Performance Evaluation of Extended Kalman Filtering for Obstacle Avoidance of Mobile Robots

Chih-Hung Wu, Wei-Zhou Hong, and Shing-Tai Pan

Abstract—Obstacle avoidance is an essential function for the navigation of mobile robots. Noise filtering improves the measurement accuracy of sensors and plays an important role for obstacle avoidance in the applications of mobile robots. This study evaluates the performance of the extended Kalman filtering (EKF) and Kalman filtering (KF) for obstacle avoidance of a two-wheeled mobile robot. EKF is an advanced version of traditional KF for signal processing. EKF is used to deal with non-linear problems that KF cannot process properly and usually has better ability of noise tolerance than KF. Due to the non-linearity and unstability of sensing results, KF has limited performance in the underlying problem. The robot used in this study carries some sonar sensors that acquire signals of obstacles periodically. EKF linearizes the estimation around the current measure using the partial derivatives of the process and measurement functions to obtain estimates of actual measurements even when non-linear relationships exist in the underlying problem. Several experiments of obstacle avoidance are conducted on the two-wheeled mobile robot and the results are analyzed. The results show that EKF provides reliable navigation information better than that from traditional KF.

Index Terms—Kalman filtering, extended Kalman filtering, sonar, obstacle avoidance, navigation, mobile robots.

I. INTRODUCTION

Mobile robots that cruise in unstructured environments and perform their designated missions autonomously have received more and more attention. Various types of sensors are installed on mobile robots for environment recognition. Infrared, sonar, gyroscope are sensors widely used for this purpose. However, due to imperfection of sensors and inaccuracy of signals, simple sensors unavoidably incorporate various types of errors and provides noisy information to mobile robots. Additionally, sensors work on a trembling platform when mobile robots patrol that introduces additional vibration and noises to sensor data and increases the complexity of controllers. An effective noise filtering method helps improve the sensing accuracy, reduces the complexity of controllers, and increase the performance of mobile robots. There are usually more than one sensors installed on mobile robots, each may incorporate various types/degrees of noise. Noise filtering is essential and important in such multisensor systems [1]. Many studies have developed methods of object tracking and obstacle avoidance for mobile robots, such as [2], [3], [4].

Kalman filtering (KF) is filtering method that performs recursively on streams of noisy signals to produce a statistically optimal estimate of the underlying system state [5]. KF is able to filter out temporary signal disturbance and commonly used in many applications for signal pre-processing [6]. Some studies reveal that KF is effective for the navigation of mobile robots [7], [8]. KF performs linear quadratic estimation and can work fine when the underlying system is linear. However, the patrol environments and the sensors used by mobile robots may not be perfect linear systems; hence the performance of mobile robots using KF may be limited.

Extended Kalman filtering (EKF) [9] is an advanced version of KF, which extends the observation models to the non-linear domain. EKF has the same structure as the traditional KF, thus preserves both the statistical optimality and the recursive computational scheme of KF. In the operation of EKF, the state transition and observation models are not necessarily to be represented as linear functions of system states but may instead be non-linear functions. EKF performs a similar recursive process but resolves the non-linear functions by partial derivatives before they are processed in the next recursions. With calculations of the non-linear functions, EKF provides better noise tolerance and performance than traditional KF. Several studies present the effectiveness of EKF for engineering applications [10], [11], [12], [13].

In the paper, we evaluate the performance of EKF and KF for obstacle avoidance that are implemented on a two-wheeled mobile robots. Simple sonar sensors are installed on the robots for collecting data of distances between the robot and obstacles. A simple controller is used to direct the rotation of the wheels. Sensor data are processed by EKF or KF before they are input to the controller. By analyzing the trajectories of the robot, the experimental results show that EKF is able to reduce the measurement errors and results in precise and efficient obstacle avoidance. The remaining part of this paper is organized as follows. Section II presents the concepts of KF and EKF. Section III presents the experimental results of obstacle avoidance using EKF and KF. Finally, Section IV concludes our study and presents the future work.

II. THE FILTERING METHODS

A. Kalman Filtering

First of all, some terms are defined for the presentation of KF.

- $A_k$: the state transition model.
- $H_k$: the observation model.
- $v_k$: the process noise.
- $w_k$: the observation noise.
Fig. 1. The processing flow of KF

- $Q_k$: the covariance of the process noise.
- $R_k$: the covariance of the observation noise.
- $\hat{x}_k$: the a priori state estimate at time $k$ given observations up to and including at time $k$.
- $\hat{x}_k$: the a posteriori state estimate at time $k$ given observations up to and including at time $k$.
- $P_k^-$: the a priori error covariance matrix.
- $P_k$: the a posteriori error covariance matrix.
- $w_k$: the true state observation (or measurement).
- $u_k$: the optional control input.

Among these terms, $w_k$ is assumed to be drawn from a zero mean multivariate normal distribution with covariance $Q_k$. Also, $u_k$ is assumed to be zero mean Gaussian white noise with covariance $R_k$. KF can be considered as two distinct phases, “predict” and “correct”, applying the following steps recursively.

Step-1: $\hat{x}_k = A_k\hat{x}_{k-1} + B_ku_{k-1}$      (1)
Step-2: $P_k^- = A_kP_{k-1}A_k^T + Q_k$      (2)
Step-3: $K_k = P_k^-(H_kP_k^-H_k^T + R_k)^{-1}$      (3)
Step-4: $\hat{x}_k = \hat{x}_{k-1} + K_k(z_k - H_k\hat{x}_{k-1})$      (4)
Step-5: $P_k = (I - K_kH_k)P_k^-$      (5)

Step-1 and Step-2 are for the predict phase and Step-3~Step-5 are for the correct phase. KF is a recursive algorithm that calculates the abovementioned equations to estimate the state of a process that minimizes the mean squared error of the estimation. Fig. 1 presents the processing flow of KF. KF supports estimations of past, present, and even future states, and it can do so even when the precise nature of the modeled system is unknown. However, systems processed by KF are assumed to be linear.

B. Extended Kalman Filtering

EKF is also a recursive algorithm that follows the processing flow of KF. In EKF, the state transition and observation models are not necessarily to be linear functions of the state but are defined as non-linear ones. The non-linear functions are linearized by partial derivatives before they are processed in the next recursion of estimation. EKF has the same structure as the traditional KF, thus preserves both the statistical optimality and the recursive computational scheme of KF. Let $f$ be a non-linear function used to describe the estimate of the underlying system states. The a priori state estimate $\hat{x}_k^-$ is described by

$$\hat{x}_k^- = f(x_{k-1}, u_{k-1}, w_{k-1}).$$

Accordingly, $\tilde{z}_k^-$ is defined as a non-linear function $h$ as

$$\tilde{z}_k^- = h(\hat{x}_k, v_k).$$

The following terms are used for defining EKF.

- $A_k$: the Jacobian matrix of partial derivatives of with respect to $x$.
- $H_k$: the Jacobian matrix of partial derivatives of with respect to $u$.
- $V_k$: the Jacobian matrix of partial derivatives of with respect to $v$.
- $W_k$: the Jacobian matrix of partial derivatives of with respect to $w$.

Among these models, $W_k$ is assumed to be drawn from a zero mean multivariate normal distribution with covariance $Q_k$ and $V_k$ is also assumed to be zero mean Gaussian white noise with covariance $R_k$. Similar to the KF process, EKF can also be conceptualized as two distinct phases: "predict" and "correct" that can be described as follows.

Step-1: $\hat{x}_k^- = f(x_{k-1}, u_{k-1}, 0)$      (8)
Step-2: $P_k^- = A_kP_{k-1}A_k^T + W_kQ_kW_k^T$      (9)
Step-3: $K_k = P_k^-H_k^{-1}(H_kP_k^-H_k^T + V_kR_kV_k^T)^{-1}$      (10)
Step-4: $\tilde{z}_k^- = \hat{x}_k^- + K_k(z_k - H_k\hat{x}_k^-)$      (11)
Step-5: $P_k = (I - K_kH_k)P_k^-$      (12)

$W_kQ_kW_k^T$ and $V_kR_kV_k^T$ represent new independent random variables having zero mean and covariance matrices. Fig. 2 presents the processing flow of EKF. For more detailed information about KF and EKF, please refer to [6].

III. EXPERIMENT

The robot is 14cm×14cm×12cm in size and is implemented with the following components. There are two wheels and 3 simple sonar sensors installed on the front, left, and right side, respectively, of the robot. The sonar sensors are Sonar-A from Innovati Co. [14]. Each wheel is connected with a servo motor that rotates forward or backward. Fig. 3 presents photos of the robot. If the robot needs to turn in-situ, one of the wheels rotates forward and the other
rotates backward at the same time. The controller of the servos connected with wheels is with the speed argument (SA). SA is an integer, 100 ≤ SA ≤ 400, representing the input signal requiring the motor to perform at a certain angular speed. The motor stops when SA = 250, rotates forward when 250 < SA ≤ 400, and rotates backward when 100 ≤ SA < 250. When SA = 400, the wheel rotates forward in full speed; conversely, the wheel rotates backward in full speed when SA = 100. The rotation speed of servo motors is not always constant when the controller is actuated. The change of SA parameter is not always linear; it depends on the burden on the robot and on the ground friction that the wheels encounter. The robot cruises around in a 1.8 × 1.8 m² environment where the ground is black and the walls are white. The three sonar sensors scan the environment periodically, measuring the distances from the robot to obstacles. Three simple IF-THEN rules are associated with each sonar sensor.

- IF d > 25 THEN do nothing
- IF 15 < d ≤ 25 THEN turn (right/left) 15 degree
- IF d ≤ 15 THEN turn (right/left) 30 degree

The symbol d denotes the distance input to the servo’s controller; the actions of turning right or left depend on the installation position of the sensor. We compare the performance of obstacle avoidance when d is pre-processed by KF/EKF or not. Fig. 4 depicts the control flow of the robot.

A. Experiment-I

The first experiment is to compare the performance of KF and EKF for measuring distances to obstacles when the robot is stationed at the starting point (see Fig. 5) with SA=250 of both wheels. The initial settings associated with KF and EKF are the followings.

- Qk = 0.001
- Rk = 0.1
- P0 = 1
- Ak = 1
- Hk = 1
- x0 (front-size) = 135
- x0 (left-size) = 45
- x0 (right-size) = 135

The sensors collect distance data for more than 100 iterations. The experimental results are presented in Fig. 6. Table I lists the average mean squared error (MSE) of all measurement iterations. Results show that both KF and EKF can filter out temporary noise; however, EKF performs better than KF.

B. Experiment-II

Next, the robot moves forward from the starting point with SA=400 of both wheels, collecting sensor data and activating the controllers for obstacle avoidance accordingly. The parameters used in Section III-A are also used in this experiment. The trajectories of the robot for obstacle avoidance of mobile robots. A two-wheeled robot is implemented with 3 simple sonar sensors installed. The robot cruises around in the test environment, collecting

![Fig. 5. The test environment and the starting point](image)

<table>
<thead>
<tr>
<th>Method</th>
<th>Front</th>
<th>Right</th>
<th>Left</th>
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<tr>
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### Table I

AVERAGE MSE
data and activation the controllers accordingly. Experiments have shown that EKF outperforms KF even simple sensors are used. The initial parameter settings may affect the performance of KF and EKF. The performance of multiple obstacle avoidance using EKF should be studied. Advanced control rules should be designed. These will be included in the future work.

**REFERENCES**


