Corrective Controllers with Switching Capability for Modeling Matching of Input/State Asynchronous Machines

Jung–Min Yang

Abstract—**A control methodology for asynchronous sequential machines is addressed in this paper. The considered machine consists of a number of input/state asynchronous machines, termed submachines, between which asynchronous switching is conducted by the switching signal. The control objective is to design a corrective controller so that the stable-state behavior of the closed-loop system can mimic a reference model. In response to the external input and state feedback, the corrective controller generates a sequence of appropriate switching signals that change the mode of the asynchronous machine to show the desirable input/state specifications. An illustrative example is provided for demonstrating the proposed corrective control scheme.**

I. Introduction

Owing to the lack of a synchronizing clock, asynchronous sequential machines are regarded more difficult to design than synchronous machines. Once implemented, on the other hand, asynchronous sequential machines exhibit significant advantages over synchronous machines such as fast transition speed, low power consumption, etc. For this reason, asynchronous sequential machines are widely used in high speed computing and parallel computing mainly having the form of digital systems, in modeling molecular biology, and in many other engineering applications [1]–[3].

In this paper, we present a control theoretic approach to asynchronous sequential machines—*corrective control theory* as it is often called [4]–[6]. If the behavior of an existent asynchronous machine is undesirable or the machine is subject to negative effects of adversarial entities or disturbances, we usually re-design the inner logic of the faulty machine. In corrective control, however, the considered machine is always intact; instead, we design a corrective controller and place it in front of the controlled machine. Receiving the external input and state or output feedback from the machine, the corrective controller provides a sequence of control inputs so that the closed-loop system can show the agreeable behavior. Corrective control is unique in that all the interactions between the controller and the machine are executed in an asynchronous mechanism. Thus the operation of the closed-loop system is interpreted in terms of the stable-state behavior. This scheme allows the asynchronous machine to show the transition features that are not inherently specified in the machine.

The main consideration of this paper is to present the model matching problem of the switched asynchronous sequential machines. The switched asynchronous sequential machine consists of multiple asynchronous machines, termed submachines or modes. The present mode of the machine is changed in response to the switching signal. The switched machine is equivalent to one of its submachines at a specific time and by change of the switching signal, it transfers to another submachine. Since each submachine is supposed to have the same input and state set in this study, the mode switching between any pair of submachines is made possible.

Recently, switched systems are gaining a prominent position in control theory, as many engineering systems can operate with different models according to the environment changes or by virtue of hardware redundancy. Researches on switched systems for the past decades mainly focused on continuous/discrete time systems [7], [8]. In contrast, the topic of switched event-driven systems, which the present study may belong to, has been attracting little interest. Among few results, switched Boolean control networks (BCN), a specific type of switched event-driven systems, haven been studied by several research groups to describe the model of gene regulatory networks in cells [9], [10].

The corrective controller studied in this paper generates the switching signal as its output value. This control structure differs from the prior works since the previous corrective controllers generate a sequence of input characters as its output. Our control goal is to derive the existence condition and design procedure for a corrective controller so that the closed-loop system follows the behavior of a reference model. This problem, called *model matching*, has been extensively studied in both continuous time domain and event driven systems [11]–[13]. A novel scheme of interweaving the switching and state feedback operations is addressed in this paper, and a simple example is provided to demonstrate the applicability of the proposed scheme.

Pioneering results on corrective control for single asynchronous sequential machines are found in [14], [15] where critical races lying in the input/state asynchronous machines are invalidated by corrective controllers. These studies have been extended to input/output asynchronous machines in [16], [17]. Corrective control has been also employed to realize fault tolerance in asynchronous machines with various fault circumstances [5], [6], [18]–[26]. Other topics include the model matching problem for input/output asynchronous machines [27], elimination of infinite cycles in the machines [28], adaptive corrective control for asynchronous machines

This work was supported in part by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (No. NRF-2010-0007271), and in part by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIP) (No. NRF-2012R1A2A2A01003419).

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ISBN: 978-988-19253-9-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

with model uncertainties, and corrective control with description of semi-tensor product (STP) [30].

II. Preliminaries

A. Switched Asynchronous Sequential Machines

A switched asynchronous sequential machine with m submachines is represented as

$$\Sigma_\sigma := (A, X, x_0, f_\sigma). \tag{1}$$

$\sigma \in M$ is the switching signal generated by the controller where $M := \{1, \ldots, m\}$. Σ_i, $i \in M$, is the ith submachine modeled as an input/state asynchronous machine in which the current state is given as the output. A and X are the input and state set, respectively, $x_0 \in X$ is the initial state, and

$$f_i : X \times A \rightarrow X$$

is the state transition function partially defined on $X \times A$. Note that all m submachines have the same input and state set; only their state transition characteristics are different with each other.

Σ_i complies with the feature of asynchronous machines. A valid state and input pair $(x, v) \in X \times A$ is said to be a stable combination and x a stable state if $f_i(x, v) = x$. Since no global clock exists that synchronizes the change of all the variables, Σ_i lingers at (x, v) infinitely as long as the external input remains fixed. Else if $f_i(x, v) \neq x$, (x, v) is called a transient combination and x a transient state. Σ_i falls into a transition combination from a stable combination by a change of the input, for example, from v' to v where (x, v') is a stable combination. Due to the absence of a synchronizing clock, a transient combination causes Σ_i to pass through a chain of transient combinations, e.g.,

$$\begin{gathered} f_i(x, v) = x_1, \\ f_i(x_1, v) = x_2, \\ \vdots \end{gathered}$$

Note that the input value v remains unchanged throughout the transitions. This chain of transitions may or may not terminate. It it does not terminate, $(x, v), (x_1, v), \ldots$ are part of an infinite cycle. In this study, we restrict our attention to asynchronous machines that have no infinite cycles. Thus Σ_i reaches another stable combination (x', v) as the result of the transitions. x' is termed the next stable state of (x, v).

A significant feature of asynchronous sequential machines is that the speed of transient transitions is very fast. This means the duration that Σ_i stays at transient states is almost zero time and imperceptible by outer users. Thus it is convenient to depict the transitions of Σ_i only in terms of stable states. For this purpose, we introduce the *stable recursion function* $s_i : X \times A \rightarrow X$ [15] as

$$s_i(x, v) := x'$$

where x' is the next stable state of (x, v). If (x, v) is a stable combination, $s(x, v) = x$. The domain of s_i can be extended from input characters to strings recursively. For $x \in X$ and $v_1 v_2 \cdots v_k \in A^+$, where A^+ is the set of all non-empty strings of characters of A_n, we define

$$s_i(x, v_1 v_2 \cdots v_k) := s(s(x, v_1), v_2 \cdots v_k).$$

Definition 1. *Let Σ_i be a submachine of the switched asynchronous machine (1). A chain of transient transitions from one stable state to another is termed a stable transition in Σ_i, e.g., from x to x' in response to v characterized by $s_i(x, v) = x'$. If there exists an input string $t \in A^+$ such that $x' = s_i(x, t)$, x' is said to be stably reachable from x in Σ_i [15].*

B. Closed-Loop System

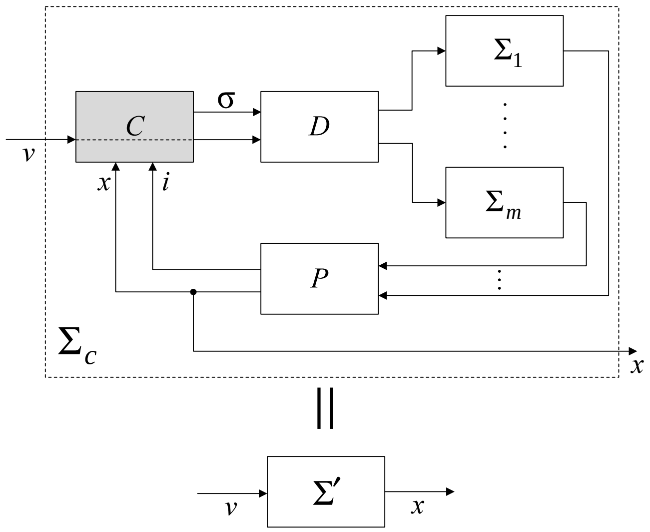

Fig. 1. Corrective control system Σ_c for model matching with Σ'.

Fig. 1 shows the proposed architecture of the corrective control system for the switched asynchronous machine (1). C is the corrective controller having the form of an input/output asynchronous machine, that is, its output is different from the current state. $v \in A$ is the external input, and $\sigma \in M$ is the switching signal generated by C. D and P represent the *demultiplexer* and *multiplexer*, respectively. D receives the external input v and provides it to the input channel of submachine Σ_σ that is determined by the switching signal σ. Among m state feedbacks from $\Sigma_1, \ldots, \Sigma_m$, P selects x, namely the state feedback of the active submachine. x is delivered to C along with $i \in M$, the index of the active submachine.

Referring to Fig. 1, C is described as the following finite-state machine.

$$C := (M \times X \times A, M, \Xi, \xi_0, \phi, \eta) \tag{2}$$

where Ξ is the state set, $\xi_0 \in \Xi$ is the initial state, and ϕ and η are the state transition function and output function, respectively, having the mapping

$$\begin{aligned} &\phi : \Xi \times M \times X \times A \rightarrow \Xi \\ &\eta : \Xi \rightarrow M. \end{aligned}$$

Note that the corrective controller C differs from the prior works [5], [15], [16] in that it generates the switching signal instead of the control input character. Although C can read the external input v, it does not alter its value in our setting as illustrated in Fig. 1. In other words, C controls the switched asynchronous machine (1) only by changing its mode in a desirable way. The closed-loop system represented by the diagram of Fig. 1 is denoted by Σ_c.

ISBN: 978-988-19253-9-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

The control objective is to present the existence condition and design algorithm for C for which the stable-state behavior of the closed-loop system Σ_c can match that of a reference model Σ' defined as

$$\Sigma' := (A, X, x_0, s').$$

Model matching between Σ_c and Σ' is regarded accomplished if their stable-state behaviors are equivalent with other other, that is, for a valid state and input pair, they reach the same next stable state. Hence only the stable recursion function s' of Σ' is given without loss of generality.

To prevent asynchronous sequential machines from doing unpredictable behaviors, one must design the machines so as to preserve the principle of the fundamental mode operation [31], namely to prohibit changes to the input while a machine is in transition. Assume on the contrary that the external input changes its value while the asynchronous machine undergoes transient transitions. Due to the lack of a synchronizing clock, it is indeterminate which transient state the machine has at the moment of the input change. Thus the next transition would be uncertain, impeding the deterministic operation. Based on the former studies on the system configuration [18], [29], we derive the following conditions for the closed-loop system Σ_c of Fig. 1 to preserve the principle of the fundamental mode operation.

Condition 1. *The closed-loop system Σ_c of Fig. 1 operates in fundamental mode when all the following conditions are valid:*

(i) *Among C and $\Sigma_1, \ldots, \Sigma_m$, whenever one machine goes on transient transitions, the other machines are at stable combinations.*
(ii) *The inputs σ and v to the demultiplexer D must change their values only one at a time.*
(iii) *The outputs i and x of the multiplexer P must change their values only one at a time.*

These conditions must be implemented during the design of the closed-loop system Σ_c. The reason for ensuring part (ii) is obvious. If the inputs σ and v change simultaneously, the next operation of D would be ambiguous, i.e., whether D must change the mode of the switched asynchronous machine to Σ_σ or it must change the current input value to v. Part (ii) can be satisfied by setting the operation sequence of C in the following way: Whenever the switching signal σ must be activated in response to the change of v, C provides σ before transmitting the changed v. This is possible because v always passes through C as shown in Fig. 1. Furthermore, to guarantee part (iii), every switching operation must be conducted in such a way that the present mode and state are not changed simultaneously.

III. Model Matching

A. Controller Existence

Let us first review the prior results (e.g., [15], [28]) on the model matching problem of a single asynchronous machine $\Sigma_1 = (A, X, x_0, f_1)$, or $m = 1$ in terms of (1). Let s_1 be the stable recursion function of Σ_1. Assume that for a valid pair $(x, v) \in X \times A$ of the model $\Sigma' = (A, X, x_0, s')$, $s'(x, v) = x'$ whereas $s_1(x, v) \neq x'$. Since the stable-state behaviors of Σ_1 and Σ' mismatch each other at (x, v), the correction procedure must be activated when Σ_1 has the pair (x, v). The necessary and sufficient condition for the existence of a corrective controller is that the goal state x' is stably reachable from x, namely there exists an input string $t \in A^+$ such that $s_1(x, t) = x'$ (refer to Definition 1). t is used by the corrective controller as the control input sequence. Upon receiving the external input v while Σ_1 staying at x, the controller provides asynchronously the sequence of input characters of t while suppressing v. As a result, Σ_1 will undergo a chain of stable transitions from x to x'. Since the transient transitions of Σ_1 and the generation of control inputs by the corrective controller are conducted in an asynchronous manner, the closed-loop system will seem to transfer from x directly to x' in response to v, thus accomplishing the matching performance.

As mentioned before, the proposed corrective controller C in Fig. 1 is endowed with only switching capability. Hence the foregoing control scheme applied to a single asynchronous machine cannot be used in our study. Instead, C must provide the switching signal that changes the present mode to a submachine having the same reachability as Σ'. To describe this in formal terms, we first define sets $A'(x), A_i(x) \subseteq A$ for $x \in X$ as

$$\begin{aligned} A'(x) &:= \{v \in A | s'(x, v) \text{ is defined}\}, \\ A_i(x) &:= \{v \in A | s_i(x, v) \text{ is defined}\}. \end{aligned}$$

$A'(x)$ and $A_i(x)$ are the set of input characters that make a valid pair with the state x in the model Σ' and submachine Σ_i, respectively. Further, we define subsets of $A'(x)$ and $A_i(x)$ as follows.

$$\begin{aligned} U'(x) &:= \{v \in A'(x) | s'(x, v) = x\}, \\ U_i(x) &:= \{v \in A_i(x) | s_i(x, v) = x\}, \\ T'(x) &:= \{v \in A'(x) | s'(x, v) \neq x\}, \\ T_i(x) &:= \{v \in A_i(x) | s_i(x, v) \neq x\}. \end{aligned} \tag{3}$$

$U'(x)$ and $U_i(x)$ denote the set of input characters that make a stable combination with x in Σ' and Σ_i, respectively. Similarly, $T'(x)$ and $T_i(x)$ denote the set of input characters that make a transient combination with x in Σ' and Σ_i. Clearly, for all $x \in X$ and all $i \in M$,

$$\begin{aligned} U'(x) \cap T'(x) &= \varnothing, \\ U_i(x) \cap T_i(x) &= \varnothing, \\ U'(x) \cup T'(x) &= A'(x), \\ U_i(x) \cup T_i(x) &= A_i(x). \end{aligned}$$

Since all the asynchronous sequential machines are supposed to have no infinite cycles in any case, we have the following.

Condition 2. *For every state $x \in X$ of Σ_i, $U_i(x) \neq \varnothing$, $i = 1, \ldots, m$. For every state $x \in X$ of Σ', $U'(x) \neq \varnothing$.*

To materialize model matching with Σ', at least one submachine must have the same transition characteristics as Σ' for each transient combination $(x, v) \in X \times T'(x)$. The latter condition is written as

$$\begin{aligned} &\forall x \in X \text{ and } \forall v \in T'(x), \\ &\exists i \in M \text{ such that } s_i(x, v) = s'(x, v). \end{aligned} \tag{4}$$

ISBN: 978-988-19253-9-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

While (4) is the necessary condition for materializing model matching between (1) and Σ', it is not sufficient. For example, suppose $v_1 \in T'(x)$ and $s_i(x, v_1) = s'(x, v_1)$ for some $i \in M$. Assume that model matching has been well implemented thus so far and that the present mode of the switched asynchronous machine (1) is Σ_k while Σ_k stays at a stable combination $(x, a) \in X \times U_k(x)$. Assume further that the external input changes to v_1. If $s_k(x, v_1) = s'(x, v_1)$, model matching is automatically satisfied, so no subsequent control action is required. On the other hand, if $s_k(x, v_1) \neq s'(x, v_1)$, the next state transition would violate the input/state specification of the model Σ' unless any control action is enforced before the stable transition. Since $s_i(x, v_1) = s'(x, v_1)$ by assumption, it seems that model matching can be maintained if the controller C generates the switching signal $\sigma = i$ at the moment the external input v_1 is transmitted to C.

However, an attention has to be paid to the switching operation between two submachines, since the property that an asynchronous machine cannot stay at a transient state imposes a restraint on the switching operation. Suppose that in response to the switching signal $\sigma = i$, the switched asynchronous machine changes its mode to Σ_i. Since the demultiplexer D in Fig. 1 preserves the principle of the fundamental mode operation, its inputs σ and v cannot change simultaneously. Hence, when σ switches to i, v does not change its value yet, which means that the value of the input channel provided by D is still a at the moment the present mode becomes submachine Σ_i. If $a \in U_i(x)$, the switched machine maintains the present state at x after completion of switching to Σ_i. If $a \notin U_i(x)$, on the other hand, Σ_i cannot stay at x with the external input a but initiate the chain of transient transitions from (x, a). Even though the external input v_1 may be transmitted to Σ_i right after the switching operation, the intended stable transition $s_i(x, v_1) = s'(x, v_1)$ could not be manifested since Σ_i is not at the state x any more.

We ensure that to elude this problem, a must belong to $U_i(x)$, namely a must make a stable combination with x in submachine Σ_i. In fact, a can be any input character that makes a stable combination with x in the model Σ', that is, $a \in U'(x)$, because model matching is supposed to have been well accomplished until now. Thus we must guarantee that not only a but all the characters in $U'(x)$ must be elements of $U_i(x)$ for certifying a consistent switching operation at the state x with respect to the input character v_1. The foregoing condition is described as

$$U'(x) \subseteq U_i(x). \tag{5}$$

The above condition is still insufficient to solve model matching at x, since it only elucidates the requirement for dealing with the input character v_1. When the switched asynchronous machine stays at the stable state x, the next input character can be any element of $T'(x)$. For such a character, condition (5) must be always valid. Considering (4) and (5) together, we have the following conclusion.

- For every $v_i \in T'(x)$, there must exist a submachine Σ_i for which (4) is valid, i.e., $s_i(x, v_i) = s'(x, v_i)$.
- For every submachine Σ_i that is selected for the stable transition from (x, v_i), (5) must be satisfied.

More than one submachine may emulate a stable transition of the model Σ'. In that case, (5) serves as the criterion for choosing a pertinent submachine. For instance, assume that $s_i(x, v_1) = s_j(x, v_1) = s'(x, v_1)$ for some $i, j \in M$. Assume further that $U'(x) \subseteq U_i(x)$ but $U'(x) \nsubseteq U_j(x)$. Definitely, submachine Σ_i must be selected in the correction procedure for (x, v_1).

Our discussion thus so far is summarized in the following theorem.

Theorem 1. *Given the switched asynchronous sequential machine (1), let $\Sigma' = (A, X, x_0, s')$ be the reference model and let $T'(x)$, $U'(x)$, and $U_i(x)$, $i = 1, \ldots, m$, be the subsets of A as defined in (3). In view of Fig. 1, there exists a corrective controller C that solves model matching between Σ_c and Σ' if and only if for each $x \in X$,*

$$\begin{aligned} &\forall v \in T'(x),\ \exists i \in M \text{ s.t.} \\ &s'(x, v) = s_i(x, v) \text{ and } U'(x) \subseteq U_i(x). \end{aligned} \tag{6}$$

B. Controller Design

Reminding that the form of the corrective controller is as described in (2), we design $C(z) = (M \times X \times A, M, \Xi, \xi_0, \phi, \eta)$, namely a corrective controller module that solves the model matching problem for the state $z \in X$. Once $C(z)$ is designed for every valid pair (z, v) of Σ', the overall controller C is obtained by assembling all $C(z)$'s:

$$C := \vee_{z \in X} C(z)$$

where '$\vee$' is the *join* operation that combines two corrective controller modules [18], [28].

Suppose that the switched asynchronous machine (1) satisfies all the conditions (6) described in Theorem 1 are valid with respect to the model Σ'. Assume that model matching between Σ_c and Σ' has been well accomplished, and that $C(z)$ stays at the initial state ξ_0. Assume further that Σ_c and Σ' reach a stable combination with the state z. $C(z)$ then transfers to $\xi_z \in \Xi$, namely the transition state that deals with all the transitions starting from z. To this end, we set the recursion function ϕ as

$$\begin{aligned} \phi(\xi_0, i, z, v) &= \xi_z,\ v \in U_i(z), \\ \phi(\xi_z, i, z, v) &= \xi_z,\ v \in U_i(z) \end{aligned} \tag{7}$$

where $i \in M$ denotes the present mode of the switched asynchronous machine (1). Since no actual control is executed in either ξ_0 or ξ_z, $C(z)$ maintains the present mode of the switched machine, so the output function η is defined as

$$\begin{aligned} \eta(\xi_0) &= i, \\ \eta(\xi_z) &= i. \end{aligned} \tag{8}$$

Assume now that the external input changes to another value $v' \in T'(z)$ when $C(z)$ is at ξ_z. Two cases arise with respect to the changed input character.

(i) First, if $s'(z, v') = s_i(z, v')$ where s_i is the stable recursion function of the present submachine Σ_i, no control activity is necessary because Σ_i has the same transition characteristic as Σ' at (z, v'). Hence, $C(z)$ maintains the present switching signal $\sigma = i$. When the state feedback changes to another value, say $z' := s_i(z, v')$, $C(z)$ returns

ISBN: 978-988-19253-9-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

to the initial state ξ_0. The latter behavior is implemented by the following.

$$\begin{aligned} \phi(\xi_z, i, z, v') &= \xi_z, \\ \phi(\xi_z, i, z', v') &= \xi_0. \end{aligned} \tag{9}$$

(ii) Secondly, assume $s'(z, v') \neq s_i(z, v')$. Model mismatch would occur unless corrective control is enforced. By assumption, a submachine Σ_j exists that satisfies condition (6) for the state/input pair (z, v'), that is, $s'(z, v') = s_j(z, v')$ and $U'(z) \subseteq U_j(z)$. Hence, upon receiving the changed input character v', $C(z)$ transfers to the auxiliary state $\xi_{(z,v')} \in \Xi$ and provides the switching signal $\sigma = j$ as the control input. In response to $\sigma = j$, the switched asynchronous machine (1) changes its mode to Σ_j and receives the input v', which moves Σ_j to the desirable state $z' := s'(z, v') = s_j(z, v')$. The foregoing correction procedure is implemented by the following assignment of ϕ and η.

$$\begin{aligned} \phi(\xi_z, i, z, v') &= \xi_{(z,v')}, \\ \phi(\xi_{(z,v')}, i, z, v') &= \xi_{(z,v')}, \\ \eta(\xi_{(z,v')}) &= j, \\ \phi(\xi_{(z,v')}, j, z', v') &= \xi_0. \end{aligned} \tag{10}$$

(7)–(10) serve as the design algorithm of the corrective controller $C(z)$ that solves the model matching problem for valid pairs $\{z\} \times A'(z)$ of Σ'.

- At first glimpse, (9) and (10) seem contradictory with each other. However, $C(z)$ will utilize only of them, depending on the feature of submachine Σ_i. As mentioned already, if $s'(z, v') = s_i(z, v')$, $C(z)$ employs the procedure (9); else if $s'(z, v') \neq s_i(z, v')$, it must launch the switching operation according to (10).
- $C(z)$ requires the states $\xi_0, \xi_z, \xi_{(z,v')}$ to realize the above procedure. Generalizing this procedure to all the input characters of $T'(z)$ that invoke transient transitions with z, we have the state set of $C(z)$:

$$\Xi = \{\xi_0, \xi_z\} \cup \{\xi_{(z,v')} | v' \in T'(z)\}.$$

IV. Example

Consider a switched asynchronous machine with two submachines Σ_1 and Σ_2 shown in Fig. 2, i.e., $m = 2$ and $M = \{1, 2\}$. The input and state set of the machine are

$$\begin{aligned} X &= \{x_1, x_2, x_3, x_4\}, \\ x_0 &:= x_1, \\ A &= \{a, b, c, d\}. \end{aligned}$$

For the sake of simplicity, we set

$$f_i(x, v) = s_i(x, v), \ \forall (x, u) \in X \times A, \ i = 1, 2.$$

The objective is to determine whether a corrective controller C exists such that the closed-loop system Σ_c can match the behavior of the reference model $\Sigma' = (A, X, x_0, s')$ whose state flow diagram is illustrated in Fig. 3. Applying Theorem 1, let us check whether condition (6) is satisfied for each state of X. $U'(x)$'s and $T'(x)$'s are derived from Fig. 3:

$$\begin{aligned} U'(x_1) &= \{a, d\}, \quad T'(x_1) = \{b, c\}, \\ U'(x_2) &= \{b, d\}, \quad T'(x_2) = \{a\}, \\ U'(x_3) &= \{a, c\}, \quad T'(x_3) = \{b, d\}, \\ U'(x_4) &= \{a, b\}, \quad T'(x_4) = \{d\}. \end{aligned}$$

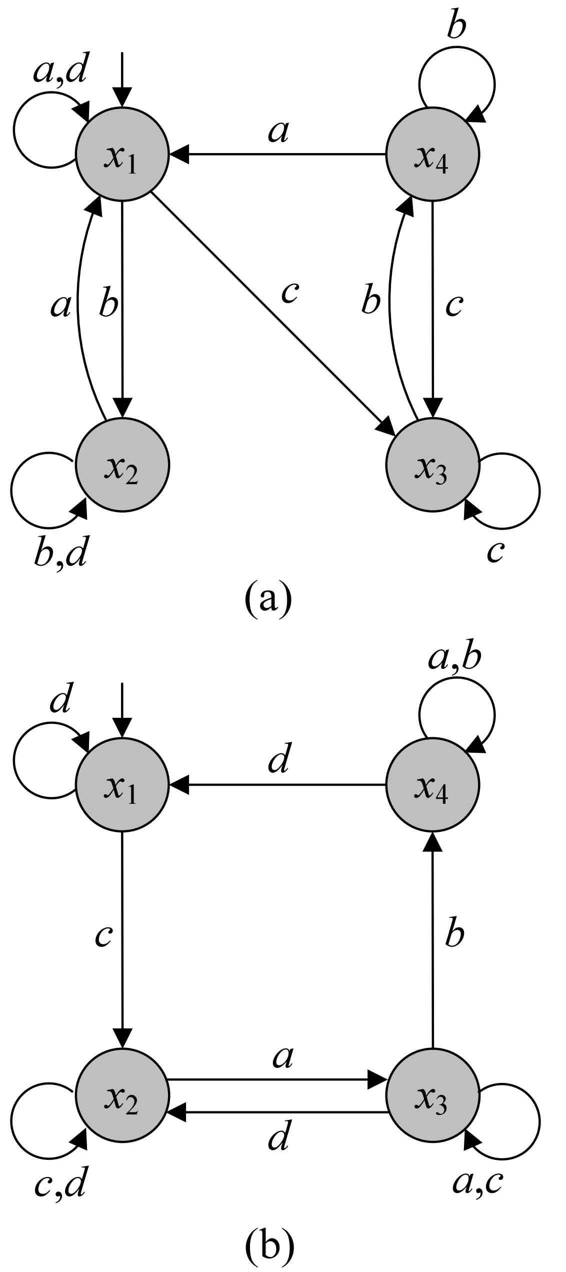

Fig. 2. Switched asynchronous machine with $m = 2$: (a) Σ_1 and (b) Σ_2.

Consider x_1 in the first. A comparison between Fig. 2 and Fig. 3 shows that x_1 of submachine Σ_1 has the same characteristics of transient transitions as x_1 of the model Σ_1. Formally, we have

$$\begin{aligned} s'(x_1, b) &= s_1(x_1, b) = x_2, \\ s'(x_1, c) &= s_1(x_1, c) = x_3. \end{aligned}$$

Also, we have

$$U_1(x_1) = \{a, d\} = U'(x_1).$$

Hence x_1 of Σ_1 satisfies condition (6) for all the input characters of $T'(x_1)$. In a similar manner to the case of x_1, we can prove that x_2 of Σ_1, x_3 and x_4 of Σ_2 emulate the corresponding states of the model Σ'. By Theorem 1, the corrective controller C exists that makes the closed-loop system Σ_c stably equivalent to the model Σ'. The construction of C can be conducted according to the algorithm (7)–(10).

ISBN: 978-988-19253-9-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

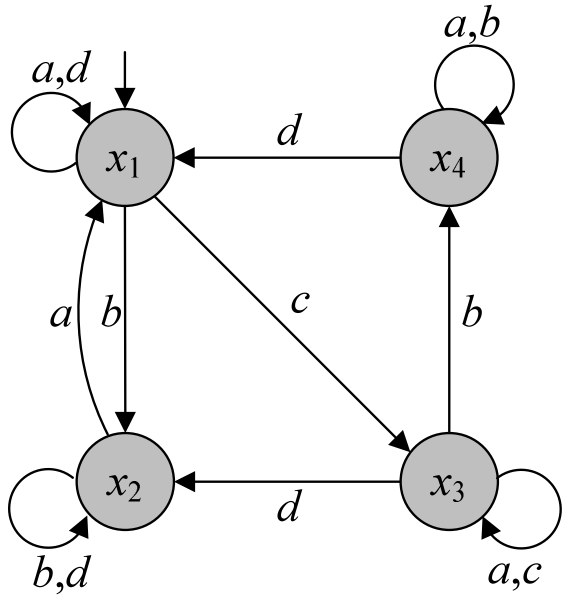

Fig. 3. Reference model Σ'.

V. Conclusion

A corrective control scheme has been presented to solve the model matching problem for switched asynchronous sequential machines. The proposed controller generates the switching signal that changes the mode of the considered switched machine. According to the incoming input value and the present state, the controller determines the next submachine so that the input/state behavior of the switched asynchronous machine can match that of a reference model. The necessary and sufficient condition for the existence of an appropriate controller has been addressed and the design algorithm for the controller has been outlined. Although the present study focuses on the corrective controller having only switching capability, it can be extended to the controllers that can provide not only the switching signal but also the control input character. The latter topic will be studied as a future research.

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ISBN: 978-988-19253-9-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)