Reliability Analysis for a Repairable System under N-policy and Imperfect Coverage

R. Sharma

***Abstract*—In this study, we deal with the reliability analysis of repairable system under N-policy, setup and imperfect coverage. The repairman is turned on only when 'N' and more failed components are accumulated in the system for providing repair. The system consists of 'M' operating, 'S' warm and 'Y' cold standby components. As any component fails, it is immediately replaced by standby component if available with coverage probability 'c'. The life and repair times of component are assumed to be exponentially distributed. Various reliability performance measures are obtained using Runge-Kutta (RK) method. Moreover, numerical results are also provided by taking an illustration. Finally, the conclusion has been given.**

***Keywords*—Imperfect coverage, N-policy, rebooting, warm standby, reliability.**

I. Introduction

RELIABILITY prediction is a key concern of the system engineer in any repairable system. Reliability can be defined for a specified period of time as the probability that component will perform its intended function under operating conditions. Complex systems are everywhere among us like as computer and telecommunication networks, manufacturing and production system, transportation, electrical appliances and many more are well-known real life examples. Designing reliable systems and determining their availability are both very important tasks for the system managers and analyst. Many empirical studies were suggested by [1] to improve the reliability of the concerned system. The performance of software reliability growth models was discussed by [2] in his study. Huang and Lyu [3] focused on software reliability with various fault detection rates. A survey on reliability modelling for distributed system was provided by Ahmed and Wu [4].

In many practical scenarios, the production may be interrupted due to the unexpected failure of the machines which brings an undesirable loss of revenue as well as goodwill in the market. Such situation can be controlled up to some extent using the support of standby units and repairs. With the help of standbys, the system remains operative and continues to perform its assigned job in case of failure. The concept of standby support has attracted the attention of several researchers working in the area of machining and reliability theory. Reliability analysis [5] has been done for the machining system with warm standbys. Machine repair problem with standby was considered by [6,7] in different frameworks. A multi-server machine repair model [8] has been developed with standby support. Recently, Jain [9] presented the transient analysis of a machining system with mixed standbys incorporating the concepts of service interruption and priority.

Reliability modeling with imperfect coverage has been investigated by many authors time to time. Due to imperfect coverage, the reliability of repairable systems cannot be enhanced unlimitedly with the increase of redundancy. The concept of coverage and its effect on a repairable system has been introduced by several authors. A simple and efficient algorithm [10] was given for computing the reliability and unreliability of systems subject to imperfect fault coverage. The status and trends of imperfect coverage models was introduced in the work of [11]. The imperfect fault coverage with a combinatorial model was studied by [12] in his study.

In many software embedded systems, the machines can be reconfigured itself temporarily by rebooting process whenever faults are undictated and recovered. The notable contributions in the area of rebooting are due to [13], [14], [15], respectively. To utilize the proper resource of repairman, the concept of N-policy has been considered by many authors in their study in different frameworks. According to this, the repairman starts repairing of failed component whenever it reaches a certain level say 'N' which number has been predefined by maintenance engineer. Reference [16] has been applied N-policy for degraded machining system with heterogeneous servers and standbys.

For providing transient solution of differential equations, R-K method has been applied by many researchers. To solve the set of differential equations, we have applied R-K method. Reference [17] dealt with transient system behavior of the queueing system using the fourth order Runge-Kutta method.

In this paper, we discuss the reliability issue of repairable system under N-policy, setup and imperfect coverage. The perfect detection and recovery of an active unit is done with probability 'c' which is known as coverage factor and imperfect detection and recovery has been done with complementary probability '1-c'. The rest of the paper is organized as follows. We formulate the model by providing requisite abbreviations and assumptions in section II. The R-K method was suggested to compute the stationary probabilities of the machining system in section III. In section IV, some performance indices are established in terms of probabilities. In section V, we have given various numerical results. Finally, the investigation comes to end with the concluding remarks in section VI.

R. Sharma is with department of Mathematics, JK Lakshmipat University, Jaipur-302026, India (phone: 141-7107-583; e-mail: aligarh.richa@gmail.com; richashrama@ jklu.edu.in).

II. Mathematical Model

In this paper, we consider multi-component machining system includes 'M' operating units, 'S' warm and 'Y' cold standbys units. Each of the operating as well as standby units (warm and cold) has an exponential time to failure distribution with rate λ. As a failure of active units occur, it is immediately dictated and replaced by a standby units with coverage probability 'c' if available. With a complimentary probability '1-c', uncovered fault is repaired during unsafe failure state with repair rate β by rebooting. The repairable system is followed the concept of FCFS (first-come first served). The failure of multi-component machining system is defined if there is less than 'K' active units where K=1, 2, 3,…,M. The failed units are repaired by a single repairman according to the N-policy. The repairman is started repairing with mean 1/μ of the failed units whenever there are 'N' or more than 'N' failed units present in the system. When the repairman is unavailable (accumulation state) then the failed units accumulate and wait for repair. The repairman takes some setup time before repairing of failed units with rate θ. The multi-component machining system is completely failed whenever the failed units become 'L' where L=M+S+Y-K+1.

Let 'n' be the number of the failed units in the system failure and service rate is defined as:

$$\lambda_n = \begin{cases} M\lambda + S\alpha, & 0 \le n \le Y-1 \\ M\lambda + (Y+S-n)\alpha, & Y \le n \le Y+S \\ (M+Y+S-n)\lambda, & Y+S+1 \le n \le L \end{cases}$$

$$\mu_n = \mu, \qquad 1 \le n \le L-1$$

Notations:

$P_{0,0}(t)$ The probability that there is no failed unit in the system at time *t*

$P_{0,n}(t)$ The probability that there are n $(1 \le n \le L)$ failed units in the system during safe state at time *t* when the repairman is in accumulation state

$P_{1,n}(t)$ The probability that there are n $(1 \le n \le L)$ failed units in the system during safe state at time *t* when the repairman is rendering repair

$P_{0,un}(t)$ The probability that there are n $(1 \le n \le L)$ failed units in the system during unsafe state at time *t* when the repairman is in accumulation state

$P_{1,un}(t)$ The probability that there are n $(1 \le n \le L)$ failed units in the system during unsafe state at time *t* when the repairman is rendering repair

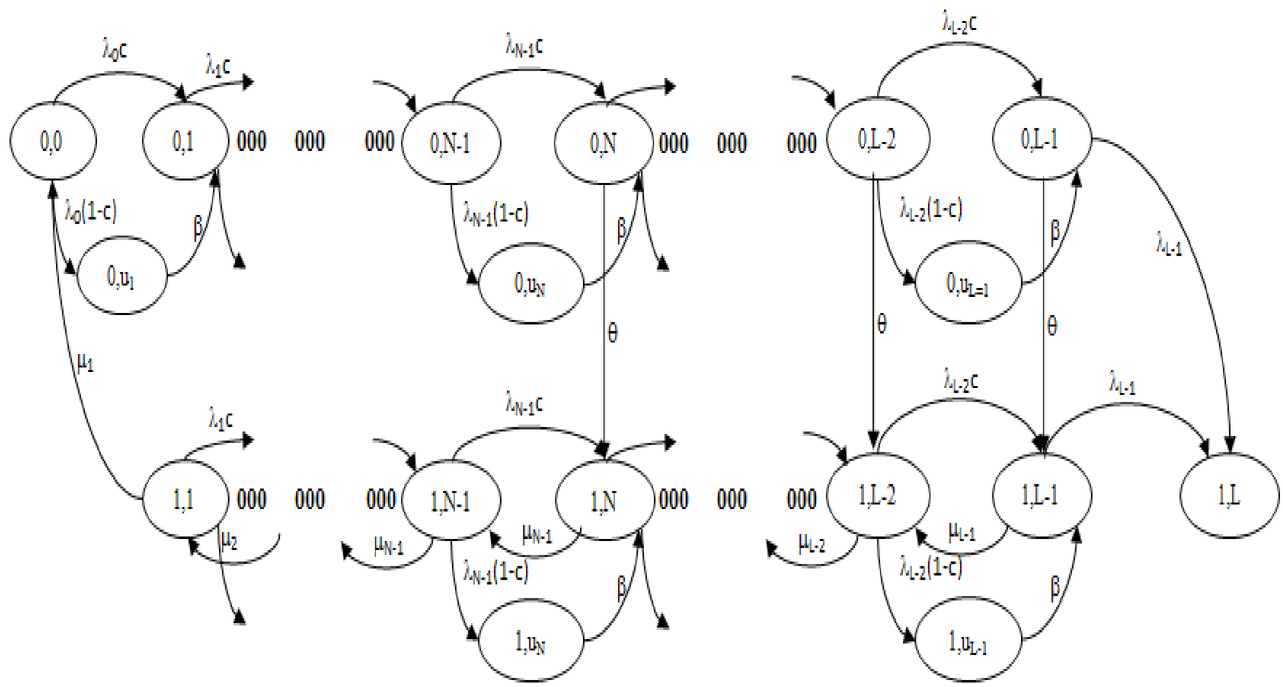

Fig. 1. State transition rate diagram.

ISBN: 978-988-19253-9-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

III. The Analysis

At the initial state, it is assumed that machining system works properly without any failure units in the system. The reliability analysis for the machining system can be done using the birth death process. Now, we provide the differential difference equations of the system using fig. 1 as under:

$$\frac{dP_{0,0}(t)}{dt} = -\lambda_0 P_{0,0}(t) + \mu_1 P_{1,1}(t) \tag{1}$$

$$\frac{dP_{0,n}(t)}{dt} = -\lambda_n P_{0,n}(t) + \lambda_{n-1} c P_{0,n-1}(t) + \beta P_{0,un}(t), \quad 1 \le n \le N-1 \tag{2}$$

$$\frac{dP_{0,n}(t)}{dt} = -(\lambda_n + \theta) P_{0,n}(t) + \lambda_{n-1} c P_{0,n-1}(t) + \beta P_{0,un}(t), \quad N \le n \le L-2 \tag{3}$$

$$\frac{dP_{0,L-1}(t)}{dt} = -(\lambda_{L-1} + \theta) P_{0,L-1}(t) + \lambda_{L-2} c P_{0,L-2}(t) + \beta P_{0,uL-1}(t), \tag{4}$$

$$\frac{dP_{1,n}(t)}{dt} = -(\lambda_n + \mu_n) P_{1,n}(t) + \mu_{n+1} P_{1,n+1}(t) + \lambda_{n-1} c P_{1,n-1}(t) + \beta P_{0,un}(t), \; 1 \le n \le N-1 \tag{5}$$

$$\frac{dP_{1,n}(t)}{dt} = -(\lambda_n + \mu_n) P_{1,n}(t) + \mu_{n+1} P_{1,n+1}(t) + \lambda_{n-1} c P_{1,n-1}(t) + \beta P_{1,un}(t) + \theta P_{0,n}(t), \quad N \le n \le L-2 \tag{6}$$

$$\frac{dP_{1,L-1}(t)}{dt} = -(\lambda_{L-1} + \mu_{L-1}) P_{1,L-1}(t) + \lambda_{L-2} c P_{1,L-2}(t) + \beta P_{1,uL-1}(t) + \theta P_{0,L-1}(t), \tag{7}$$

$$\frac{dP_{1,L}(t)}{dt} = \lambda_{L-1} P_{1,L-1}(t) + +\lambda_{L-1} P_{0,L-1}(t) \tag{8}$$

The above mentioned transient equations are solved using Runge-Kutta (R-K) method which is powerful numerical method for solving system of linear equations.

IV. Performance Measure

In this section, we establish various performance measures which are required to predict the reliability characteristics of the multi-component machining system. For this purpose, R-K technique is implemented by developing program in MATLAB software.

- The reliability of machining system at time t is given by

$$R_Y(t) = 1 - P_{1,L}(t) - \sum_{i=0}^{1} \sum_{n=1}^{L-1} P_{i,un}(t) \tag{9}$$

- Mean time to system failure is

$$MTTF = \int_{t=0}^{\infty} R_Y(t) dt = \int_{t=0}^{\infty} \left(1 - P_{1,L}(t) - \sum_{i=0}^{1} \sum_{n=1}^{L-1} P_{i,un}(t) \right) dt \tag{10}$$

- The probability that the repairman is idle at time t is

$$P_I(t) = \sum_{n=0}^{N-1} P_{0,n}(t) + \sum_{n=0}^{N-1} P_{0,un}(t) \tag{11}$$

V. Numerical Results

In this section, we present numerical experiment to demonstrate the tractability of the suggested computational approach based on R-K method. The transient results for various performance measures are shown by developing a program in 'MATLAB' software. For this purpose, the default parameters are taken as M=5, N=4, S=3, Y=2, λ_0=0.5, λ_1=0.6, μ=3, θ=0.9, β=1 and c=0.9.

Figs 2 and 3 display the trends of system reliability ($R_Y(t)$) against time (t) for different values of parameters λ_0 and λ_1, respectively. It is clear from the figures that system reliability shows decreasing pattern with time in the beginning and after reaching a certain value, it attains almost constant value. It is also noticed from figs 2 and 3 that $R_Y(t)$ decreases with the increment in λ_0 and λ_1, respectively.

.

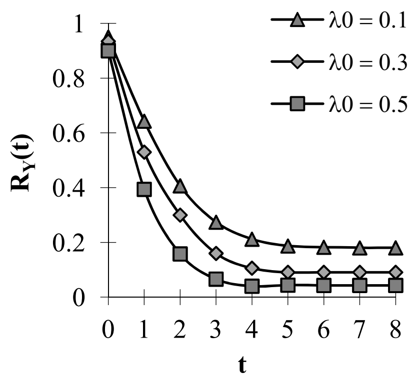

Fig. 2. System reliability vs time t by varying λ_0.

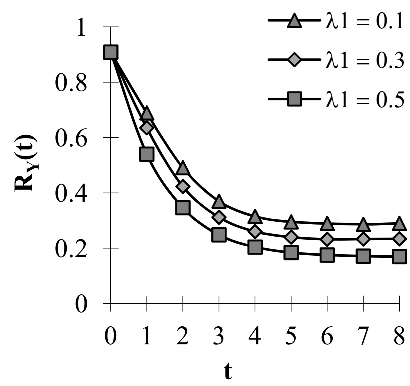

Fig. 3. System reliability vs time t by varying λ_1.

Figs 4 and 5 explore the effect of number of operating components and λ_0, λ_1 on MTTF. From theses figs it is examined that MTTF decreases rapidly for lower values of λ_0 and λ_1 then further decreases slowly until it attains a certain value and later on it tends asymptotically to zero, as λ_0 and λ_1 increase. As expected, there is an increment in MTTF as the values of operating component 'M' increases.

ISBN: 978-988-19253-9-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

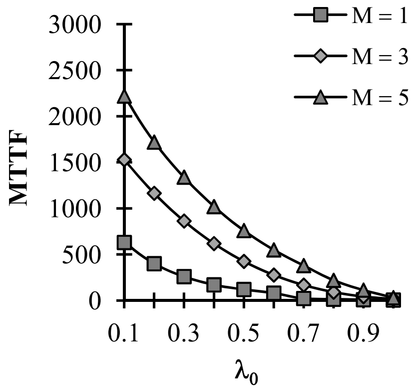

Fig. 4. Mean time-to- system failure for λ_0 by varying M.

.

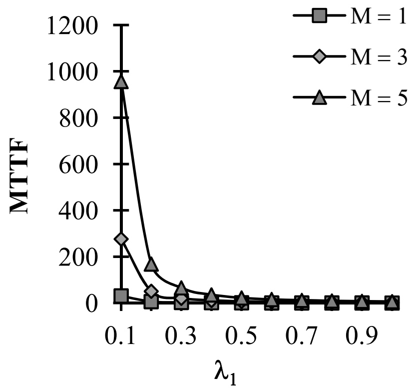

Fig. 5. Mean time-to- system failure for λ_1 by varying M.

Overall, based on numerical experiment, we conclude that

- On increasing the failure rates λ_0 and λ_1, the system reliability decreases.
- Expected life span of the system is significantly affected by the number of operating components as MTTF shows increasing trend which is quite visible.
- It is evident that the expected life time of the system can be increased significantly by incorporating more number of operating/standby components in the system but it may be costly affair.

VI. Conclusion

In the present study, we have studied multi component repairable system under N-policy, setup, standbys and imperfect coverage. The explicit expression has been provided for reliability function. It is concluded that the concept of N-policy is better for hiring a repairman in comparison to full time repairman. The developed model can be implemented to predict the performance behavior of production system, telecommunication system, flexible manufacturing system, etc.

References

[1] P. A. Keiller and D. R. Miller, "On the use and he performance of software reliability growth models," *Reliability Engineering & System Safety*, 1991, 32, pp. 95-117.

[2] K. C. Kapur and L. R. Lamberson, "Reliability in Engineering Design," John Wiley & Sons, 1977, pp 405-414.

[3] C. Y. Huang and M. R.Lyu, "Framework for modeling software reliability, using various efforts and fault-detection rates", *IEEE Transaction on Reliability*, 2001, 50, 310- 320.

[4] W. Ahmed and Y. W. Wu, "A survey on reliability in distributed systems", *Journal of Computer and System Sciences*, 2013, 79, pp. 1243-1255.

[5] B. D. Sivazlian and K. H. Wang, "Economic analysis of the M/M/R machine repair problem with warm standbys", *Microelectronics Reliability*, 1989, 29, pp. 25–35.

[6] J. C. Ke and K. H. Wang, "Vacation policies for machine repair problem with two type spares", *Applied Mathematical Modelling*, 2007, 31, pp. 880–894.

[7] Y. L. Hsu, J. C. Ke and T. H. Liu, "Standby system with general repair, reboot delay, switching failure and unreliable repair facility– A statistical standpoint", *Mathematics and Computers in Simulation*, 2011, 81, pp. 2400–2413.

[8] J. C. Ke and C. H. Wu, "Multi-server machine repair model with standbys and synchronous multiple vacation", *Computers & Industrial Engineering*, 2012, 62, pp. 296–305.

[9] M. Jain, "Transient analysis of machining systems with service interruption, mixed standbys and priority", *International Journal of Mathematics in Operational Research*, 2013, 5, pp. 604–625.

[10] S. V. Amari, J. B. Dugan and R. B. Misre, "A separable method for incorporating imperfect fault-coverage into combinatorial models", *IEEE Transactions on Reliability*, 1999, 48, pp. 267-274.

[11] S. V. Amari, A. F. Myers, A. Rauzy and K. S. Trivedi, "Imperfect coverage models: status and trends", Handbook of Performability Engineering, Springer, Berlin, 2008, 321-348.

[12] K. H. Wang, T. C. Yen and Y. C. Fang, "Comparison of availability between two systems with warm standby units and different imperfect coverage", *Quality Technology and Quantitative Management,* 2011, 9, pp. 265-282.

[13] M. S. Moustafa, " Reliability analysis of K-out-of-N: G systems with dependent failures and imperfect coverage", *Reliability Engineering and System Safety*, 1997, 58, pp. 15–17.

[14] Amari, S. V., Dugan, J. B., & Misra, R. B. (1999). Optimal reliability of systems subject to imperfect fault-coverage. IEEE Transactions on Reliability, 48, 275–284.

[15] Amari, S. V., Pham, H., & Dill, G. (2004). Optimal design of K-out of-N: G subsystems subject to imperfect fault coverage. IEEE Transactions on Reliability, 53, 567–575.

[16] M. Jain and K. Kumar, "Threshold N-policy for (M, m) degraded machining system with heterogeneous servers, standby switching failure and multiple vacation", *International Journal of Mathematics in Operational Research*, 2013, 5, pp. 423–445.

[17] H. Van As, "Transient analysis of Markov queueing systems and its application to congestion-control modeling," *IEEE J. Selected Area. Communication*, 2003, 4, pp. 891-904.