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An Iterative Algorithm for Generalized Mixed Equilibria with Variational Inequalities

Lu-Chuan Ceng *, Yung-Yih Lur † and Ching-Feng Wen ‡

Abstract—**In this paper, we shall introduce an iterative algorithm by multi-step implicit hybrid steepest-descent method for finding a common element of the set of solutions of a finite family of generalized mixed equilibrium problems, the set of solutions of a finite family of variational inequalities for inverse strongly monotone mappings and the set of fixed points of a countable family of nonexpansive mappings in a real Hilbert space. We also prove strong and weak convergence theorems for the proposed iterative algorithm under appropriate conditions. Our results improve and extend the earlier and recent results in the literature.**

Keywords: Implicit hybrid steepest-descent method; Generalized mixed equilibrium; Variational inequality; Non-expansive mapping

1 Introduction

Let H be a real Hilbert space with inner product $\langle\cdot,\cdot\rangle$ and norm $\|\cdot\|$, C be a nonempty closed convex subset of H and P_C be the metric projection of H onto C. Let $S : C \to C$ be a self-mapping on C. We denote by $\mathrm{Fix}(S)$ the set of fixed points of S and by $\mathbf{R}$ the set of all real numbers. A mapping V is called strongly positive on H if there exists a constant $\bar{\gamma} > 0$ such that

$$\langle Vx, x\rangle \geq \bar{\gamma}\|x\|^2, \quad \forall x \in H.$$

A mapping $A : C \to H$ is called L-Lipschitz continuous if there exists a constant $L \geq 0$ such that

$$\|Ax - Ay\| \leq L\|x - y\|, \quad \forall x, y \in C.$$

In particular, if $L = 1$ then A is called a nonexpansive mapping; if $L \in [0, 1)$ then A is called a contraction.

Let $A : C \to H$ be a nonlinear mapping on C. We consider the following variational inequality problem (VIP): find a point $x \in C$ such that

$$\langle Ax, y - x\rangle \geq 0, \quad \forall y \in C. \tag{1.1}$$

The solution set of VIP (1.1) is denoted by $\mathrm{VI}(C, A)$.

The VIP (1.1) was first discussed by Lions [8] and now is well known. The VIP (1.1) has many applications in computational mathematics, mathematical physics, operations research, mathematical economics, optimization theory, and other fields; see, e.g., [2,3,5,24]. Not only the existence and uniqueness of solutions are important topics in the study of VIP (1.1), but also how to actually find a solution of VIP (1.1) is important. There are a lot of different approaches towards solving VIP (1.1) in finite-dimensional and infinite-dimensional spaces, and the research is intensively continued.

In 1976, Korpelevich [1] proposed an iterative algorithm for solving the VIP (1.1) in Euclidean space $\mathbf{R}^n$:

$$\begin{cases} y_n = P_C(x_n - \tau Ax_n), \\ x_{n+1} = P_C(x_n - \tau Ay_n), \quad \forall n \geq 0, \end{cases}$$

with $\tau > 0$ a given number, which is known as the extragradient method (see also [13]). The literature on the VIP is vast and Korpelevich's extragradient method has received great attention given by many authors, who improved it in various ways; see e.g., [4,6-7,10-12,17-19,22,27-28,29-33] and references therein, to name but a few.

Let $\varphi : C \to \mathbf{R}$ be a real-valued function, $A : H \to H$ be a nonlinear mapping and $\Theta : C \times C \to \mathbf{R}$ be a bifunction. In 2008, Peng and Yao [17] introduced the following generalized mixed equilibrium problem (GMEP) of finding $x \in C$ such that

$$\Theta(x, y) + \varphi(y) - \varphi(x) + \langle Ax, y - x\rangle \geq 0, \quad \forall y \in C. \tag{1.2}$$

We denote the set of solutions of GMEP (1.2) by $\mathrm{GMEP}(\Theta, \varphi, A)$. The GMEP (1.2) is very general in the sense that it includes, as special cases, optimization problems, variational inequalities, minimax problems, Nash equilibrium problems in noncooperative games and others. The GMEP is further considered and studied; see e.g., [9,14,16,19-20,33].

We present some special cases of GMEP (1.2) as follows.

*Department of Mathematics, Shanghai Normal University, and Scientific Computing Key Laboratory of Shanghai Universities, Shanghai 200234, China. Email: zenglc@hotmail.com

†Department of Industrial Management, Vanung University, Taoyuan, Taiwan. Email:yylur@vnu.edu.tw. Research is partially supported by grants of MOST 103-2115-M-238-001.

‡Corresponding author. Center for Fundamental Science, Kaohsiung Medical University, Kaohsiung, Taiwan. Email: cfwen@kmu.edu.tw. Research is partially supported by grants of MOST 103-2115-M-037-001.

If $\varphi = 0$, then GMEP (1.2) reduces to the generalized equilibrium problem (GEP) which is to find $x \in C$ such that

$$\Theta(x, y) + \langle Ax, y - x\rangle \geq 0, \quad \forall y \in C.$$

It is introduced and studied by Takahashi and Takahashi [21]. The set of solutions of GEP is denoted by GEP(Θ, A).

If $A = 0$, then GMEP (1.2) reduces to the mixed equilibrium problem (MEP) which is to find $x \in C$ such that

$$\Theta(x, y) + \varphi(y) - \varphi(x) \geq 0, \quad \forall y \in C.$$

It is considered and studied in [15]. The set of solutions of MEP is denoted by MEP(Θ, φ).

If $\varphi = 0,\ A = 0$, then GMEP (1.2) reduces to the equilibrium problem (EP) which is to find $x \in C$ such that

$$\Theta(x, y) \geq 0, \quad \forall y \in C.$$

It is considered and studied in [25]. The set of solutions of EP is denoted by EP(Θ). It is worth to mention that the EP is an unified model of several problems, namely, variational inequality problems, optimization problems, saddle point problems, complementarity problems, fixed point problems, Nash equilibrium problems, etc.

Throughout this paper, it is assumed as in [17] that $\Theta : C\times C \to \mathbf{R}$ is a bifunction satisfying conditions (A1)-(A4) and $\varphi : C \to \mathbf{R}$ is a lower semicontinuous and convex function with restriction (B1) or (B2), where

(A1) $\Theta(x, x) = 0$ for all $x \in C$;

(A2) Θ is monotone, i.e., $\Theta(x, y) + \Theta(y, x) \leq 0$ for any $x, y \in C$;

(A3) Θ is upper-hemicontinuous, i.e., for each $x, y, z \in C$,

$$\limsup_{t\to 0^+} \Theta(tz + (1 - t)x, y) \leq \Theta(x, y);$$

(A4) $\Theta(x, \cdot)$ is convex and lower semicontinuous for each $x \in C$;

(B1) for each $x \in H$ and $r > 0$, there exists a bounded subset $D_x \subset C$ and $y_x \in C$ such that for any $z \in C \setminus D_x$,

$$\Theta(z, y_x) + \varphi(y_x) - \varphi(z) + \frac{1}{r}\langle y_x - z, z - x\rangle < 0;$$

(B2) C is a bounded set.

Next we list some elementary conclusions for the MEP.

Proposition 1.1 (see [15]). Assume that $\Theta : C\times C \to \mathbf{R}$ satisfies (A1)-(A4) and let $\varphi : C \to \mathbf{R}$ be a proper lower semicontinuous and convex function. Assume that either (B1) or (B2) holds. For $r > 0$ and $x \in H$, define a mapping $T_r^{(\Theta,\varphi)} : H \to C$ as follows:

$$\begin{aligned} T_r^{(\Theta,\varphi)}(x) \quad = \quad & \{z \in C : \Theta(z, y) + \varphi(y) - \varphi(z) + \\ & \frac{1}{r}\langle y - z, z - x\rangle \geq 0, \forall y \in C\} \end{aligned}$$

for all $x \in H$. Then the following hold:

(i) for each $x \in H,\ T_r^{(\Theta,\varphi)}(x) \neq \emptyset$;

(ii $T_r^{(\Theta,\varphi)}$ is single-valued;

(iii) $T_r^{(\Theta,\varphi)}$ is firmly nonexpansive, that is, for any $x, y \in H$,

$$\|T_r^{(\Theta,\varphi)}x - T_r^{(\Theta,\varphi)}y\|^2 \leq \langle T_r^{(\Theta,\varphi)}x - T_r^{(\Theta,\varphi)}y, x - y\rangle;$$

(iv) Fix$(T_r^{(\Theta,\varphi)}) =$ MEP(Θ, φ);

(v) MEP(Θ, φ) is closed and convex.

Combining the hybrid steepest-descent method in [26] and hybrid viscosity approximation method in [23], Ceng *et al.* [20] proposed and analyzed an iterative method for finding a common element of $\cap_{i=1}^N \mathrm{Fix}(S_i) \cap$ GMEP(Θ, φ, A), the set of solutions of GMEP (1.2) and the set of fixed points of a finite family of nonexpansive mappings $\{S_i\}_{i=1}^N$.

In this paper, we shall introduce an iterative algorithm by multi-step implicit hybrid steepest-descent method for finding a common element of the set of solutions of a finite family of generalized mixed equilibrium problems, the set of solutions of a finite family of variational inequalities for inverse strongly monotone mappings and the set of fixed points of a countable family of nonexpansive mappings in a real Hilbert space. We also prove strong and weak convergence theorems for the proposed iterative algorithm under appropriate conditions. Our results improve and extend the corresponding results announced in Ceng *et al.* [20].

2 Main Results

For the remainder of this paper, we let C be a nonempty closed convex subset of a real Hilbert space H. Recall that a mapping $A : C \to H$ is called

(i) monotone if

$$\langle Ax - Ay, x - y\rangle \geq 0, \quad \forall x, y \in C;$$

(ii) η-strongly monotone if there exists a constant $\eta > 0$ such that

$$\langle Ax - Ay, x - y\rangle \geq \eta\|x - y\|^2, \quad \forall x, y \in C;$$

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(iii) α-inverse-strongly monotone if there exists a constant $\alpha > 0$ such that

$$\langle Ax - Ay, x - y\rangle \geq \alpha\|Ax - Ay\|^2, \quad \forall x, y \in C.$$

Let $\{T_n\}_{n=1}^{\infty}$ be an infinite family of nonexpansive self-mappings on H and $\{\lambda_n\}_{n=1}^{\infty}$ be a sequence of nonnegative numbers in $[0,1]$. For any $n \geq 1$, define a self-mapping W_n on H as follows:

$$\begin{cases} U_{n,n+1} = I, \\ U_{n,n} = \lambda_n T_n U_{n,n+1} + (1-\lambda_n)I, \\ U_{n,n-1} = \lambda_{n-1} T_{n-1} U_{n,n} + (1-\lambda_{n-1})I, \\ \cdots \\ U_{n,k} = \lambda_k T_k U_{n,k+1} + (1-\lambda_k)I, \\ U_{n,k-1} = \lambda_{k-1} T_{k-1} U_{n,k} + (1-\lambda_{k-1})I, \\ \cdots \\ U_{n,2} = \lambda_2 T_2 U_{n,3} + (1-\lambda_2)I, \\ W_n = U_{n,1} = \lambda_1 T_1 U_{n,2} + (1-\lambda_1)I. \end{cases}$$

Such a mapping W_n is called the W-mapping generated by $T_n, T_{n-1}, ..., T_1$ and $\lambda_n, \lambda_{n-1}, ..., \lambda_1$.

Let M, N be two positive integers. We also adopt the following notations:

- For $k \in \{1,2,...,M\}$, Θ_k is a bifunction from $C \times C$ to $\mathbf{R}$ satisfying (A1)-(A4).
- For $k \in \{1,2,...,M\}$, $\varphi_k : C \to \mathbf{R} \cup \{+\infty\}$ is a proper lower semicontinuous and convex function.
- $A_k : H \to H$ and $B_i : C \to H$ are μ_k-inverse strongly monotone and η_i-inverse strongly monotone, respectively, where $k \in \{1,2,...,M\}$, $i \in \{1,2,...,N\}$.
- $F : H \to H$ is a κ-Lipschitzian and η-strongly monotone operator with positive constants $\kappa, \eta > 0$.
- μ and τ are two constants such that $0 < \mu < \frac{2\eta}{\kappa^2}$ and $\tau = 1 - \sqrt{1 - \mu(2\eta - \mu\kappa^2)}$.
- $f : H \to H$ is an l-Lipschitzian mapping with $0 \leq \gamma l < \tau$.
- V is a $\bar{\gamma}$-strongly positive bounded linear operator with $\gamma l < \bar{\gamma}$.

Let $\{T_n\}_{n=1}^{\infty}$ be a sequence of nonexpansive self-mappings on H and $\{\lambda_n\}$ be a sequence in $(0, b]$ for some $b \in (0,1)$. Let W_n be the W-mapping generated by $T_n, T_{n-1}, ..., T_1$ and $\lambda_n, \lambda_{n-1}, ..., \lambda_1$. For arbitrarily given $x_1 \in H$, let $\{x_n\}$ be a sequence generated by the following algorithm:

$$\begin{cases} u_n &= T_{r_{M,n}}^{(\Theta_M,\varphi_M)}(I - r_{M,n}A_M) \cdot \\ & \quad T_{r_{M-1,n}}^{(\Theta_{M-1},\varphi_{M-1})}(I - r_{M-1,n}A_{M-1}) \cdots \\ & \quad T_{r_{1,n}}^{(\Theta_1,\varphi_1)}(I - r_{1,n}A_1)x_n, \\ z_n &= P_C(I - \lambda_{N,n}B_N)P_C(I - \lambda_{N-1,n}B_{N-1}) \cdots \\ & \quad P_C(I - \lambda_{2,n}B_2)P_C(I - \lambda_{1,n}B_1)u_n, \\ y_n &= \alpha_n\gamma f(y_n) + \beta_n z_n + \\ & \quad ((1-\beta_n)I - \alpha_n V)W_n y_n, \\ x_{n+1} &= \sigma_n\gamma f(y_n) + (I - \sigma_n\mu F)W_n y_n, \quad \forall n \geq 1, \end{cases} \tag{2.1}$$

where $\{\lambda_{i,n}\} \subset [a_i, b_i] \subset (0, 2\eta_i)$, $\{r_{k,n}\} \subset [e_k, f_k] \subset (0, 2\mu_k)$, $i \in \{1,2,...,N\}, k \in \{1,2,...,M\}$, and $\{\alpha_n\}, \{\beta_n\}$ and $\{\sigma_n\}$ are three sequences in $(0,1)$. Then we have the following results.

Theorem 2.1. Assume that $\Omega := \cap_{n=1}^{\infty}\text{Fix}(T_n) \cap \cap_{k=1}^{M}\text{GMEP}(\Theta_k, \varphi_k, A_k) \cap \cap_{i=1}^{N}\text{VI}(C, B_i)$ is nonempty and that either (B1) or (B2) holds. Let $\{x_n\}$ be the sequence generated by the algorithm defined in (2.1).

(I) If $\lim_{n\to\infty}\|x_n - x_{n+1}\| = 0$, and the sequences $\{\alpha_n\}, \{\beta_n\}$ and $\{\sigma_n\}$ satisfy the following conditions:

(1) $\lim_{n\to\infty}\sigma_n = 0$ and $\sum_{n=1}^{\infty}\sigma_n = \infty$;

(2) $\lim_{n\to\infty}\frac{\alpha_n}{\sigma_n} = 0$ and $0 < \liminf_{n\to\infty}\beta_n \leq \limsup_{n\to\infty}\beta_n < 1$.

Then the sequence $\{x_n\}$ converges strongly to $x^* \in \Omega$, where $x^* = P_{\Omega}(I - (\mu F - \gamma f))x^*$ is a unique solution of the VIP:

$$\langle(\gamma f - \mu F)x^*, y - x^*\rangle \leq 0, \quad \forall y \in \Omega.$$

(II) If the sequences $\{\alpha_n\}, \{\beta_n\}$ and $\{\sigma_n\}$ satisfy the following conditions:

(1) $\sum_{n=1}^{\infty}\alpha_n < \infty$ and $\sum_{n=1}^{\infty}\sigma_n < \infty$;

(2) $0 < \liminf_{n\to\infty}\beta_n \leq \limsup_{n\to\infty}\beta_n < 1$.

Then the sequence $\{x_n\}$ converges weakly to some $w \in \Omega$.

3 Concluding Remarks

In this paper, we have developed an iterative algorithm by multi-step implicit hybrid steepest-descent method for finding a common element of the set of solutions of a finite family of generalized mixed equilibrium problems, the set of solutions of a finite family of variational inequalities for inverse strongly monotone mappings and the set of fixed points of a countable family of nonexpansive mappings in a real Hilbert space. Our Theorem 2.1 extends, improves and supplements Ceng *et al.* [20] in the following aspects:

(i) The problem of finding a point

$$x^* \in \cap_{n=1}^{\infty}\text{Fix}(T_n) \cap \cap_{k=1}^{M}\text{GMEP}(\Theta_k, \varphi_k, B_k) \cap \cap_{i=1}^{N}\text{VI}(C, A_i)$$

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in our Theorem 2.1 is very different from the problem of finding a point

$$x^* \in \cap_{n=1}^N \mathrm{Fix}(T_n) \cap \mathrm{GMEP}(\Theta, \varphi, A)$$

in Ceng *et al.* [20, Theorem 3.1]. There is no doubt that our problem is more general and more subtle than the problem proposed in Ceng *et al.* [20].

(ii)The iterative scheme in our Theorem 2.1 is more advantageous and more flexible than the iterative scheme in Ceng *et al.* [20, Theorem 3.1] because it involves solving three problems: a finite family of GMEPs, a finite family of VIPs, and the fixed point problem of a countable family of nonexpansive mappings.

(iii) The iterative scheme in our Theorem 2.1 is very different from the iterative scheme in [20, Theorem 3.1] because the iterative scheme in our Theorem 2.1 involves Korpelevich's extragradient method and hybrid steepest-descent method.

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