Proceedings of the International MultiConference of Engineers and Computer Scientists 2015 Vol I,
IMECS 2015, March 18 - 20, 2015, Hong Kong

On the Capability of a Fuzzy Inference System with Improved Interpretability

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***Abstract*—Many studies on modeling of fuzzy inference systems have been made. The issue of these studies is to construct automatically fuzzy systems with interpretability and accuracy from learning data based on meta-heuristic methods. Since accuracy and interpretability are contradicting issues, there are some disadvantages for self-tuning method. Obvious drawbacks of the method are lack of interpretability and getting stuck in a shallow local minimum. Therefore, the conventional learning methods with multi-objective fuzzy modeling and fuzzy modeling with constrained parameters of the ranges have become popular. However, there are little studies on effective learning methods of fuzzy inference systems dealing with interpretability and accuracy. In this paper, we will propose a fuzzy inference system with interpretability. Firstly, it is proved that the proposed model is an universal approximator of continuous functions. Further, the capability of the proposed model learned by the steepest descend method is compared with the conventional models using function approximation problems. Lastly, the proposed model is applied to obstacle avoidance and the capability of interpretability is shown.**

***Index Terms*—fuzzy set, fuzzy inference, interpretability, universal approximator, obstacle avoidance.**

I. Introduction

FUZZY inference systems are widely used in system modeling for the fields of classification, regression, decision support system and control [1], [2]. Therefore, many studies on modeling of fuzzy inference systems have been made. The issue of these studies is to construct automatically fuzzy systems with interpretability and accuracy from learning data based on meta-heuristic methods [4], [5], [9], [12], [14]. Since accuracy and interpretability are contradicting issues, there are some disadvantages for self-tuning method. Obvious drawbacks of the method are lack of interpretability and getting stuck in a shallow local minimum [6]. As the meta-heuristic methods, some novel methods have been developed which 1) use GA and PSO to determine the structure of the fuzzy model [6], [7], 2) use generalized objective functions [8], 3) use fuzzy inference systems composed of small number of input rule modules, such as SIRMs and DIRMs methods [10], [11], 4) use a self-organization or a vector quantization technique to determine the initial assignment and 5) use combined methods of them [5], [14]. Since accuracy and interpretability are conflicting goals, the conventional learning methods with multi-objective fuzzy modeling and fuzzy modeling with constrained parameters of the ranges have become popular. However, there are little studies on effective learning methods of fuzzy inference systems dealing with interpretability and accuracy. On the other hand, fuzzy modeling with preserving interpretability is proposed by Shi [15]. However, there are no studies on the detailed capability of this type of model.

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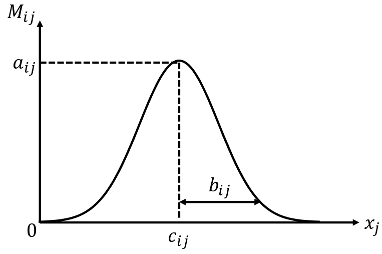


Fig. 1. The Gaussian membership function

In this paper, we will propose a fuzzy inference system with interpretability. Firstly, it is proved that the proposed model is a universal approximator of continuous functions. Further, the capability of the proposed model learned by the steepest descend method is compared with the conventional models using function approximation problems. Lastly, the proposed model is applied to obstacle avoidance and the capability of interpretability is shown.

II. Preliminaries

A. The conventional fuzzy inference model

The conventional fuzzy inference model is described [1]. Let $Z_j = \{1, \cdots, j\}$ for the positive integer j. Let $\boldsymbol{R}$ be the set of real numbers. Let $\boldsymbol{x} = (x_1, \cdots, x_m)$ and y^r be input and output data, respectively, where $x_j \in \boldsymbol{R}$ for $j \in Z_m$ and $y^r \in \boldsymbol{R}$. Then the rule of fuzzy inference model is expressed as

$$R_i \ : \ \text{if } x_1 \text{ is } M_{i1} \text{ and } \cdots \text{ and } x_m \text{ is } M_{im} \quad \text{then } y \text{ is } f_i(x_1, \cdots, x_m) \tag{1}$$

, where $i \in Z_n$ is a rule number, $j \in Z_m$ is a variable number, M_{ij} is a membership function of the antecedent part, and $f_i(x_1, \cdots, x_m)$ is the function of the consequent part.

A membership value of the antecedent part μ_j for input $\boldsymbol{x}$ is expressed as

$$\mu_i = \prod_{j=1}^{m} M_{ij}(x_j). \tag{2}$$

If Gaussian membership function is used, then M_{ij} is expressed as follow (See Fig.1):

$$M_{ij} = a_{ij} \exp\left(-\frac{1}{2}\left(\frac{x_j - c_{ij}}{b_{ij}}\right)^2\right). \tag{3}$$

, where a_{ij}, c_{ij} and b_{ij} are the amplitude, the center and the width values of M_{ij}, respectively.

The output y^* of fuzzy inference is calculated by the following equation:

$$y^* = \frac{\sum_{i=1}^{n} \mu_i \cdot f_i}{\sum_{i=1}^{n} \mu_i}. \tag{4}$$

Specifically, simplified fuzzy inference model is known as one with $f_i(x_1, \cdots, x_m) = w_i$ for $i \in Z_n$, where $w_i \in \boldsymbol{R}$ is a real number. The simplified fuzzy inference model is called Model 1.

B. Learning algorithm for the conventional model

In order to construct the effective model, the conventional learning method is introduced. The objective function E is defined to evaluate the inference error between the desirable output y^r and the inference output y^*. In this section, we describe the conventional learning algorithm. Let $\boldsymbol{D} = \{(x_1^p, \cdots, x_m^p, y_p^r) | p \in Z_P\}$ be the set of learning data. The objective of learning is to minimize the following mean square error(MSE):

$$E = \frac{1}{P} \sum_{p=1}^{P} (y_p^* - y_p^r)^2. \tag{5}$$

In order to minimize the objective function E, the parameters $\alpha \in \{a_{ij}, c_{ij}, b_{ij}, w_i\}$ are updated based on the descent method as follows [1]:

$$\alpha(t+1) = \alpha(t) - K_\alpha \frac{\partial E}{\partial \alpha} \tag{6}$$

where t is iteration time and K_α is a constant. When Gaussian membership function with $a_{ij} = 1$ for $i \in Z_n$ and $j \in Z_m$ are used, the following relation holds [6].

$$\frac{\partial E}{\partial c_{ij}} = \frac{\mu_j}{\sum_{i=1}^{n} \mu_i} \cdot (y^* - y^r) \cdot (w_i - y^*) \cdot \frac{x_j - c_{ij}}{b_{ij}^2} \tag{7}$$

$$\frac{\partial E}{\partial b_{ij}} = \frac{\mu_i}{\sum_{i=1}^{n} \mu_i} \cdot (y^* - y^r) \cdot (w_i - y^*) \cdot \frac{(x_j - c_{ij})^2}{b_{ij}^3} \tag{8}$$

$$\frac{\partial E}{\partial w_i} = \frac{\mu_i}{\sum_{i=1}^{n} \mu_i} \cdot (y^* - y^r) \tag{9}$$

Then, the conventional learning algorithm is shown as below [1], [2], [6].

Learning Algorithm A

Step A1 : The threshold θ of inference error and the maximum number of learning time T_{max} are given. The initial assignment of fuzzy rules is to equally intervals. Let n be the number of rules and $n = d^m$ for an integer d. Let $t = 1$.

Step A2 : The parameters b_{ij}, c_{ij} and w_i are set to the initial values.

Step A3 : Let $p = 1$.

Step A4 : A data $(x_1^p, \cdots, x_m^p, y_p^r) \in \boldsymbol{D}$ is given.

Step A5 : From Eqs.(2) and (4), μ_i and y^* are computed.

Step A6 : Parameters c_{ij}, b_{ij} and w_i are updated by Eqs.(7), (8) and (9).

Step A7: If $p = P$ then go to Step A8 and if $p < P$ then go to Step A3 with $p \leftarrow p+1$.

Step A8: Let $E(t)$ be inference error at step t calculated by Eq.(5). If $E(t) > \theta$ and $t < T_{max}$ then go to Step A2 with $t \leftarrow t+1$ else if $E(t) \leq \theta$ and $t \leq T_{max}$ then the algorithm terminates.

Step A9: If $t > T_{max}$ and $E(t) > \theta$ then go to Step A3 with $n = d^m$ as $d \leftarrow d+1$ and $t = 1$.

C. The proposed model

It is known that Model 1 is effective, because all the parameters are adjusted by learning. On the other hand, all the parameters move freely, so interpretability capability is low. Therefore, we propose the following model.

$$R^{i_1 \cdots i_m} : \text{ if } x_1 \text{ is } M_{i_1 1} \text{ and } \cdots \text{ and } x_m \text{ is } M_{i_m m} \text{ then } y \text{ is } f_{i_1 \cdots i_m}(x_1, \cdots, x_m) \tag{10}$$

, where $1 \leq i_j \leq l_j$, $j \in Z_m$.

$$\mu_{i_1 \cdots i_m} = \prod_{j=1}^{m} M_{i_j j}(x_j) = M_{i_1 1}(x_1) \cdots \cdot M_{i_m m}(x_m) \tag{11}$$

$$y = \frac{\sum_{i_1} \cdots \sum_{i_m} \mu_{i_1 \cdots i_m} f_{i_1 \cdots i_m}(x_1, \cdots, x_m)}{\sum_{i_1} \cdots \sum_{i_m} \mu_{i_1 \cdots i_m}} \tag{12}$$

In this case, the model with $f_{i_1, \cdots, i_m}(x_1 \cdots, x_m) = w_{i_1, \cdots, i_m}$ and triangular membership function has already proposed in the Ref. [15].

We will consider a model with $f_{i_1, \cdots, i_m}(x_1, \cdots, x_m) = w_{i_1, \cdots, i_m}$ and Gaussian membership functions. The model is called Model 2(See Fig.2). Remark that Model 2 is one that the parameters of membership function for each variable are adjusted by learning.

Learning equation for Model 2 is obtained as follows:

$$\frac{\partial E}{\partial M_{i_j j}} = \sum_{i_1} \cdots \sum_{i_{j-1}} \sum_{i_{j+1}} \cdots \sum_{i_m} \frac{\mu_{i_1 \cdots i_m}}{\sum_{i_1} \cdots \sum_{i_m} \mu_{i_1 \cdots i_m}} \cdot (y^* - y^r) \cdot (y^* - w_{i_1 \cdots i_m}) \tag{13}$$

$$\frac{\partial E}{\partial w_{i_1 \cdots i_m}} = \frac{\mu_{i_1 \cdots i_m}}{\sum_{i_1} \cdots \sum_{i_m} \mu_{i_1 \cdots i_m}} \cdot (y^* - y^r) \tag{14}$$

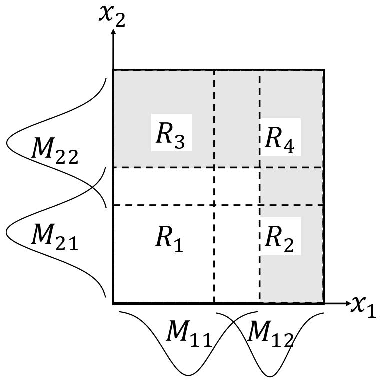
When Eq.(15) is used as a membership function, the following equations for $a_{i_j j}$, $c_{i_j j}$ and $b_{i_j j}$ are obtained;

$$M_{i_j j}(x_j) = a_{i_j j} \exp\left(-\frac{1}{2} \sum_{j=1}^{m} \frac{(x_j - c_{i_j j})^2}{b_{i_j j}^2}\right) \tag{15}$$

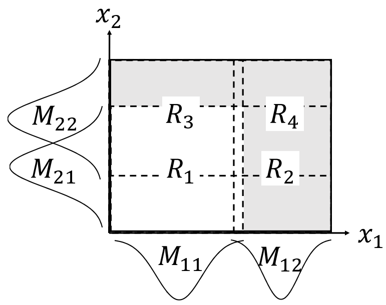
$$\frac{\partial M_{i_j j}}{\partial a_{i_j j}} = \exp\left(-\frac{1}{2} \sum_{j=1}^{m} \frac{(x_j - c_{i_j j})^2}{b_{i_j j}^2}\right) \tag{16}$$

$$\frac{\partial M_{i_j j}}{\partial c_{i_j j}} = \frac{(x_j - c_{i_j j})}{b_{i_j j}^2} \exp\left(-\frac{1}{2} \sum_{j=1}^{m} \frac{(x_j - c_{i_j j})^2}{b_{i_j j}^2}\right) \tag{17}$$

$$\frac{\partial M_{i_j j}}{\partial b_{i_j j}} = \frac{(x_j - c_{i_j j})^2}{b_{i_j j}^3} \exp\left(-\frac{1}{2} \sum_{j=1}^{m} \frac{(x_j - c_{i_j j})^2}{b_{i_j j}^2}\right) \tag{18}$$



(a)Before learning.



(b)After learning.

Fig. 2. The figure to explain Model 2 with $m = 2$ and $i_1 = i_2 = 2$. The assignment (a) of fuzzy rules for Model 2 is transformed into the assignment (b) after learning.

Mamdani type model is special case of Model 2 [7]. It is the model with the fixed parameters of antecedent part of fuzzy rule and membership function assigned to equally intervals [12], [14]. The model is called Model 3. It has good interpretable capability , but the accuracy capability is law. Therefore, TSK model with the weight of linear function $f_i(x_1, \cdots, x_m)$ is introduced as a generalized model of Model 3 [3]. The model is called Model 4.

III. Fuzzy inference system as Universal Approximator

In this section, the universal approximation capabilities of Model 1, 2, 3 and 4 are discussed using the well-known Stone-Weierstrass Theorem. See Ref. [2] about the mathematical terms.

[Stone-Weierstrass Theorem] [2]

Let S be a compact set with m dimensions, and $C(S)$ be a set of all continuous real-valued functions on S. Let Ω be the set of continuous real-valued functions satisfying the conditions:

(i) Identity function : The constant function $f(\boldsymbol{x}) = 1$ is in Ω.

(ii) Separability : For any two points $\boldsymbol{x}_1, \boldsymbol{x}_2 \in S$ and $\boldsymbol{x}_1 \neq \boldsymbol{x}_2$, there exists a $f \in \Omega$ such that $f(\boldsymbol{x}_1) \neq f(\boldsymbol{x}_2)$.

(iii) Algebraic closure : For any $f, g \in \Omega$ and $\alpha, \beta \in \boldsymbol{R}$, the function $f \cdot g$ and $\alpha f + \beta g$ are in Ω.

Then, Ω is dense in $C(S)$. In other words, for any $\varepsilon > 0$ and any function $g \in C(S)$, there is a function $f \in \Omega$ such that

$$|g(\boldsymbol{x}) - f(\boldsymbol{x})| < \varepsilon$$

for all $\boldsymbol{x} \in S$.□

It means that the set Ω satisfying the above conditions can approximate any continuous function with any accuracy. Since the sets of RBF and Model 1 are satisfied with the conditions of Stone-Weierstrass Theorem, they hold for universal approximation capabilities [2], [16]. Further, we can show the result about Model 2 in the following.

[Theorem]

Let Φ be the set of all functions that can be computed by Model 2 on a compact set $S \in \boldsymbol{R}^m$ as follows:

Let

$$\Phi_{l_1 \cdots l_m} = \left\{ f(\boldsymbol{x}) = \frac{\sum_{i_1} \cdots \sum_{i_m} \prod_j M_{i_j j}(x_j) w_{i_1 \cdots i_m}}{\sum_{i_1} \cdots \sum_{i_m} M_{i_j j}(x_j)} \right.$$
$$\left. , w_{i_1 \cdots w_m}, a_{i_j j}, c_{i_j j}, b_{i_j j} \in \boldsymbol{R}, \boldsymbol{x} \in S \right\}$$

for $M_{i_j j}(x_j) = a_{i_j j} \exp\left(-\frac{1}{2}\left(\frac{x_j - c_{i_j j}}{b_{i_j j}}\right)^2\right)$ and

$$\Phi = \bigcup_{l_1=1}^{\infty} \cdots \bigcup_{l_m=1}^{\infty} \Phi_{l_1 \cdots l_m} \tag{19}$$

Then Φ is dense in $C(S)$.

[Outline of Proof]

Let f and g be two functions in Φ and be represented as

$$f(\boldsymbol{x}) = \frac{\sum_{i_1} \cdots \sum_{i_m} \prod_j M^f_{i_j j}(x_j) w^f_{i_1 \cdots i_m}}{\sum_{i_1} \cdots \sum_{i_m} \prod_j M^f_{i_j j}(x_j)} \tag{20}$$

$$g(\boldsymbol{x}) = \frac{\sum_{l_1} \cdots \sum_{l_m} \prod_j M^g_{l_j j}(x_j) w^g_{l_1 \cdots l_m}}{\sum_{l_1} \cdots \sum_{l_m} \prod_j M^g_{l_j j}(x_j)} \tag{21}$$

for

$$M^f_{i_j j}(x_j) = a^f_{i_j j} \exp\left(-\frac{1}{2}\left(\frac{x_j - c^f_{i_j j}}{b^f_{i_j j}}\right)^2\right) \tag{22}$$

$$M^g_{l_j j}(x_j) = a^g_{l_j j} \exp\left(-\frac{1}{2}\left(\frac{x_j - c^g_{l_j j}}{b^g_{l_j j}}\right)^2\right) \tag{23}$$

Then, we will show only $\alpha f + \beta g \in \Phi$ and $f \cdot g \in \Phi$.

We define

$$\begin{aligned} M^{fg}_{i_j l_j j}(x_j) &= M^f_{i_j j}(x_j) \cdot M^g_{l_j}(x_j) \\ &= a^{fg}_{i_j l_j j} \exp\left(-\frac{1}{2}\left(\frac{x_j - c^{fg}_{i_j l_j j}}{b^{fg}_{i_j l_j j}}\right)^2\right) \end{aligned} \tag{24}$$

$$w^{fg1}_{i_1 \cdots i_m l_1 \cdots l_m} = w^f_{i_1 \cdots i_m} + w^g_{l_1 \cdots l_m} \tag{25}$$

$$w^{fg2}_{i_1 \cdots i_m l_1 \cdots l_m} = w^f_{i_1 \cdots i_m} \cdot w^g_{l_1 \cdot l_m} \tag{26}$$

, where $a^{fg}_{i_j l_j j}, c^{fg}_{i_j l_j j}, b^{fg}_{i_j l_j j}, w^{fg1}_{i_1 \cdots w_{i_m} l_1 \cdots l_m}, w^{fg2}_{i_1 \cdots i_m l_1 \cdots l_m} \in \boldsymbol{R}$. By using these values,

$$\alpha f + \beta g$$
$$= \frac{\sum_{i_1} \cdots \sum_{i_m} \sum_{l_1} \cdots \sum_{l_m} \prod_j M^{fg}_{i_j l_j j}(x_j) w^{fg1}_{i_1 \cdots i_m l_1 \cdots l_m}}{\sum_{i_1} \cdots \sum_{i_m} \sum_{l_1} \cdots \sum_{l_m} \prod_j M^{fg}_{i_j l_j j}(x_j)} \tag{27}$$

$$f \cdot g$$
$$= \frac{\sum_{i_1} \cdots \sum_{i_m} \sum_{l_1} \cdots \sum_{l_m} \prod_j M^{fg}_{i_j l_j j}(x_j) w^{fg2}_{i_1 \cdots i_m l_1 \cdots l_m}}{\sum_{i_1} \cdots \sum_{i_m} \sum_{l_1} \cdots \sum_{l_m} \prod_j M^{fg}_{i_j l_j j}(x_j)} \tag{28}$$

ISBN: 978-988-19253-2-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

TABLE I
INITIAL CONDITION FOR SIMULATION OF FUNCTION APPROXIMATION.

	Model 1	Model 2	Model 3	Model 4
T_{max}	50000	50000	50000	50000
K_c	0.01	0.01	0.0	0.0
K_b	0.01	0.01	0.0	0.0
K_w	0.1			
d	3	7	4	6
Initial c_{ij}	equal intervals			
Initial b_{ij}	$\frac{1}{2(d-1)}\times$(the domain of input)			
Initial w_{ij}	random on $[0,1]$			

Further, Eqs.(27) and (28) have the same form as Model 2. Therefore, $(\alpha f+\beta g)$ and $f\cdot g\in\Phi$ hold.□

[Remarks]
Remark that the results using Stone-Weierstrass Theorem hold only for Model 1 and 2 with $f_i(x_1\cdots x_m)=w_i$ and Gaussian membership function. On the other hand, Stone-Weierstrass Theorem does not always hold for Model 3, 4 and the models with triangular membership function, because the multiplicative condition fails. Further, it is an existence theorem and there is another problem whether we can get the system with high accuracy. Therefore, we need effective learning algorithm. Learning Algorithm A is a learning algorithm based on the steepest descend method.

IV. NUMERICAL SIMULATIONS

In this section, two kinds of simulations are performed to compare with the capabilities of models for learning method based on steepest descend method. In the simulations, let $a_{ij}=1$ and $a_{i_j j}=1$ for $i\in Z_n$ and $j\in Z_m$.

A. Function approximation

This simulation uses four systems specified by the following functions with $[0,1]\times[0,1]$.

$$y = \sin(\pi x_1^3)\cdot x_2 \quad (29)$$
$$y = \frac{\sin(2\pi x_1^3)\cdot\cos(\pi x_2)+1}{2} \quad (30)$$
$$y = \frac{1.9(1.35+e^{x_1}\sin(13(x_1-0.6)^2)\cdot e^{-x_2}\sin(7x_2))}{7} \quad (31)$$
$$y = \frac{\sin(10(x_1-0.5)^2+10(x_2-0.5)^2)+1.0}{2} \quad (32)$$

The condition of the simulation is shown in Table I. The value θ is 1.0×10^{-5} and the numbers of learning and test data selected randomly are 200 and 2500, respectively. Table II shows the results on comparison among four models. In Table II, Mean Square Error(MSE) of learning($\times10^{-4}$) and MSE of test($\times10^{-4}$) are shown. The result of simulation is the average value from ten trials. Table II shows that Model 1 and 2 have almost the same capability in this simulation, where ♯parameter means the number of parameters.

B. Obstacle avoidance and arriving at designated point

In order to show interpretability, let us perform simulation of control problem for Model 1 and Model 2 [11]. As shown in Fig.3, the distance r_1 and the angle θ_1 between mobile object and obstacle and the distance r_2 and the angle θ_2 between mobile object and the designated place are selected as input variables, where θ_1 and θ_2 are normalized.

TABLE II
RESULTS FOR SIMULATION OF FUNCTION APPROXIMATION.

		Eq.(29)	Eq.(30)	Eq.(31)	Eq.(32)
Model 1	learning	0.10	0.44	2.25	0.30
	test	0.30	2.16	3.84	0.88
	♯parameter	45	45	45	45
Model 2	learning	0.10	1.47	0.10	0.42
	test	0.93	5.46	0.39	2.03
	♯parameter	48	48	48	48
Model 3	learning	1.10	13.33	1.32	4.83
	test	4.50	50.29	4.40	29.23
	♯parameter	49	49	49	49
Model 4	learning	0.21	2.52	0.71	2.87
	test	0.98	9.68	1.95	10.97
	♯parameter	48	48	48	48

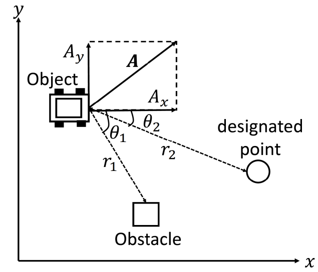


Fig. 3. Simulation on obstacle avoidance and arriving at the designated point.

The problem is to construct fuzzy inference system that mobile object avoids obstacle and arrives at the designated point. From (operation) data, fuzzy inference rules for Model 1 and Model 2 are constructed from learning for data of 400 points shown in Fig. 4. An obstacle is placed at $(0.5,0.5)$ and a designated point is placed at $(1.0,0.5)$. The number of rules for each model is 81 and the number of attributes is 3. Then, the numbers of parameters for Model 1 and 2 are 729 and 93, respectively. The mobile object moves with the vector $\boldsymbol{A}$ at each step, where A_x of $\boldsymbol{A}$ is constant and A_y of $\boldsymbol{A}$ is output variable. Learning for two models are successful and the following tests are performed.
(1)Test 1 is simulation for obstacle avoidance and arriving at the designated place when the mobile object stars from various places (See Fig.5). Fig.5 shows the results of moves of mobile object for starting places at $(0.1,0),(0.2,0),\cdots,(0.8,0),(0.9,0)$ after learning. As shown in Fig.5, the test simulations are successful for both

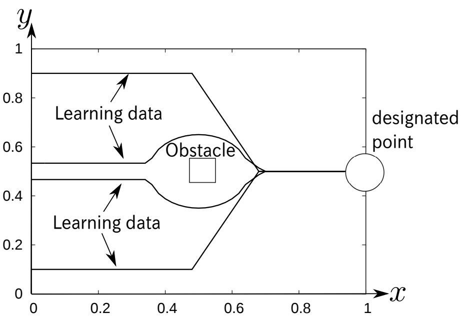
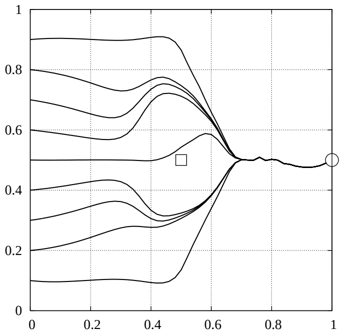


Fig. 4. Learning data to avoid obstacle and arrive at the designated point (1.0, 0.5).

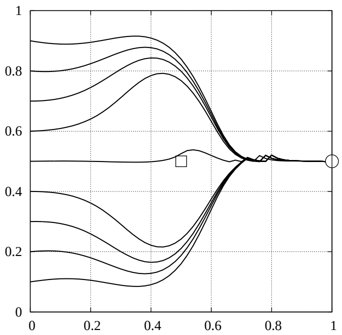
ISBN: 978-988-19253-2-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

TABLE III
INITIAL CONDITION FOR SIMULATION OF OBSTACLE AVOIDANCE.

	Model 1	Model 2
T_{max}	50000	50000
K_c	0.001	0.001
K_b	0.001	0.001
K_w	0.01	
d	3	3
Initial c_{ij}	equal intervals	
Initial b_{ij}	$\frac{1}{2(d-1)}\times$(the domain of input)	
Initial w_{ij}	0.0	



(a)Model 1

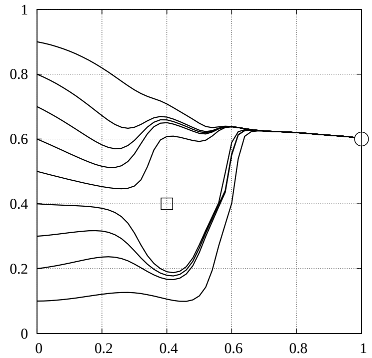


(b)Model 2

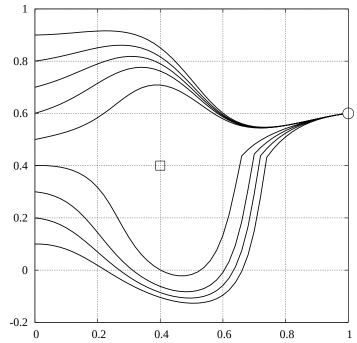
Fig. 5. Simulation for obstacle avoidance and arriving at the designated place starting from various places after learning.

models.
(2)Test 2 is simulation for the case where the mobile object avoids obstacle placed at different place and arrives at the different designated place. Simulations with obstacle placed at the place $(0.4, 0.4)$ and arriving at the designated place $(1, 0.6)$ are performed for two models. The results are successful as shown in Fig.6.
(3)Test 3 is simulation for the case where obstacle moves with the fixed speed. Simulations with obstacle moving with the speed $(0.01, 0.02)$ from the place $(0.3, 1.0)$ to the place $(0.8, 0.0)$ and arriving at the place $(1, 0.6)$ are performed. The results are successful as shown in Fig.7.
(4)Test 4 is simulation for the case where obstacle moves randomly as shown in Fig.8, where $|\boldsymbol{B}|$ is constant, and the angle θ_b is determined randomly at each step. Simulations with obstacle moving from the point $(0.5, 1.0)$ are perfurmed for two models. The results are successful as shown in Fig.9.

Lastly, let us consider interpretability for the proposed model. From fuzzy rules constructed for Model 2 by learning, we can find interpretable rules. Assume that three attributes

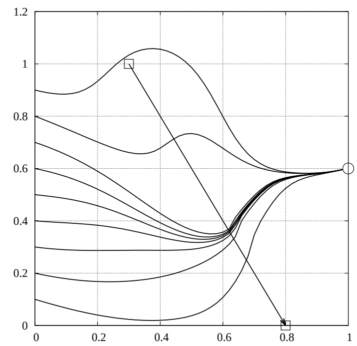


(a)Model 1

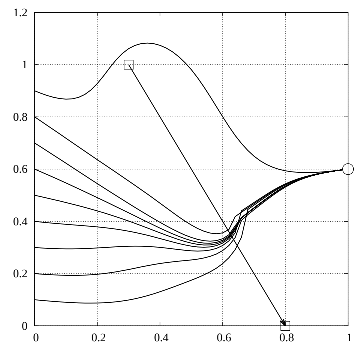


(b)Model 2

Fig. 6. Simulation for obstacle placed at the different position $(0.4, 0.4)$ and arriving at the different place $(1.0, 0.6)$.



(a)Model 1



(b)Model 2

Fig. 7. Simulation for moving obstacle avoidance with fixed speed and the different designated place $(1.0, 0.6)$.

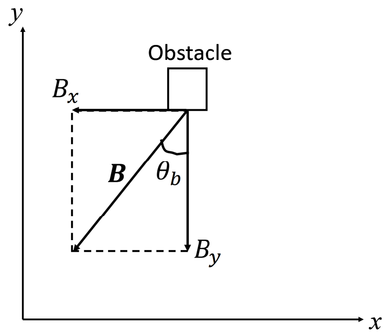
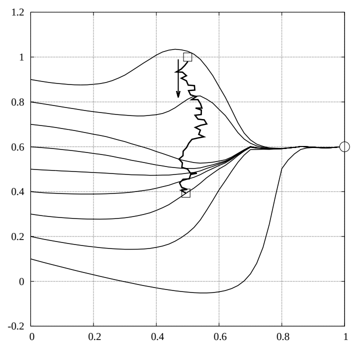
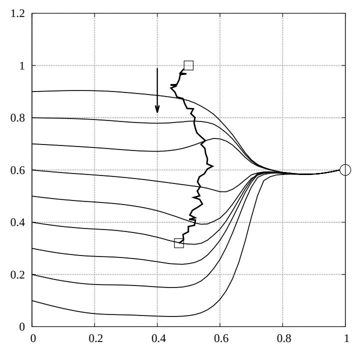


Fig. 8. The obstacle moves with the vector $\boldsymbol{B}$, where $|\boldsymbol{B}|$ is constant and θ_b is selected randomly.



(a)Model 1



(b)Model 2

Fig. 9. Simulation for moving obstacle randomly and the different designated place $(1.0, 0.6)$.

are short, middle and long for d_1 and d_2, minus, central and plus for θ_1 and θ_2 and left, center and right for the direction of A_y, respectively. Then, the principal fuzzy rules for Model 2 are constructed as shown in TableIV. They are similar to human activity to solve the problem. On the other hand, fuzzy rules for Model 1 are not so clear.

TABLE IV
THE PRINCIPAL FUZZY RULES FOR MODEL 2.

	d_1	d_2	θ_1	θ_2	A_y
Rule 1	short	long	plus	center	right
Rule 2			minus		left
Rule 3		middle		middle	right
Rule 4			plus	plus	left
Rule 5	middle				left
Rule 6			minus	minus	right

V. CONCLUSION

In this paper, a theoretical result and some numerical simulations including obstacle avoidance are presented in order to compare the proposed model with the conventional models. It is shown that Model 1 and Model 2 with Gaussian membership function and $f_i(x_1, \cdots, x_m) = w_i$ are satisfied with the conditions of Stone-Weierstrass Theorem, so both models are universal approximators of continuous functions. Further, in order to compare the capability of learning algorithms for models, numerical simulation of function approximation is performed. Lastly, some simulations on obstacle avoidance are performed. In the simulations, it is shown that both models are successful in all trials. Specifically it is shown that Model 2 with the small number of parameters and interpretability is constructed.

In future work, we will find an effective learning method for Model 2.

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