})\,erfc\left(\eta\sqrt{Sc}+\sqrt{Kt}\right)\right\} \tag{13}$$

$$\begin{aligned}\theta=&\frac{1}{2\left(\sqrt{R}-1\right)}\left\{\exp(2\eta\sqrt{\Pr ct})\,erfc\left(\eta\sqrt{\Pr}+\sqrt{ct}\right)+\exp(-2\eta\sqrt{\Pr ct})\,erfc\left(\eta\sqrt{\Pr}-\sqrt{ct}\right)\right\}\\ &-\frac{\Pr}{1-R}\left\{\frac{\exp(\eta^2\Pr-ct)}{\sqrt{\Pr\pi t}}erfc\left(\eta\sqrt{\Pr}+\sqrt{ct}\right)\right\}\end{aligned} \tag{14}$$

$$\begin{aligned}U=&\exp(at)/2\begin{pmatrix}\exp(2\eta\sqrt{at})\,erfc(\eta+\sqrt{at})\\ +\exp(-2\eta\sqrt{at})\,erfc(\eta-\sqrt{at})\end{pmatrix}\\
&+\left(\frac{Gc}{b(1-Sc)}-\frac{Gr}{d(1-\Pr)(1-\sqrt{R})}\right)erfc(\eta)\\
&+\frac{Gr\exp(dt)}{2d(1-\Pr)(1-\sqrt{R+\Pr d})}\\
&\left\{\begin{array}{l}\exp(2\eta\sqrt{dt})\,erfc\left(\eta+\sqrt{dt}\right)\\ +\left(\exp(-2\sqrt{dt})\,erfc\left(\eta-\sqrt{dt}\right)\right)\\ -\exp(2\eta\sqrt{\Pr(d+c)t})\,erfc\left(\eta\sqrt{\Pr}+\sqrt{(d+c)t}\right)+\\ \exp(-2\eta\sqrt{\Pr(d+c)t})\,erfc\left(\eta\sqrt{\Pr}-\sqrt{(d+c)t}\right)\end{array}\right\}\\
&+\frac{Gr\left(\Pr\right)^2}{\left(1-\Pr\right)\left(1-R\right)\left(1-R-\Pr d\right)}\\
&\left\{\left(\begin{array}{l}\frac{\exp(-\eta^2)}{\sqrt{\pi t}}\,erfc(\eta\sqrt{\Pr}+\sqrt{ct})-\\ \left(\frac{\exp(\eta^2\Pr-ct)}{\sqrt{\Pr\pi t}}erfc(\eta\sqrt{\Pr}+\sqrt{ct})\right)\end{array}\right)\right\}\\
&+\frac{Gr}{2a\left(1-\Pr\right)\left(1-\sqrt{R}\right)}\left\{\begin{array}{l}\exp(2\sqrt{\Pr ct})\,erfc\left(\eta\sqrt{\Pr}+\sqrt{ct}\right)\\ +\exp(-2\eta\sqrt{\Pr ct})\,erfc\left(\eta\sqrt{\Pr}-\sqrt{ct}\right)\end{array}\right\}\\
&+\frac{Gc\exp(bt)}{2b\left(1-Sc\right)}\left\{\begin{array}{l}\left(\begin{array}{l}\exp(2\eta\sqrt{Sc(K+b)t})\,erfc\left(\eta\sqrt{Sc}+\sqrt{(K+b)t}\right)+\\ \exp(-2\eta\sqrt{Sc(K+b)t})\,erfc\left(\eta\sqrt{Sc}-\sqrt{(K+b)t}\right)\end{array}\right)\\ -\left(\begin{array}{l}\exp(2\eta\sqrt{bt})\,erfc\left(\eta+\sqrt{bt}\right)+\\ \left(\exp(-2\eta\sqrt{bt})\,erfc\left(\eta-\sqrt{bt}\right)\right)\end{array}\right)\end{array}\right\}\end{aligned} \tag{15}$$

Where $b=\dfrac{ScK}{1-\ Sc}$ $\quad c=\dfrac{R}{\Pr}$ $\quad d=\dfrac{R}{1-\ \Pr}$

IV. RESULTS AND DISCUSSIONS

In order to get the physical insight to the problem numerical computations are carried out for different physical parameters such as mass Grashof number Gc, thermal Grashof number Gr, Schmidt number Sc, Prandtl number Pr, chemical reaction parameter K, radiation parameter R, acceleration a, and time t. The value of Schmidt number Sc, is taken to be 0.6, which correspond to water vapour. Also the value of Prandtl number are chosen such that they represent air (Pr = 0.71).

Temperature Profiles.

The temperature profiles of different values of Radiation parameter (R = 5, 10, 15, 20), Pr = 0.71, t = 0.2, is shown in figure 1. It is observed that the temperature decreases with increase in values of R. When radiation is present, the thermal boundary layer always found to thicken, this shows that the radiation provides additional means to diffuse energy. The effect of temperature for different values of Prandtl number (Pr = 0.71, 0.85, 1.0), R = 7, t = 0.2, are studied and presented in figure 2. It shows that there is slight decrease in temperature with increase in Pr.

Velocity Profiles.

The velocity profile for different values of mass Grashof number (Gc) and thermal Grashof number (Gr) are presented in figure 3 and 4. It is observed that the velocity increase with increasing values of Gc and Gr. This is possible because Gr and Gc increase the contribution from the buoyancy force near the plate become significant and hence rise in velocity near the plate is observed. In figure 5, the effects of parameter (a) on velocity in the presence of radiation on exponentially accelerated plate is presented. It is observed that the velocity increase with increasing values of a. Figure 6 illustrated the effects of time t on velocity profiles. It is observed that velocity increase with increasing values of t. The profiles are shown in figure 7, 8 and 9 for different values of radiation parameter R, Schmidt number Sc and chemical reaction parameter K respectively. The velocity decrease in increasing the values of all the three parameters that is R,Sc and K. Physically this means that when radiation is present the momentum boundary layer was found thicken which is in agreement with observation.

Concentration Profiles

The effects of various parameters had been demonstrated in figures 10, 11, and 12. In figure 10, the concentration profile decreases with increase in the values of Sc. Also in figure 11, various values of K decrease the concentration profile. Furthermore, figure 12 shows that increase in values t results in decrease in concentration profile.

V. CONCLUSION

The study of radiation effects on heat and mass transfer on exponentially accelerated infinite vertical plate with chemical reaction. The dimensionless governing equations are solved analytically by usual Laplace-transform method. The effect of various parameters like thermal Grashof number, mass Gashof number, Schmidt number and time are studied and discussed. The conclusions of the study are as follows;

The temperature increases with decreasing radiation R and Prandtl number Pr.

The velocity increases with increasing values of Gr, Gc, a and t. The velocity decreases with increase in Sc and R.

The concentration increases with increasing Schmidt number and decreases with increasing chemical reaction parameter K and time t.

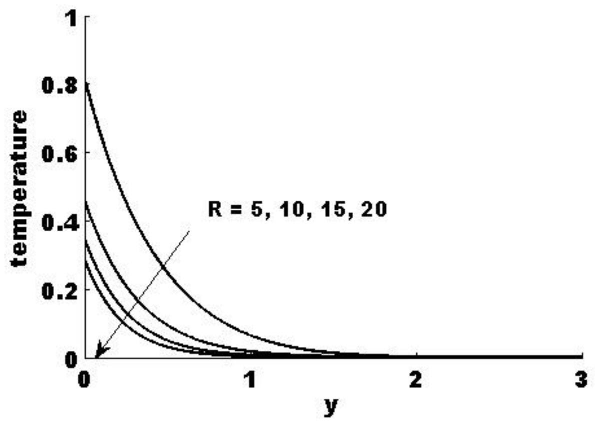

Fig. 1 Effects of Radiation (R) on temperature profiles

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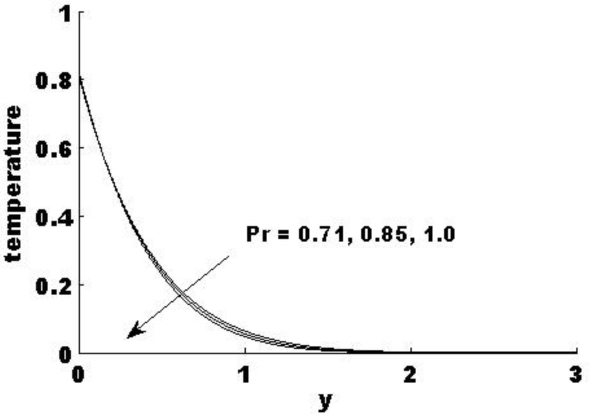

Fig. 2 Effects of Prandtl number Pr on temperature profiles

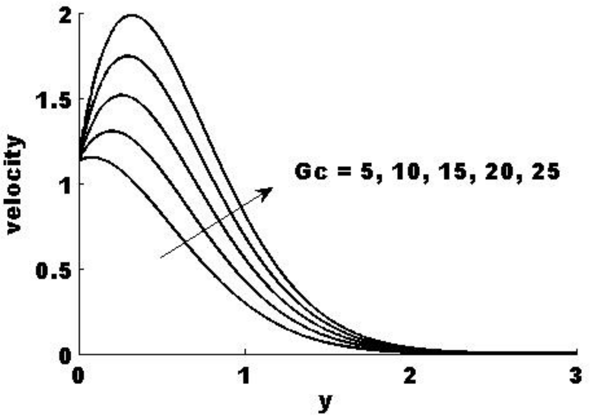

Fig. 3 Effects of Mass Grashof number (Gc) on velocity profiles

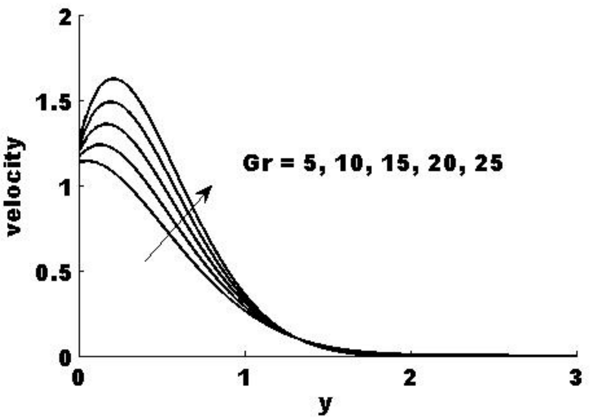

Fig. 4 Effects of Thermal Grashof number (Gr) on velocity profiles

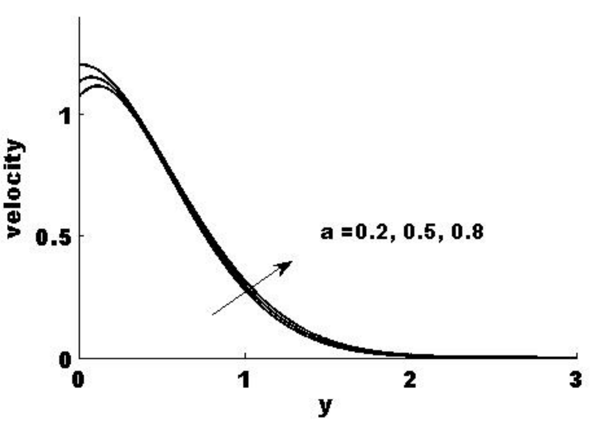

Fig. 5 Effects of a on velocity profiles

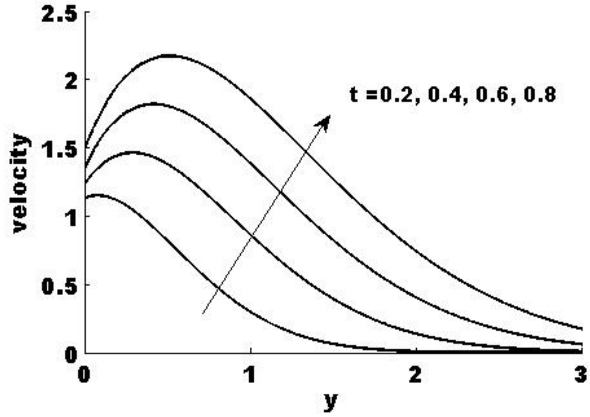

Fig. 6 Effects of Time (t) on velocity profiles

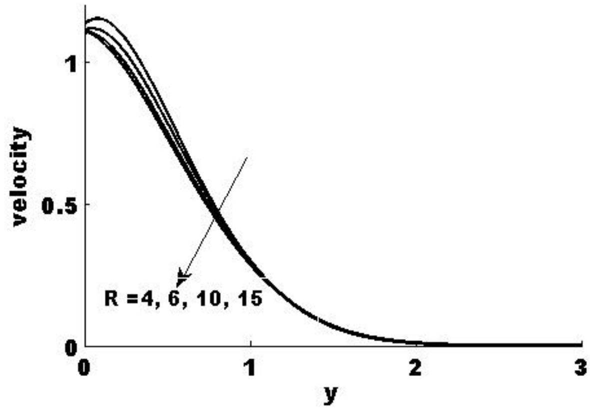

Fig. 7 Effects of Radiation (R) on velocity profiles

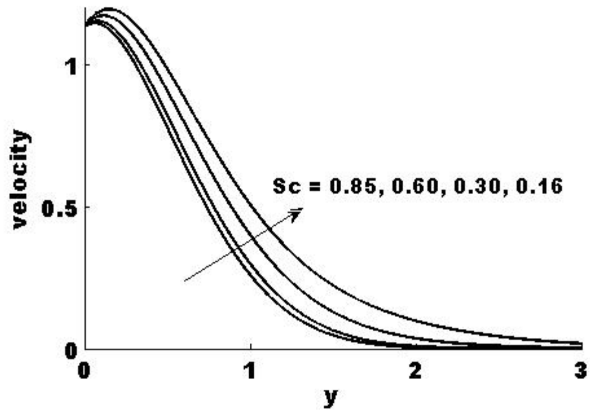

Fig. 8 Effects of Schmidt number (Sc) on velocity profiles

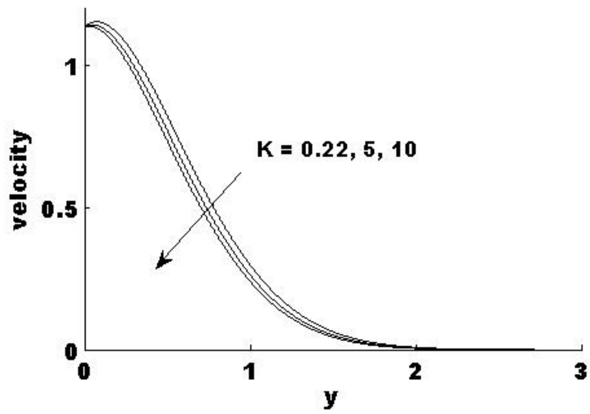

Fig. 9 Effects of Chemical reaction parameter (K) on velocity profiles

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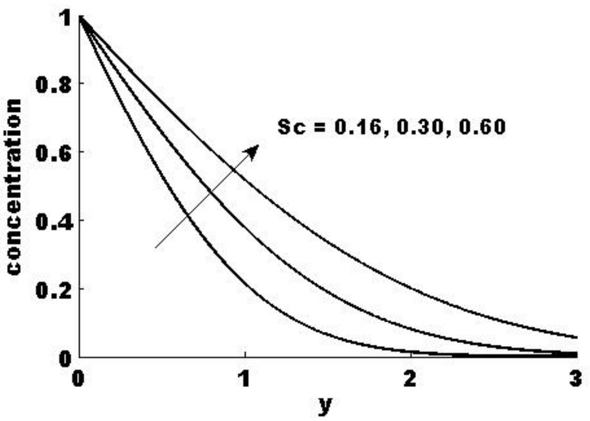

Fig.10 Effects of Schmidt number (Sc) on concentration profiles

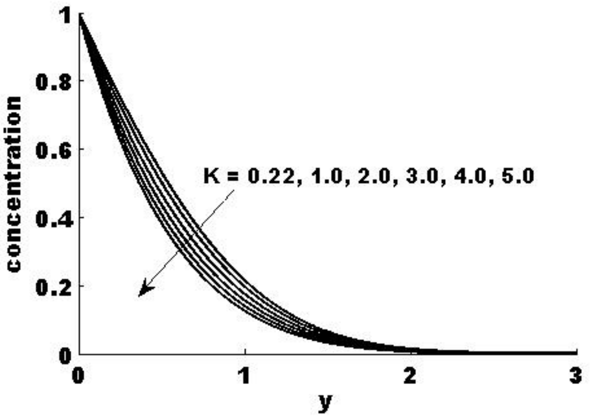

Fig. 11 Effects of Chemical reaction parameter (K) on concentration profiles

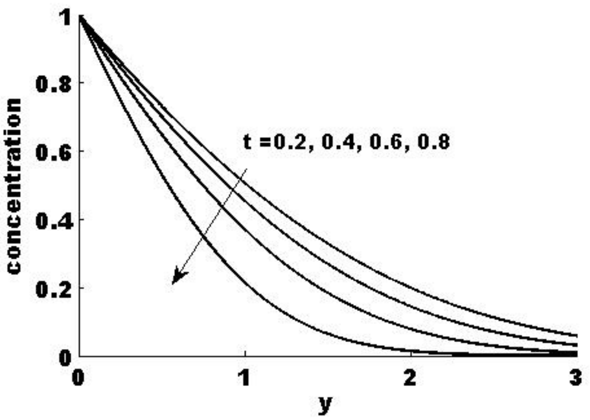

Figure 12. Effects of Time (t) on concentration profiles

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