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# Performance Evaluation of Photon Diffusion Models Configured in Planar Source Illumination Geometry for Quantifying the Optical Properties of Turbid Media

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***Abstract*—In this study, we employed the planar source illumination (PSI) geometry to quantify optical properties of turbid media. Delta-P1 and P1 photon diffusion models were utilized to calculate the reflectance in the PSI geometry. The performance of the P1 and Delta-P1 diffusion models was evaluated using Monte Carlo simulation and liquid phantom measurements. Our results revealed that Delta-P1 diffusion model is more accurate than P1 diffusion model in recovering the absorption and scattering properties of turbid media.**

***Index Terms*—Diffusion theory, Frequency Domain Photon Migration, Monte Carlo simulation, Tissue optics.**

## I. INTRODUCTION

Diffuse Reflectance Spectroscopy (DRS) can be used to determine the optical properties of biological tissues efficiently and noninvasively. Tissue optical properties, absorption ($\mu_a$) and the reduced scattering coefficients ($\mu_s'$), are markers of tissue function and structure, such as distribution of water and utilization of oxygen [1]. Most DRS systems are based on the standard diffusion equation (SDE) derived from the radiative transfer equation as the forward model. Typically, DRS systems work in conjunction with the SDE are applied to study high scattering deep tissues. For example, Tromberg *et al.* showed that their DRS system with source-detector separation larger than 20 mm can be employed to derive the optical properties of samples at depths greater than 10 mm [2]. However, more than 80% of cancers originate from epithelial tissues [3]. Thus, properly modifying the SDE or measurement geometry for detecting shallower depth is necessary. In this study, we demonstrate the use of a uniform planar light source to replace a point or pencil source, as shown in Fig.1, to increase the efficiency in measurement and theoretical modelling. Pham *et al.* had demonstrated the use of a frequency domain photon migration DRS system and the PSI probing geometry for determining the optical properties of turbid samples [4]. However, our previous work indicated that the PSI geometry working with a SDE (in P1 approximation) could not reliably recover the properties of the superficial samples [5]. In this study, we employed the delta-Eddington phase function in the SDE derivation and obtained the Delta-P1 diffusion model in the PSI geometry. To compare the differences between P1 and Delta-P1 models, we used them as the forward models to fit Monte Carlo simulation generated reflectance. The performance comparison between P1 and Delta-P1 models will be discussed in this paper. In addition, we measured the liquid phantoms in the PSI geometry and recovered their optical properties using P1 and Delta-P1 diffusion models. The experimental performance of the two models will also be investigated.

## II. MATERIALS AND METHODS

### A. Frequency domain photon migration system

A frequency domain photon migration (FDPM) instrument is used to measure the amplitude and phase of photon density waves from 10MHz to 200MHz. The FDPM instrument setup is shown in Fig.1. The FDPM instrument is composed of a laser diode at 658nm, a laser diode mount (TCLDM9, THORLABS), a laser diode controller (LDC-3908, ILX Lightwave), a network analyzer (N5230C, Agilent Technologies), and a temperature-stabilized avalanche photodiode (APD) (C5658, Hamamatsu). The network analyzer generates sinusoidal RF power at 201 frequencies over the range of 10 -200 MHz. The RF current is mixed with the DC current from a laser diode controller by a laser diode mount. A 658-nm laser diode is derived by the mixed RF and DC currents. The sinusoidal intensity-modulated light is collimated to form a uniform 3-cm-diameter illumination spot. The 1-mm detecting fiber is normal to the surface of liquid phantoms ($\mu_{a1}$=0.0186mm$^{-1}$, $\mu_{s1'}$=1.2797mm$^{-1}$, $\mu_{a2}$=0.0392mm$^{-1}$, $\mu_{s2'}$=1.2797mm$^{-1}$ @ 658nm) as shown in Fig.1. The detected light is converted to a RF signal by the APD and the RF signal is sent to the network analyzer to determine the phase delay and amplitude demodulation introduced by the sample under investigation.

### B. Diffusion Equation

Light propagation in scattering media can be described by the radiative transfer equation (RTE). Starting from the time-dependent RTE:

$$\frac{\partial L(\vec{r},\hat{s},t)}{c\cdot\partial t} = -\hat{s}\nabla L(\vec{r},\hat{s},t) - \mu_t L(\vec{r},\hat{s},t) + \mu_s \int_{4\pi} L(\vec{r},\hat{s},t)\cdot p(\hat{s}',\hat{s})\, d\Omega' + \mathrm{S}, \tag{1}$$

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where

$$L(\vec{r},\hat{s},t)=\frac{1}{4\pi}\left[\phi(\vec{r},t)+3j(\vec{r},t)\cdot\hat{s}\right], \tag{2}$$

is the radiance, $c$ is the light speed in the medium, and S is the source term. Under specific approximations and limitations, the radiative transfer equation is simplified to the time-domain standard diffusion equation (SDE):

$$\left[\nabla^2-3\mu_a\mu_{tr}-\frac{3\mu_{tr}}{c}\left(1+\frac{\mu_a}{\mu_{tr}}\right)\frac{\partial}{\partial t}-\frac{3}{c^2}\frac{\partial^2}{\partial t^2}\right]\phi=q, \tag{3}$$

where $\mu_{tr}=\mu_a+\mu_s'$ is the transport constant, $\phi$ and q are the fluence rate and source term, respectively.

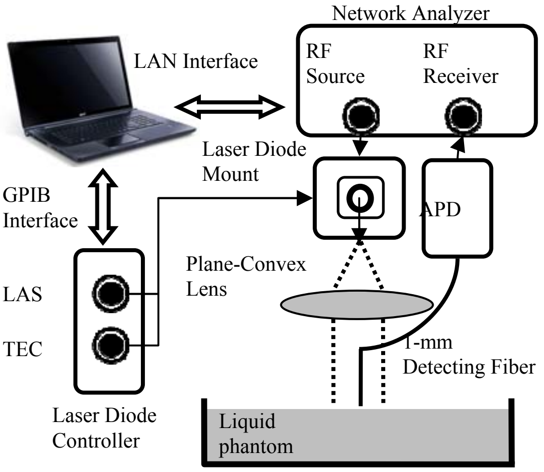

Fig.1. FDPM system with the planar source illumination (PSI) geometry

A photon diffusion model that employs a uniform planar source to irradiate onto a semi-infinite, homogeneous medium was previously disclosed by Pham *et al.* [4]. We refer Pham's diffusion model here as the P1 diffusion model. For a planar source, Eq. 3 can be reduced to a one-dimensional equation with z-direction dependence only. In our previous work, this PSI diffusion model demonstrated a recovery error approaching 100% in some cases [5]. To increase the model accuracy, the anisotropic scattering is considered here by introducing the delta-Eiddington phase function in derivation of the diffusion equation for the planar source illumination geometry. This model is refer to as the Delta-P1 diffusion model. Eqs. 4 and 5, respectively, show the source terms employed in the P1 and Delta-P1 models:

$$q_1=-3\mu_s'P_0(\mu_{tr}+\frac{i\omega}{c})\exp\left[-\left(\mu_{tr}+i\frac{\omega}{c}\right)z\right], \tag{4}$$

$$\begin{aligned}q_2&=-3\mu_s^{*}P_0\left[(\mu_{tr}+\mu_{tr}^{*}g^{*})+\left(1+g^{*}\right)\frac{i\omega}{c}\right]\\&\times\exp\left[-\left(\mu_{tr}^{*}+\frac{i\omega}{c}\right)z\right],\end{aligned} \tag{5}$$

where $g^{*}=\frac{g}{1+g}$, $P_0$ is the power of the incident light source. It should be noted that when g approaches zero, the Delta-P1 model converges to the P1 model. The source terms are assumed to attenuate exponentially along the z-direction. The planar source has infinite lateral dimensions and the diffuse reflectance is collected at the surface of sample. For the diffusion equation, the fluence rate can be determined by applying the following boundary condition:

$$j\big|_{z=0^+}=\frac{1-R_{eff}}{2\left(1+R_{eff}\right)}\phi\big|_{z=0^+}, \tag{6}$$

where $R_{eff}$ is the effective Fresnel reflection coefficient. The diffuse reflectance R can be obtained using following equation:

$$R=\frac{1}{4\pi}\int_{\Omega}\left[1-R_{fres}\left(\theta\right)\right]L\cos\theta\,d\Omega, \tag{7}$$

### C. Monte Carlo Simulation

The Monte Carlo simulation is employed here as a benchmark method to evaluate the performance of diffusion models, either derived from P1 or Delta-P1 approximation. Our MC computer code was adopted from the one developed by Wang *et al.* [6]. In the MC model, the detector is located at the origin of the system and its diameter and refractive index are set to 100 μm and 1.4, respectively. Our previous results indicated that the 2-cm-diameter circular source is sufficient to meet the infinitely wide and planar source assumption [5]. Thus, a 2-cm-diameter circular source and a sample radius of $10^8$ cm are employed in our Monte Carlo model to simulate a planar source illuminated on a semi-infinite sample. Our Monte Carlo code used the Henyey-Greenstein phase function with an antisotropy factor of g=0.8. In the MC simulations, the frequency was limited in the range from 10 to 200 MHz corresponding to the frequency range of experiments.

## III. RESULTS AND DISCUSION

To understand the theoretical performance of Delta-P1 and P1 diffusion models, Monte Carlo simulation was utilized to determine the frequency domain reflectance. To observe the effect of the samples optical properties on the reflectance, we designed seven samples of different optical properties as shown in Table I. We used the diffusion model as a forward model to fit the MC model generated reflectance by performing the MATLAB “lsqcurvefit” least-square curve fitting.

In the Monte Carlo simulations, we included seven samples from low to high absorption and from high to low scattering as listed in Table I. Sample three that had optical properties similar to those of *in-vivo* human skin was also included in the experiment. The recovered optical properties and errors are shown in Table I. In general, most of P1 diffusion model recovered values have recovery errors larger than 17% except for samples with low absorption and high scattering coefficients. In contrast, the errors from Delta-P1 models are within 16% except for the sample three. It can be seen that the recovery results reveal a noticeable trend that Delta-P1 diffusion model can accurately recover the sample optical properties with percent errors smaller than those obtained from P1 diffusion model.

TABLE I: OPTICAL PROPERTIES OF SAMPLES RECOVERED USING P1 AND DELTA-P1 DIFFUSION MODELS WITH PSI GEOMETRY

| **True/ recovered values(mm$^{-1}$)** | **P1** | | **Delta-P1** | |
|---|---|---|---|---|
| | ***(mm$^{-1}$)*** | ***Error (%)*** | ***(mm$^{-1}$)*** | ***Error (%)*** |
| $\mu_a$=0.01<br>$\mu_{s'}$=10 | $\mu_a$= 0.010<br>$\mu_{s'}$= 8.875 | 6.2<br>-11.2 | $\mu_a$= 0.010<br>$\mu_{s'}$= 9.212 | 5.3<br>-7.8 |
| $\mu_a$=0.003<br>$\mu_{s'}$=10 | $\mu_a$= 0.003<br>$\mu_{s'}$= 9.087 | 15.8<br>-9.1 | $\mu_a$= 0.003<br>$\mu_{s'}$= 9.298 | 15.0<br>-7.0 |
| $\mu_a$=0.03<br>$\mu_{s'}$=1 | $\mu_a$= 0.035<br>$\mu_{s'}$= 0.541 | 17.1<br>-45.8 | $\mu_a$= 0.032<br>$\mu_{s'}$= 0.765 | 9.6<br>-23.4 |
| $\mu_a$=0.01<br>$\mu_{s'}$=1 | $\mu_a$= 0.011<br>$\mu_{s'}$= 0.718 | 15.6<br>-28.1 | $\mu_a$= 0.010<br>$\mu_{s'}$= 0.843 | 11.6<br>-15.6 |
| $\mu_a$=0.1<br>$\mu_{s'}$=0.1 | $\mu_a$= 0.085<br>$\mu_{s'}$= 0.011 | -14.9<br>-88.2 | $\mu_a$= 0.090<br>$\mu_{s'}$= 0.085 | -9.9<br>-14.6 |
| $\mu_a$=1<br>$\mu_{s'}$=1 | $\mu_a$= 0.730<br>$\mu_{s'}$= 0.685 | -26.9<br>-31.4 | $\mu_a$= 0.904<br>$\mu_{s'}$= 0.904 | -9.5<br>-9.5 |
| $\mu_a$=10<br>$\mu_{s'}$=10 | $\mu_a$= 7.116<br>$\mu_{s'}$= 7.116 | -28.8<br>-28.8 | $\mu_a$= 8.895<br>$\mu_{s'}$= 8.895 | -11.0<br>-11.0 |

Furthermore, we carried out phantom measurements to study the experimental performance of the two diffusion model in the PSI geometry. The PSI probe was placed on the surface of the phantom. A homogeneous liquid phantom was fabricated by mixing 20% Lipofundin (B. Braun), Nigrosin (MP Biomedicals, Inc.) and de-ionized water. We obtained instrument response by measuring a liquid phantom with known optical properties and subsequently used it to calibrate the raw data from our liquid phantom. The frequency domain diffusion reflectance in terms of amplitude demodulation and phase delay are obtained. Table II shows the recovery values of the two liquid phantoms that have optical properties similar to the samples three and four. It is seen that the errors of the P1 model are in general higher than the Delta-P1 model. This experiment trend agrees with that of Monte Carlo simulation.

TABLE II: OPTICAL PROPERTIES OF LIQUID PHANTOMS RECOVERED USING DELTA-P1 AND P1 DIFFUSION MODELS

| **True/ recovered values(mm$^{-1}$)** | **P1** | | **Delta-P1** | |
|---|---|---|---|---|
| | ***(mm$^{-1}$)*** | ***Error (%)*** | ***(mm$^{-1}$)*** | ***Error (%)*** |
| $\mu_a$=0.018<br>$\mu_{s'}$=1.27 | $\mu_a$= 0.015<br>$\mu_{s'}$= 1.313 | -15.3<br>2.6 | $\mu_a$= 0.016<br>$\mu_{s'}$= 1.276 | -12.9<br>-0.2 |
| $\mu_a$=0.039<br>$\mu_{s'}$=1.27 | $\mu_a$= 0.047<br>$\mu_{s'}$= 1.364 | 22.2<br>6.60 | $\mu_a$= 0.046<br>$\mu_{s'}$= 1.381 | 18.8<br>7.9 |

The measured results of the amplitude demodulation and phase decay are shown in Fig.2. In Fig. 2, the discrete points are the measured values, the solid lines are the Delta-P1 diffusion model fit, and the dashed lines are the P1 diffusion model fit. Overall, from the data shown in Fig. 2, it can be concluded that Delta-P1 diffusion model is more accurate in recovering the absorption and scattering properties of tissues than P1 diffusion model. The Delta-P1 diffusion is advantageous for quantifying optical properties of tissue in the planar source illumination geometry.

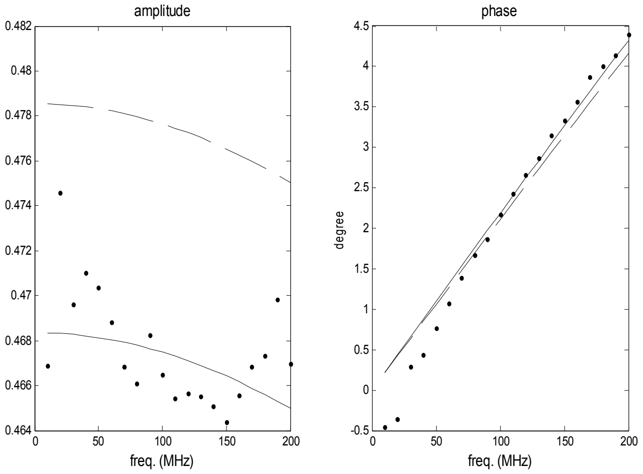

Fig.2. FDPM system with the planar source illumination (PSI) geometry

## IV. CONCLUSION

In our previous study, we have shown that the PSI geometry with P1 diffusion model is not able to accurately quantify the optical properties of surficial tissue [5]. In this paper, we derived the Delta-P1 diffusion model and introduced the MC stimulation results as reference to characterize the performances of the P1 and Delta-P1 diffusion models in PSI geometry. It is found that the theoretical errors of the Delta-P1 model are smaller than those of P1 model. In addition, we carried out phantom measurements to support our simulation results. Finally, our results indicated that the Delta-P1 model is able to more accurately determine the absorption and scattering coefficients of turbid samples. Our future research will include measuring phantoms which covers absorption and reduced scattering coefficients to a greater extent to understand the performance of the Delta-P1 diffusion model in detail. In addition we will fabricate a PSI geometry based clinical probe to work in conjunction with the Delta-P1 model to investigate the properties of various biological tissues.

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