

# Research in Modeling of Complex Adaptive Petri Net

WANG Jun<sup>1, 2, a</sup>, YU Weidong<sup>1, 3, b</sup>

**Abstract--In this article, a Complex Adaptive Petri Net (CAPN) model was proposed, formalized definition and classified through analyzing the difficulties of Complex Adaptive System (CAS). The properties of activity, reliability and boundedness of CAPN were analyzed by method of reachability tree. On the basis of this, a model of CAPN complexity was constructed. The result of analysis shown that this CAPN could not only solve the process characterization of flexible production and prediction of quality integrated intelligent formal modeling and analysis, but also provide a solving framework for a real-time feedback about intelligent control optimization solution during the complex production process.**

**Key Word-- CAS, CAPN, Reachability Tree, Complexity**

## I. PETRI NET

Petri net is a network for description about events and conditions proposed by German scholar Carl A. Petri in his doctoral thesis in 1962, which was used a simple graph to represent the relationship of concurrency, synchronization and causal<sup>[1]</sup>. At present, Petri net has been applied<sup>[2]</sup> in various model, such as in finite state machine modeling<sup>[3]</sup>, communication protocol,<sup>[4]</sup> synchronization control, production system<sup>[5]</sup>, the form of language, multi-processor systems<sup>[6]</sup> and so on. The development of Petri Net<sup>[7]</sup> has been experienced three stages. The first stage is namely special net theory stage in the 1960s, which is aim to seek analysis technology and application method on the basis of the alone net system as object. The second stage is namely general net theory stage in the 1970s. During this period, the theory systems mainly including concurrent theory, theory of synchronous logic and network topology were developed by studying its classification and relationship between

various networks on the basis of the whole network system as object. The last stage is namely comprehensive development stage since the 1980s. The main content of this stage are the combination of theory and application and the development of the aided tools of computer<sup>[8]</sup>.

The most basic Petri net is defined as follows:

Definition 1: Petri net  $H = \{P, T, F, M_0\}$

- (1) P: P is nonempty finite set of place,
- (2) T: T is nonempty finite set of transition,
- (3) F: F is directed arc set,  $F = P \times T \cup T \times P$ , P and T also meet the condition of  $P \cap T = \emptyset$  and  $P \cup T \neq \emptyset$ ;
- (4)  $M_0$ :  $M_0$  is the initial state.

$N = \{P, T, F\}$  is the basic net. In the Petri network, "○" denotes library, "□" denotes changes. The arrows from library to changes or opposite indicate an arc with concentration. The dynamic behavior of Petri net was described by token distribution denoted with "●" in this figure. The operation of Petri net depends on the token distribution in the Petri net.

The condition of changes trigger is that the number of tokens contained in each output library is more than the number of directed arc from library to changes<sup>[9]</sup>.

## II. CAPN

The complexity of manufacturing systems may lead to quite complex of Petri net. The above multi state interleaving is easy to produce space state explosion. Combined the characteristics of complex adaptive system and the problem encountered during characterization of present production process and in the limitations of Petri net, complex adaptive Petri net (CAPN) theory was proposed in this article. In one hand, this theory could make the complex adaptive process in flexible production process characterized by modeling systematic. In the other hand, it could not only solve the difficult characterization of complex adaptive during the process of modeling, but also solve the resource waste condition caused by applying the other type Petri net.

A. CAPN is Defined as Follows:

Definition 2: CAPN  $H = \{P, T, F, A, C, R, D, \omega\}$ ,

<sup>1</sup> Textile college, Donghua University, Shanghai 201620, P. R. China,

<sup>2</sup> Zhengzhou University of Industrial Technology, Henan 450000, P. R. China,

<sup>3</sup> Key Laboratory of Textile Science & Technology, Ministry of Education, Donghua University, Shanghai 201620, P.R. China

E-mail: <sup>a</sup> cnwjun@gmail.com, <sup>b</sup> wdyu@dhu.edu.cn

$M_0\}$

- (1) P: P is nonempty finite set of place,  $P=\{p_1, p_2, \dots, p_1, \dots, p_m\}$ ;
- (2) T: T is nonempty finite set of transition,  $T=\{t_1, t_2, \dots, t_i, \dots, t_n\}$ ;
- (3) F: F is directed arc set,  $F=P \times T \cup T \times P$ , P and T also meet the condition of  $P \cap T = \emptyset$  and  $P \cup T \neq \emptyset$ ;
- (4) A: A is attribute set,  $A=\{A_1, A_2, \dots, A_m, \dots, A_n\}$ ,  $A \notin \emptyset$ ;
- (5) C: C is control function set,  $C \notin \emptyset$ ;
- (6) R: R is feedback function set,  $R \notin \emptyset$ ;
- (7)  $\omega$ :  $\omega$  is adjust function set,  $\omega \notin \emptyset$ ;
- (8) D: D is mapping of  $C \cup R \rightarrow A$ ;
- (9)  $M_0$ :  $M_0$  is the initial state;

Definition 3: In the library of P, there are two types of place with input P ( $P_i$ ) and output P ( $P_o$ ). The Petri net with strong connection could be obtained, if a transition, of T was joined between  $P_i$  and  $P_o$ .

Definition 4: In the change of T, there are two categories of transition, with information processing transition, T ( $T_p$ ) and information transition, T ( $T_i$ ).

According to the need and different of information processing, processing transition, can be divided into adaptive production and processing transition, ( $T_p$ ), production forecast change ( $T_f$ ) and production feedback transition, ( $T_t$ ).

Definition 5: In directed arc set, there must be one or more directed arc of  $P \times T$ . This directed arc T located the relative front position of the network. The change P located the relative end position of the network.

Definition 6: In label set, there must be two types of label with real label “●” and virtual label “◎”. Virtual label only existed in initial state, which used to activate the changes and transmit the information of virtual label to complex adaptive module, in order to make the change implement with reason and target.

### B. Classification and Basic Unit of CAPN

In the flexible production, especially in textile production process, the final product was determined by many factors. In this process, the processing links are not continuous, which must go through multiple processes. The products may be different, if any information in the controllable production process changed. It is depended by the real condition to decide that which processes would need the adaptive processing. We will classify the complex adaptive Petri net.

#### a. Same Domain

Figure 1 is the same domain of CAPN. In this CAPN, the real label and the virtual label are in the same manufacture process. In this process, real label and virtual label respectively appear in the port of input and output in the adaptive change. This way could realize adaptive execution processing by fully using of the information in output.

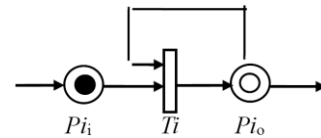


Fig. 1. The same domain of CAPN

#### b. Domain Class

Figure 2 is the domain class of CAPN. Although  $T_i$  and  $T_{(i+1)}$  are the different processes,  $T_i$  need the output results of  $T_{(i+1)}$  to carry out the adaptive manufacture. As a result, the real label and the virtual label must respectively lay at the input of  $T_i$  and the output of  $T_{(i+1)}$ . Act as the change of information transformation, Change  $T_i$  made the between different connect effectively. The domain class of CAPN not only represents the adjacent two processes, but also refers to any two differently intermediate processes.

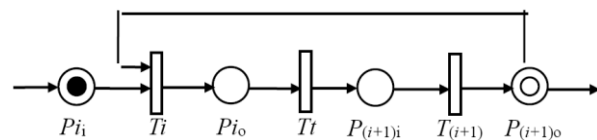


Fig. 2. The domain class of CAPN

#### c. Basic Unit of CAPN

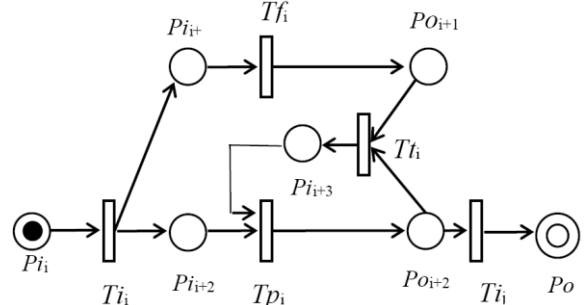


Fig. 3. Basic unit of CAPN

Figure 3 is a basic unit of CAPN. This basic unit is the smallest size of intelligent model to realize control, feedback, self-adaption during the flexible processing cycle. From  $P_{ii}$  to  $P_{ii+3}$  are belong to place and attribute.  $P_{oi+1}$  acts as quality output for intelligent predict.  $P_{oi+2}$  acts as output for production process.  $T_{ii}$  acts as transition for delivery.  $T_{pi}$  acts as transition for adaptive manufacture.  $T_{fi}$

acts as transition for intelligent predict.  $T_{ti}$  acts as transition for feedback control.  $T_{ij}$  acts as transition for virtual distribution.  $P_{oj}$  is virtual label output to meet the quality standard. In this paper,  $P_i$  is the identification of input place,  $P_o$  is the identification of output place;  $T_i$ ,  $T_p$ ,  $T_f$ ,  $T_t$  are different identification of transition,  $i$  is the sequence identification of CAPN Uint ( $i=1,2,3,\dots,n$ ).

### III. BASIC UNIT ANALYSIS AND WORKING MECHANISM OF CAPN

#### A. Basic Unit Analysis of CAPN

Based on the basic unit of CAPN and basic character of Petri net, this model shown the character of basic Petri net including of sequence, concurrency, synchronization and other features, which could effectively resolve the characteristics of conflict and disturbance. In this model,  $P_i$  to  $P_{i+3}$  act as concurrency,  $\{P_{o_{i+1}}, P_{o_{i+2}}, P_{o_{i+3}}, T_{ti}\}$  acts as synchronization,  $\{P_{o_{i+3}}, P_{o_{i+2}}, T_{pi}\}$  acts as priority. Through detailed analysis to complex system, Basic unit of CAPN could not only avoid the conflict during the process of sending mult changes, but also avoid the disorder phenomenon of conflict and concurrent coexistence.

#### B. Working Mechanism of CAPN

The work steps about basic unit of CAPN in figure 3 are as follows:

- The state set  $P$  and the attribute set  $A$  with timestamp are input through  $P_i$ .
- The data set input by  $P_i$  is delivered to  $P_{i+1}$  and  $P_{i+2}$  via  $T_i$ .
- Based intelligent prediction model (such as artificial neural network, genetic algorithm and hybrid intelligent forecasting algorithm), the optimal property  $P_{o_{i+1}}$  for attribute set  $A$  in  $P_{i+1}$  could be predicted via  $T_{fi}$ .
- Firstly, the first output  $P_{o_{i+2}}$  is obtained by the way of controllable production of control function set  $C$  after receiving the attribute set and the state set in  $P_{i+2}$  by  $T_{pi}$ . Secondly, it could be adaptively adjusted to optimal production process through  $\omega$  (adjustment function set) after inputting by  $P_{i+3}$ . In this step, the priority of  $P_{i+3}$  is higher than that of  $P_{i+2}$ .
- Combined with the analysis result of  $P_{o_{i+1}}$  and  $P_{o_{i+2}}$ , the optimal state set and the attribute set  $P_{i+3}$  is obtained by feedback function set  $R$  via  $T_{ti}$ . If  $P_{o_{i+1}}$  and  $P_{o_{i+2}}$  uniform,  $T_{ti}$  would exercise distribution function.
- The results obtained through adaptive adjustment after the production and processing are output by  $T_{pj}$  and distributed to virtual label by  $T_{ij}$ .

### IV. REACHABILITY TREE ANALYSIS FOR BASIC UNIT OF CAPN

#### A. Analysis step of reachability tree

- $m_0$  is identified as "tree root" (also as "new" identifier (ID)).
- Choosing a "new" ID  $m$ , if the  $m$  is the same as the other ID in the tree, it would be recorded as "old", then turn to other "new" ID. If there is no change enable in  $m$ ,  $m$  would be recorded as "dead end". Otherwise, it would be terminated.
- All changes  $t$  with enable in  $m$  are excited to generate ID  $M'$ . If there is an  $m''$  on the way from root to  $m'$ , it would make the  $m'$  cover  $m''$ . However, this  $m''$  is not equal to  $m'$  ( $m' > m''$ ). For those  $p$ , which make  $m'(P) > m''(P)$ , could be make the  $\omega$  replace  $m''(p)$ . Making  $m$  as a node, a directed line is draw from  $m$  to  $m'$ , then mark it as  $t$ , and mark  $m'$  as "new";
- Remove the "new" mark of  $m$ , then go back to step (2).

#### B. Basic unit of CAPN represented by reachability tree

According to mathematical meaning of CAPN reachability tree, as the reachability tree label vector of CAPN basic unit,  $m$  is expressed as follows:  $m_0=(1\ 0\ 0\ 0\ 0\ 0\ 0)^T$ ,  $m_1=(0\ 1\ 1\ 0\ 0\ 0\ 0)^T$ ,  $m_2=(0\ 0\ 0\ 1\ 1\ 0\ 0)^T$ ,  $m_3=(0\ 0\ 0\ 0\ 0\ 1\ 0)^T$ ,  $m_4=(0\ 0\ 0\ 0\ 1\ 0\ 0)^T$ ,  $m_5=(0\ 0\ 0\ 0\ 0\ 0\ 1)^T$ . Figure 4 is basic unit of reachability tree.

Analysis for boundedness: If and only if there is no  $\omega$  among all the nodes in the tree, CAPN is bounded. According to reachability tree, the condition above is satisfied, so this model is bounded.

Analysis for safety: There only contains 0 or 1 on the all nodes of reachability tree, so CAPN is safety.

Analysis of activity: There is also a directed path with all changes between any 2 nodes given in the tree, so CAPN is activity.

Analysis of reversibility: There is also a directed path from any node to the root node, so CAPN is reversibility.

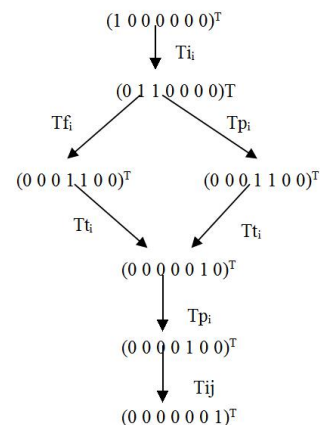


Fig. 4. Chart of reachability tree for basic unit of CAPN

## V. ANALYSIS FOR PERFORMANCE DEFINITION OF CAPN

The Establishment of Petri network for the purpose of realizing the performance analysis for the model using by analysis method. This analysis not only allows us to thoroughly understand the dynamic behavior of the simulated system, but also verifies the reliability of the system. It generally divided into two method, including the reachability tree and correlation matrix method. Essentially, the method of reachability tree is enumeration of all reachable labels. It is suitable for all types of Petri net. However, it is only suitable for smaller Petri net because of "state space explosion". The method of correlation matrix is calculated by the incidence matrix and state equation. The ability of calculation of this method is very strong. It is suitable for some special cases of Petri net. CAPN is the type of correlation matrix method. In this article, we use correlation matrix method for analysis.

The defined of correlation matrix method is as follows:

Definition 7: (definition of reliability) If and only if a Petri net is activity and boundedness, the Petri net would be considered for reliability.

In Petri net, the directed arc represents the relationship between library and change. Assuming that the number of library and change were  $m$  and  $n$  respectively. They could be represented by available output matrix  $O$  and input matrix  $I$  with matrix of  $(m \times n)$ . In the matrix of  $(m \times n)$ , row means library, column means change. According to the input and output of each change of Petri net model, the matrices of  $O$  and  $I$  could be obtained, then the correlation matrix  $C=O-I$  could be obtained. If  $m_k$  acts as the label of Petri net after  $k$  ( $k \geq 0$ ) cycle running, the label of Petri net after  $k+1$  cycle running could be  $m_{k+1}=m_k+Cu_k$ ,  $k \geq 0$ , named as state equation of Petri net. Act as  $u_k$  excitation count vectors,  $u_k$  is a vector of  $(m \times 1)$ , in which  $i$  represents the number of excitation for change  $t_i$  during the running of  $k+1$  cycle. Because the number of tokens in all library is non negative, it ensured  $m_k+Cu_k \geq 0$  for all number of  $k \geq 0$ .

Definition 8: (definition of activity) Supposing, the formula (4-1) as  $m_k=m_0+Cu_k$ ,  $m_k$  as the label of termination,  $m_0$  as initial label, when  $u_k$  was calculated as a non-negative integer based on the correlation matrix  $C$ , the CAPN is active. In the above condition, the non-negative elements of  $u_k$  is the number of corresponding change excitation from the initial label  $m_0$  to terminal  $m_k$  in transition sequence.

Definition 9: (definition of boundedness) According to the correlation matrix  $C$  in definition 8, If  $\exists Y > 0$  is right,

$CY \leq 0$  is always right, the CAPN is boundedness.

## VI. COMPLEXITY MODEL OF CAPN

The complexity for CAPN characterization reflects the complexity of the type, quantity and their about relationship for composed agents in system. In addition, the external environment is also as a quite key to influence the complexity of the system. The complexity of system in CAPN model is divided to three parts, including agent complexity  $C_a$ , relationship complexity  $C_r$  and environment complexity  $C_e$ , which expressed as  $C=\{C_a, C_r, C_e\}$ . From the state of the system, it is divided into static complexity and dynamic complexity. The static complexity in this article could be regarded as a special case of dynamic complexity with the complexity of external influence as 0.

According to the different distribution for complex of systemic composition, the complexity of above three aspects was given different weights. If  $\alpha$ ,  $\beta$  and  $\gamma$  acted as influence coefficient for  $C_a$ ,  $C_r$  and  $C_e$  respectively, the formula of  $C=\alpha C_a+\beta C_r+\gamma C_e$  is right with  $\alpha+\beta+\gamma=1$ .

In the model of CAPN, the number of nodes can intuitively reflect the number of agents. The definition of system can also reflect the types of agents during the application. The connection among the systemic agents is the directed arc. The number of arc can suggest the complexity between the internal relationships in system. Some of the systemic token change occurred at the initial state, the others came from the output. The token from output can reflect the influence condition to system by external environment. It also can judge the complexity influenced by external environment from the types of token. In generally, the more numbers and types of system agents, the value of  $C_a$  is larger in the certainty of other factors. In the same reason, the more numbers of directed arc, the value of  $C_r$  is larger, the more types of token, the value of  $C_e$  is larger.

Then:

$$C_a = f(|P| + |T| + |F|), C_r = f(|F|), C_e = f(A).$$

$$C_a = 1 - \sqrt{\frac{1}{|F| * (|P| + |T|)}}$$

If the formula of

$$C_r = \frac{\text{Max}(|X| + |X'|)}{\text{Max}(|X| + |X'|) + \text{Min}(|X| + |X'|)}$$

$$C_e = \frac{\text{Max}(K)}{\sum_{i=1}^{|P|} K_i}$$

are right. In this formula,  $X$  and  $X'$  are the input and output arc set of  $X$  node.

If the formula of

$$\alpha = \frac{Ca}{Ca + Cr + Ce}, \beta = \frac{Cr}{Ca + Cr + Ce},$$

$$\gamma = \frac{Ce}{Ca + Cr + Ce}$$

are right.

Then the complexity of system can be expressed by :

$$C = \left( \frac{Ca}{Ca + Cr + Ce} \right) \left( 1 - \sqrt{\frac{1}{|F| * (|P| + |T|)}} \right) +$$

$$\left( \frac{Cr}{Ca + Cr + Ce} \right) \frac{\text{Max} (|X| + |X'|)}{\text{Max} (|X| + |X'|) + \text{Min} (|X| + |X'|)}$$

$$+ \left( \frac{Ce}{Ca + Cr + Ce} \right) \frac{\text{Max} (K)}{\sum_{i=1}^{|P|} Ki}$$

Based on the above formula, the complexity base of basic model of Petri can be calculated as follow: Ca=0.55, Cr=0.5 and C=0.526. In the basic unit of CAPN static case, the complexity can be calculated used by the above formula as follow: Ca=0.916, Cr=0.75 and C=0.84. From the basic implementation of CAPN, this model can attain the complex adaptive systems modeling through the minimal complexity (basic PN complexity).

## VII. CHARACTERISTIC AND ADVANTAGE OF CAPN

The usual Petri nets have no concept about type and module, only have one type of token, so these nets are flat. Although colored Petri net (CPN) can use multiple types of token to data operation, it cannot solve the intelligent operation for heterogeneous types token and complex data set. In theory, complex adaptive Petri network kept the basic contents of definition for Petri net and combined with the specific features of the complex adaptive system, which made it can not only solve the complexly heterogeneous data operation, but also provide parallel computational intelligence operation of three kinds of data in the network (attribute data, state data and control parameters).

The specific advantages of CAPN are as follows:

- a. Each place has a timestamp containing data set P, called state set for this place.
- b. Each token has an attribute set, called attributes of this token.
- c. Transitions include lots of model libraries such as intelligent forecasting model Tf, intelligent control model Tp, automatic distribution model Ti, adaptive feedback control models Tt and so on.

## VIII. CONCLUSION

In this article, a Complex Adaptive Petri Net (CAPN)

model with new FMS process and controllable characterization was proposed. The mechanism of CAPN was also researched. The properties of reliability, boundedness and activity of CAPN were analyzed by method of reachability tree.

Using the CAPN model, on the one hand, the common input and output location and migration from the loop problem in the feedback system CAPN model can be solved by introducing four kinds of change. On the other hand, it can not only solve the mixed FMS characterization, but also improve system performing performance through the state set with time stamp. It provided the analysis method for the modeling complexity of complex adaptive system through construction complexity model of system. This CAPN model could not only solve the process characterization of flexible production and prediction of quality integrated intelligent formal modeling and analysis, but also provide a solving framework for a real-time feedback about intelligent control optimization solution during the complex production process.

## REFERENCE

- [1] W.M. Zuberek, "Timed Petri Nets Definitions Properties and Applications," *Microelectron Reliab.* 1991, Vol31, No4, p 627-644,
- [2] J. L. Ball, H. Alla, R. David, "Asymptotic Continuous Petri Nets. Discrete Event Dynamic Systems," *Theory and Applications.* 1993, (2) p235-263.
- [3] C. G. Looney, "Fuzzy Petri net for rule-based decision making," *IEEE Trans. Syst. Man Cyber.* vol. 14, 1988. p178-183
- [4] H. Monika, V. Giorgio, W. Dietmar, "A Petri net based methodology to integrate qualitative and quantitative analysis." *Information and Software Technology*, 1994, 36 (7) p435-441.
- [5] R. Davidrajuh, "Activity-Oriented Petri Net for Scheduling of Resources," *Proc. IEEE International Conference on Systems, Man, and Cybernetics (IEEE SMC 2012)*, October 14-17, 2012, Seoul, Korea
- [6] P. Matej, K. Matej, P. Janez, M. Gasper, V. Goran, K. Stanislav, "Analysis of multi-agent activity using petri nets," *Pattern Recognition*, 43 (2010) p1491-1501.
- [7] T. Murata, "Petri nets: Properties, analysis and applications", *Proc. IEEE*, vol77, no. 4, 1989 .p541 -580
- [8] W.S. Tommy, J. Y. Li, "Higher-order Petri net models based on artificial neural networks," *Artificial Intelligence*, 92 (1997) 289-300