Stochastic Decision Model of the Remanufactured Product with Warranty

Swee S. Kuik, Toshiya Kaihara, and Nobutada Fujii

Abstract— **Due to an increase of the used product disposal, the end of life (EOL) management has drawn significant attention by manufacturers. One of the critical concerns is the reliability and quality of the remanufactured product. In order to increase consumer satisfaction, the warranty services management has to provide sort of quality assurance for its product post-use life. However, higher warranty price tags may often reduce consumers' purchasing power and, therefore manufacturers need to determine the appropriate price tags for their manufactured products in order to gain significant market competitiveness. This article presents the mathematical models to examine two types of the proposed extended warranty policies for manufacturers to make the comparisons of their possible gained profit of remanufactured products by manufacturers. The optimization models were numerically solved by sequential quadratic programming. An outcome of the numerical example showed that manufacturer's warranty type-I policy with time factor was generally more desirable than warranty type-II policy with failure counts for implementation.**

Index Terms—**sustainable manufacturing, product recovery, reliability, warranty**

I. Introduction

The consumer markets for purchasing remanufactured products have increased significantly due to the recent changes in environmental legislative regulations and as well as the used product disposal requirements [1]. Numerous customers are now willing to consider for purchasing various reliable remanufactured products [2].

In today's market, the remanufactured products are also sold with time limit warranty. A common product warranty is usually an agreement made to a purchaser for the specified period during product use stage [3, 4]. Under this specified period, manufacturers are fully responsible for its product replacement, repair and/or refund. Sometimes, manufacturers can also provide certain compensations to a purchaser for a product's major failure [5]. In practice, this specified time limit for a remanufactured product is legally bounded in 3 and/or 12 months of the base warranty. For the extended period of the product servicing life cover, purchasers are required to pay some additional extended warranty costs [3].

For the operational terms and conditions, manufacturers usually have the sole authorization to release and impose their pre-defined specifications of the extended care and operating maintenance procedures for remanufactured products. In fact, purchasers/users must comply with it, otherwise the product warranty may be void without any further consideration. In real life, most purchasers also have their own preference on acquiring the long-lasting remanufactured products with a longer extended period of servicing life by manufacturers [3]. For such a long period of servicing life, it may impact on the associated operating and warranty servicing costs of the remanufactured products. Manufacturers are now very keen on developing an effective strategy to reduce this type of warranty servicing costs [4]. Therefore, the purpose of this article is to study on the extended warranty service period for remanufactured products that may be offered and provided by manufacturers to consumers and to compare their profits gained with different warranty policies.

II. Literature Review

A. Background

The remanufacturing markets have been expanded rapidly in the past decades. There are numerous consumer products that are being produced with those mixed-remanufactured components and/or modules. An overview of the product recovery with a closed loop system is illustrated in Fig. 1, where, in general, there are four alternative disposition decisions, such as those components and/or parts are to be reused, remanufactured and recycled and totally disposed for landfills. The remanufactured product usually consists of various multiple components reuse, remanufacture and recycle [2, 6]. These recovery strategies have been proven as the most effective way to decrease virgin materials usage, as well as to reduce waste disposal treatments [6]. For example, those currently available remanufactured products include air conditioners, heavy machineries, power bearings, water pumps, electrics motors, etc. [6].

In the case of utilizing remanufactured products, most customers are often uncertain for the actual performance and physical reliability during its post-use lifecycle stage. The introduction of the extended warranty period may increase customers' confidence level in its post-use and market competiveness. Further, it also provides the quality assurance to users and/or customers when purchasing remanufactured products. Wu et al. [3] stated that one of the significant operating expenses when producing a consumer product would be warranty servicing costs. In practice, manufacturers must provide free servicing jobs for consumers, whenever their remanufactured products fail to meet the essential requirements of consumers. Huang et al. [7] recommended

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S. Kuik is with the Graduate School of System Informatics, Kobe University, 1-1 Rokkodai, Nada, Kobe, Hyogo, 657-8501 Japan (corresponding author to provide phone: +81-78-803-6250; fax: +81-78-803-6250; e-mail: kuik@ kaede.cs.kobe-u.ac.jp).
T. Kaihara is with the Graduate School of System Informatics, Kobe University, 1-1 Rokkodai, Nada, Kobe, Hyogo, 657-8501 Japan
N. Fujii is with the Graduate School of System Informatics, Kobe University, 1-1 Rokkodai, Nada, Kobe, Hyogo, 657-8501 Japan

the appropriate coordination with faulty product returns and warranty services by purchasers is an important factor to reduce potential conflicts among various parties in reverse supply chain networks. Therefore, the remanufactured product warranty management is one of the significant research fields in quality and reliability improvement [3].

B. Remanufactured Product

In reverse supply chain management, the product recovery strategy is usually established based on various kinds of components reuse, remanufacture and recycle. This has become an increasingly common research focus for the last 20 years, in response to higher costs of waste treatment and increased landfill burdens [2, 6]. In the lifecycle management, the aim of recovery is to rebuild consumer products for sales upon receiving from returns streams. Despite the good product recovery strategy to minimize used product disposal to landfills, there are still many shortcomings that have been identified by researchers. One of the most critical facts is indeed product reliability and warranty that has been faced by most manufacturers for improvement [6].

C. Product Returns Management

One obstruction to the profitability of product recovery for manufacturers is the uncertainty and variation of returns and collection rate of used products in the market. The product returns management is known as the bridge between reverse logistics system and production planning as a whole [2]. Another significant factor is also related to the demand fluctuation due to consumer confidence level of using remanufactured products [2, 6]. Till today, a large number of consumers' preference is still on purchasing a new product, which is fabricated by use of virgin materials and/or partially recycled materials than recovery due to the reliability and quality issues. By providing a certain quality assurance to consumers by manufacturers, consumers can choose to use remanufactured products without any worries [8, 9].

In recent years, manufacturers are still concerned with the increased recovery values for producing remanufactured products in the market and at the same time, minimize the warranty servicing costs. Researchers [2, 6] also stressed on the increased returns incentive and implementation of the

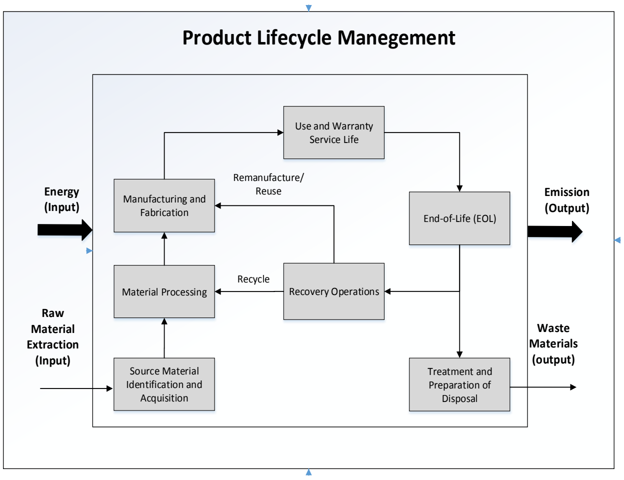

Fig. 1. An overview of the manufactured product with recovery operations

effective transparent returns procedures by manufacturers that could improve used product returns management.

Furthermore, the European environmental sustainability committee has now emphasized on the extended producer responsibility (EPR) [1]. The EPR policy may stimulate manufacturers to focus on the recovery value within returns streams when the remanufactured products are produced.

Several research studies also considered the modelling of product recovery problems, including the optimization of design for assembling and/or disassembling, maximization of recovery weight upon returns, minimization of warranty services costs for second-hand products, and maintenance strategies for used products [1, 2, 6]. However, most of them focused on the deterministic modeling scenario in product returns with recovery operations.

In addition, the increased value of recovery savings does not guarantee on the actual gained profits by manufacturers as the product warranty servicing cost is also one of the most significant operating expenses within manufacturing system during its post-use stage [3, 7]. Therefore, there are numerous research studies that focus on the warranty servicing costs for used products to gain market competiveness but none of them focuses on how the recovery configuration option may impact on the extended product warranty during its post-use life. This research area still remains as a non-resolved issue in current literature.

III. Model Formulation

This section presents the analytical modeling to determine warranty services associated costs with the remanufactured products and sales to the buyers. In this article, there are two general classifications of the warranty policies that are considered for further analyses.

First, an analysis of the product's servicing life with the Type-I warranty service offer to the buyer, which means that after base warranty service that has been expired and the extended period of use stage is introduced. The manufacturer has obligations to repair or replace any part/product under the extended period if there is product's failure during use stage.

Second, an analysis of the product's servicing life with Type-II warranty policy considers the number of failures of a remanufactured product in its use stage that are considered under the obligations by the manufacturer to repair and/or replace failure part/product. This simply implied that if the number of failures exceeded anytime, the manufacturer had no obligation for offering free product's servicing job.

For the estimation of failure scenario, it is usually assumed as Weibull distribution. In the modeling, the Weibull probability density function is applied and expressed as follows [6, 7]:

$$R(t) = \frac{\psi}{\theta}\left(\frac{t}{\theta}\right)^{\psi-1} \exp\left(-\left(\frac{t}{\theta}\right)\right)^{\psi} \tag{1a}$$

where the failure rate is quantitatively defined as follows:

$$s(t) = \frac{\psi}{\theta}\left(\frac{t}{\theta}\right)^{\psi-1} \tag{1b}$$

For the estimation of the cost of producing remanufactured product with different recovery configuration options for each of the used component (i.e. virgin, reuse, remanufacture, and recycle), the associated equations are mathematically expressed as follows (2)-(5):

$$C_{\mathrm{V}ir} = X_{1,i}\left(C_8 + \sum_{op\in\{1\ldots3\}} C_{op,i}\right) \tag{2}$$

$$C_{\mathrm{Re}use} = X_{2,i}\left(\sum_{op\in\{3\ldots5\}} C_{op,i}\right) \tag{3}$$

$$C_{\mathrm{Re}man} = X_{3,i}\left(\sum_{op\in\{3\ldots6\}} C_{op,i}\right) \tag{4}$$

$$C_{\mathrm{Re}cycl} = X_{4,i}\left(C_{7,i} + \sum_{op\in\{2\ldots5\}} C_{op,i}\right) \tag{5}$$

Therefore, the cost of producing a remanufactured product, C_p, is calculated as the summation of all the associated costs with the options of virgin, reuse, remanufacture and recycle for i^{th} component.

A. Notations

The mathematical notations used in this study to formulate the stochastic decision model of remanufactured product with different warranty policies are summarized in Table I.

TABLE I
NOTATIONS AND DESCRIPTION

Symbol	Description
ψ	*>0, Weibull shape parameter*
θ	*>0, Weibull scale parameter*
i	*Product component index, where i=1, 2…*
$X_{1,i}$	*=1 if i^{th} component is virgin, otherwise =0*
$X_{2,i}$	*=1 if i^{th} component is reused, otherwise =0*
$X_{3,i}$	*=1 if i^{th} component is remanufactured, otherwise =0*
$X_{4,i}$	*=1 if i^{th} component is recycled, otherwise =0*
op	*op^{th} operating process*
$C_{1,i}$	*Raw material acquisition cost for i^{th} component*
$C_{2,i}$	*Manufacturing cost for i^{th} component*
$C_{3,i}$	*Assembly cost for i^{th} component*
$C_{4,i}$	*Direct reuse associated cost for i^{th} component*
$C_{5,i}$	*Disassembly cost for i^{th} component*
$C_{6,i}$	*Remanufacturing cost for i^{th} component*
$C_{7,i}$	*Recycling cost for i^{th} component*
$C_{8,i}$	*Disposal cost for i^{th} component*
C_s	*Selling price of remanufactured product*
C_p	*Costs of remanufacturing product*
$C_{EX,j}$	*Extended warranty costs with policy j*
C_{mi}	*Servicing costs of minor related failure for the manufacturer*
ϕ	*>0, selling price flexibility ratio of remanufactured product*
σ	*Warranty period flexibility ratio in [0,1]*
κ_j	*>0, extended warranty period flexibility ratio for policy, j*
δ	*>0, extended warranty price flexibility ratio*
v_1	*≥0, demand expansion factor*
v_2	*>0, warranty length constant*
$\tilde{v}_j$	*≥0, demand expansion factor with extended warranty for policy, j*
$E[y_j]$	*Renewal warranty policy, j, i.e. if j=1, policy with time factor and if j=2, policy with failure counts*
T	*Extended warranty for the Type-I warranty policy, time period*
u	*Extended warranty for Type II warranty policy, number of failure counts*
p	*Probability with minor failure for the remanufactured product of its use stage*
q	*Probability with minor failure for the remanufactured product of its use stage*
$s(t)$	*Failure rate for a remanufactured product*
l_w	*Base service warranty length of its use stage*
$R(t)$	*pdf of failure with respect to time of a remanufactured product*
D	*Demand function using Glickman-Berger equation*
$D_{p,j}$	*Demand function with policy j as Glickman-Berger equation*
$D_{p,j}^{EX}$	*Demand function with extended warranty under policy j using Glickman-Berger equation*
$G(t)$	*Expected number of the failures in t*
$AVC_{m,j}$	*Average cost per lifecycle with policy j by manufacturer*
$AVC_{b,j}$	*Average cost per lifecycle with policy j for buyer*
$AVC_{mp,j}$	*Average cost with policy j for manufacturer, where the remanufactured product is purchased*
$AVC_{bp,j}$	*Average cost with policy j for buyer, where the remanufactured product is purchased*
$AVC_{mp,j}^{EX}$	*Average cost with policy j for manufacturer, where the remanufactured product with extended warranty is purchased*
$AVP_{pc,j}$	*Average profit per lifecycle with policy j by the manufacturer*
$AVP_{mp,j}$	*Average earned profit by the manufacturer with policy j, where only the remanufactured product is purchased*
$AVP_{mp,j}^{EX}$	*Average earned profit by the manufacturer with policy j, where the remanufactured product with extended warranty is purchased*
TP_j	*Total gained profit with policy j by the manufacturer*

B. Description of the Warranty Policy

As mentioned previously, in this study, there are two types of warranty servicing policies for buyers that are offered by manufacturers when purchasing the remanufactured product. For simplification, the demand function, to be considered in this study, is applied from the Glickman-Berger equation [5]. It is written as follows:

$$D = v_1 C_s^{-\phi}(l_w + v_2)^{\sigma} \tag{6}$$

The above demand function as shown in Eq. (6) is modified to suit for the case of the extended warranty periods with remanufactured product. The modified demand function is then expressed as follows:

$$D_{p,j}^{EX}\left(E\left[y_j\right]+C_{EX,j}\right)=\tilde{v}_j C_s^{-\phi}(l_w+v_2)^{\sigma} E(y_j)^{\kappa_j} C_{EX,j}^{-\delta} \tag{7}$$

For Type-I and Type-II warranty policies, these are expressed as $E[y_1]=T$ and $E[y_2]=u$ respectively. In the following section, the details of mathematical derivations with different product warranty policies, which are offered by the manufacturers, are presented.

C. *Type-I Warranty Policy*

In this section, the cost equations of the Type-I warranty policy are formulated by considering the manufacturer's and buyer's perspectives.

For simplification, the average related cost for a buyer, who is interested to purchase and/or use the remanufactured product, is expressed as the selling price of a remanufactured product over the base warranty period as shown in Eq. (8).

$$AVC_{bp,1}=\frac{C_s}{l_w} \tag{8}$$

For the remanufactured product without the extended warranty, the average cost for a manufacturer is expressed as the associated costs when producing a remanufactured product and servicing costs, which may be incurred for a product's failure (either it is the minor failure, C_{mi} or the major failure, C_{ma}) over the base warranty length. $G(t)$ is defined as the expected number of failures at the time interval $\left[0,l_w\right]$ as shown in Eq. (9).

$$AVC_{mp,1}=\frac{C_p+G(t)(pC_{mi}+qC_{ma})}{l_w} \tag{9}$$

With fixed period of the extended warranty, the average cost for a buyer is then expressed as the selling price of a remanufactured product with extended warranty cover for the base and extended periods as shown in Eq. (10).

$$AVC_{bp,1}^{EX}=\frac{C_s+C_{EW,1}}{E[y_1]+l_w} \tag{10}$$

In addition of the fixed period warranty for a buyer, the average costs for a manufacturer, who has obligation to repair and/or replace minor or major failures for a remanufactured product, is calculated as follows:

$$AVC_{mp,1}^{EX}=\frac{C_p+G(t)(pC_{mi}+qC_{ma})}{E\left[y_1\right]+l_w} \tag{11}$$

In the mathematical derivation, it is also assumed that the failure rate is constant and the minor failure, m_i happened during the time interval of $[0,t_k]$, where t_k is the time taken before next major failure occurs, $k\in\{1,2,3,\dots n\}$ and next minor failure happened during the time interval of $[0,t_c]$, where it is at t_c for $c\in\{1,2,3,\dots m_k\}$.

As defined clearly in the notation, if $s_{mi}(t)=p\cdot s(t)$, it is known as the steady failure rate of a remanufactured product for minor failure and if $s_{ma}(t)=q\cdot s(t)$, it is known as the steady failure rate of a remanufactured product for major failure. The average cost incurred by a manufacturer is then expressed as follows:

$$AVC_{mp,1}=\frac{C_p+C_A}{l_w} \tag{12}$$

Furthermore, the average costs of the extended warranty to be purchased by a buyer during the time period of $E[y_1]+l_w$ is expressed as follows:

$$AVC_{mp,1}^{EX}=\frac{C_p+C_A}{E\left[y_1\right]+l_w} \tag{13}$$

where the associated costs of the Type-I warranty policy for a manufacturer, C_A is expressed as follows:

$$C_A=\sum_{k=1}^{n}\left[C_{mi}\sum_{c=1}^{m_k}\int_{t_c-1}^{t_c} s_{mi}(t)dt+C_{ma}\int_0^{t_k} s_{ma}(t)dt\right] \tag{14}$$

D. *Type-II Warranty Policy*

In this type of warranty policy, the failure number count, u for servicing jobs are considered. The average cost with extended warranty by the number of failure counts. It is expressed as follows:

$$AVC_{mp,2}^{EX}=\frac{C_p+C_B}{E[y_2]+l_w} \tag{15}$$

where the associated costs of Type-II warranty policy for a manufacturer, C_B is expressed as follows:

$$C_B=\sum_{k=1}^{n}\left[C_{mi}\int_0^{t_k} s_{mi}(t)dt+C_{ma}\int_0^{t_k} s_{ma}(t)dt\right]+E[y_2](pC_{mi}+qC_{ma}) \tag{16}$$

Meanwhile, the average cost with extended warranty based on the number of failure counts for a buyer is expressed as follows:

$$AVC_{bp,2}^{EX}=\frac{C_s+C_{EW,2}}{E[y_2]+l_w} \tag{17}$$

E. *Profit Gained by Manufacturers*

In this section, the generic decision stochastic model for the gained profit evaluation of a manufacturer is developed in order to examine the Type-I and Type-II warranty scenario for remanufactured products.

The total profit gained by a manufacturer is calculated as the multiplication of the total sales volume with or without inclusion of the extended warranty period under policy, j.

The equations for both profit gained for a manufacturer with Type-I and II warranty policies are derived as follows:

$$TP_1=AVP_{mp,1}^{EX}D_{EX,1}+AVP_{mp,1}\left(D-D_{EX,1}\right) \tag{18}$$

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$$TP_2 = AVP^{EX}_{mp,2} D_{EX,2} + AVP_{mp,2}\left(D - D_{EX,2}\right) \tag{19}$$

where (a) the average profit gained for a manufacturer is calculated as the difference between the average profit per product lifecycle for a buyer and a manufacturer, as shown in (20), (b) the average profit gained without extended warranty for the manufacturer as shown in (21), and (c) the average profit gained with the extended warranty for a manufacturer as shown in (22).

$$AVP_{pc,j} = AVC_{b,j} - AVC_{m,j} \tag{20}$$

$$AVP_{mp,j} = AVC_{bp,j} - AVC_{mp,j} \tag{21}$$

$$AVP^{EX}_{mp,j} = AVC^{EX}_{bp,j} - AVC^{EX}_{mp,j} \tag{22}$$

F. Quadratic programming

This section presents the constrained optimization models for profit gained by a manufacturer with Type-I and II warranty policies and solution methods using sequential quadratic programming (SQP).

SQP is a useful solution method for analyzing non-linear maximization problems. As shown in (18) and (19), both of them are non-linear objective functions. The solution method for solving non-linear optimization models by SQP is presented here.

For Type-I warranty policy, the objective function for the profit gained by a manufacturer in relation to time factor is derived as follows:

$$\textit{Maximize} \quad AVP^{EX}_{mp,1} D_{EX,1} + AVP_{mp,1}\left(D - D_{EX,1}\right) \tag{23}$$

subject to

$$\begin{aligned} &C_{EX,1} - C_s \le 0, && \text{for } C_s, C_{EX,2} \in R \\ &T - \tilde{T} \le 0, && \text{for } T, \tilde{T} \in N \\ &C_{EX,1} \ge 0 \\ &T \ge 0 \end{aligned}$$

For Type-II warranty policy, the objective function for the profit gained by a manufacturer in relation to the number of failure counts is derived as follows:

$$\textit{Maximize} \quad AVP^{EX}_{mp,2} D_{EX,2} + AVP_{mp,2}\left(D - D_{EX,2}\right) \tag{24}$$

subject to

$$\begin{aligned} &C_{EX,2} - C_s \le 0, && \text{for } C_s, C_{EX,2} \in R \\ &u - \tilde{u} \le 0, && \text{for } u, \tilde{u} \in N \\ &C_{EX,2} \ge 0, \\ &u \ge 0 \end{aligned}$$

The SQP solution method is then proposed in order to solve numerically with the constrained non-linear optimization problem, provided that the requirements are satisfied with the concave function, *f(x)* and convex function, *g(x)* under the Karush-Kuhn-Tucker optimality conditions (KKT).

A general expression of the non-linear optimization model with the equality and non-equality constraints are expressed as follows:

$$\textit{Minimize} \quad f(x) \text{ for } x \in \mathbb{R}^n \tag{25}$$

subject to

$$\begin{aligned} &g_i(x) = 0 \quad i = 1,2,3,\ldots,m_e \\ &g_i(x) \le 0 \quad i = 1,2,3,\ldots,m \end{aligned}$$

In case of the inequality constraints that exist for this type of optimization problem, the appropriate function is then identified using interior penalty approach. This function is under the conditions of non-differentiable and discontinues. It is written as follows:

$$\textit{Minimize} \quad f(x) + \lambda(x) \tag{26}$$

where the appropriate function is expressed as:

$$\lambda(x) = \gamma \sum_{i=1}^{m} g_i(x) \tag{27}$$

Therefore, the objective function of this study by considering the absolute-value function is defined as follows:

$$\textit{Minimize} \quad f(x) + \gamma \sum_{i=1}^{m} g_i(x) \tag{28}$$

By applying the solution method, it may generate a series of the interior points and then converge to a specific global optimum solution in the non-linear optimization problem. In this study, the Matlab platform is used for calculations. The solver is known as *'fmincon'* and is used for SQP.

IV. Numerical Example

This section presents the numerical example for comparing Type-I and Type-II warranty policies. In practice, most remanufactured products with the extended period are being offered by the manufacturers. For this study, the experimental simulations are restricted to the remanufactured products with failure behavior within short lifespan. Even though numerous commercial remanufactured products have the base warranty period of approximately 3-12 months, for this simulation, the extended period length for the warranty is fixed at 12-month, which is about 1 year.

TABLE II
USED PARAMETERS FOR SIMULATION

Symbol	Value	Symbol	Value
l_w	1	C_s	260
ϕ	3.8	σ	0.76
κ_1	1.32	κ_2	1.16
ν_1	1.10×10^{15}	ν_2	0.10
$\tilde{\nu}_1$	1.18×10^{17}	$\tilde{\nu}_2$	5.21×10^{16}
θ	2	ψ	1.8
δ	1.3		

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The parameters are listed in Table II and III for the analysis of Type-I and Type-II warranty policies. In this scenario, the manufacturer requires to utilize a total of 20-separate components (product component no. CP-1 to CP-20) for producing a remanufactured product. Each of these separate components has different disposition decisions. These components can be either virgin, reused, remanufactured, or recycled. Two selected recovery configurations, which are named as "TA" and "TB", are examined in this study. The comparative studies of both warranty servicing policies are also presented. In this simulation, the upper and lower boundaries are chosen as T is between 1 to 5, u is between 2 to 6 and the condition of $C_{EX,j}$ is used between 10 and 260.

TABLE III
COSTS WITH RECOVERY OPTIONS FOR 20 SEPARATE COMPONENTS

CP	C_{Vir}	C_{Reuse}	C_{Reman}	C_{Recycl}
1	*8.21*	*4.92*	*8.13*	*7.52*
2	*6.91*	*1.84*	*5.19*	*4.42*
3	*6.65*	*1.69*	*4.35*	*4.52*
4	*5.74*	*1.94*	*4.30*	*4.49*
5	*5.56*	*0.84*	*5.09*	*3.48*
6	*4.35*	*0.92*	*4.04*	*1.47*
7	*3.99*	*0.97*	*2.99*	*2.25*
8	*3.84*	*1.29*	*2.61*	*2.44*
9	*4.71*	*1.58*	*3.79*	*3.12*
10	*7.34*	*1.71*	*5.36*	*5.07*
11	*7.32*	*2.25*	*7.21*	*6.46*
12	*8.44*	*2.27*	*5.49*	*5.99*
13	*5.94*	*1.38*	*5.33*	*5.86*
14	*6.19*	*1.23*	*5.88*	*2.54*
15	*4.55*	*1.23*	*4.29*	*2.34*
16	*5.17*	*1.44*	*4.48*	*3.02*
17	*6.23*	*2.01*	*5.95*	*5.47*
18	*5.81*	*1.60*	*4.44*	*5.43*
19	*6.56*	*2.45*	*5.32*	*5.83*
20	*6.36*	*3.80*	*6.27*	*5.69*

V. RESULTS AND DISCUSSIONS

In this example, the "TA" recovery configuration consists of 2-reused, 2-remanufactured, 4-recycled and 18-virgin components when producing a remanufactured product. The obtained results in Table IV showed that Type-I warranty policy (i.e. warranty price index of $103.03) was slightly better than Type-II warranty policy (i.e. warranty price index of $186.25) for a manufacturer.

TABLE IV
OPTIMUM SOLUTIONS FOR REMANUFACTURED "TA" PRODUCT

Warranty Policy Type I	Value	Warranty Policy Type II	Value
$C_{EX,1}$	103.0254	$C_{EX,2}$	186.2541
T	12	u	2
TP_1	2.425x10^5	TP_2	2.312x10^5
$D_{p,1}^{EX}$	8.725x10^3	$D_{p,2}^{EX}$	3.498x10^3

Next comparison is to analyze another possible recovery configuration option (i.e. configuration of "TB"), which has proposed to utilize 2-reused, 4-remanufactured, 3-recycled, 17-vrigin components when producing a remanufactured product. The obtained results in Table V also revealed that Type-I warranty policy (i.e. warranty price index of $125.52) was still outperformed than Type-II warranty policy (i.e. warranty price index of $214.76) in this comparative study.

TABLE V
OPTIMUM SOLUTIONS FOR REMANUFACTURED "TB" PRODUCT

Warranty Policy Type I	Value	Warranty Policy Type II	Value
$C_{EX,1}$	125.5214	$C_{EX,2}$	214.7548
T	12	u	2
TP_1	3.191x10^5	TP_2	3.751x10^5
$D_{p,1}^{EX}$	8.218x10^3	$D_{p,2}^{EX}$	3.117x10^3

Overall, the profit gained by a manufacturer for Type-I warranty policy is much higher than Type-II warranty policy for both configurations. Furthermore, the selection of recovery configuration options for a remanufactured product is also regarded as a critical aspect in manufacturing industries. Especially, the virgin material supply associated costs, and used product disposal treatment costs have been increased significantly in recent years. This study analyzed different recovery configurations (i.e. "TA" & "TB") for the proposed warranty service policies to be considered by a manufacturer. The results showed that warranty policy Type-I was more desirable than Type-II warranty policy.

VI. CONCLUDING REMARKS

In summary, this study has demonstrated the analysis of warranty servicing costs and profit gained for remanufactured product based on Type-I and Type-II warranty policies. Both are considered as quite effective warranty policies to be offered by manufacturers to consumers. Results of these warranty servicing policies also showed the profit gained for manufacturers. In addition, the obtained results revealed that the difference of recovery configuration options may have direct impact on the profit gained by a manufacturer. For future research works, various recovery configuration options with different quality and reliability will be examined for the costs of producing remanufactured products and warranty costs while maximizing profits gained for manufacturers.

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